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KINEMATICS OF INERTIAL NAVIGATION SYSTEMS

by M. I. Zakharin and F. M. Zakharin

"Mashinostroyeniye" Press
Moscow, 1968

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By M. I. Zakharin and F. M. Zakharin

Translation of "Kinematika Inertsial'nykh Sistem Navigatsii." "Mashinostroyeniye" Press, Moscow, 1968

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the National Technical Information Service, Springfield, Virginia 22151 — Price $3.00

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KINEMATICS OF INERTIAL NAVIGATION SYSTEMS

M. I. Zakharin and F. M. Zakharin

ABSTRACT. This book is devoted to methods of obtaining the required navigational information on board objects of motion by means of inertial navigation systems (INS). These systems are classified and their kinematic equations are derived in this work. Practical recommendations are given in conjunction with the nonsphericity of the Earth's surface, eccentricity and anomalies in the gravitational field of the Earth.

INS design principles are examined with a "free" gyroplatform and without a gyroplatform, in which the instantaneous vertical is modeled mathematically.

Simplified INS equations are derived, depending on the ranges of the change in kinematic parameters of specific objects in motion, and equations are formulated for systematic and instrumental errors in the systems examined. A method is shown for the approximation of errors with the aid of a calculating machine, which permitted the investigation of the influence of errors in individual elements as well as any combination of these on system accuracy as a whole. A comparative evaluation of these systems is given.

The book is intended for scientific workers, engineer-designers and will be useful to engineering college students.

There are five tables, seventy-eight illustrations, and thirty-seven bibliographical entries included.

FOREWORD

In the problem of automatic flight control, an important place is occupied by the determination of the instantaneous values of the kinematic parameters of the motion of an aircraft with respect to a chosen system of calculation. /3*

In order to solve this problem navigational systems may be employed in which operation is based on various physical principles. In this book methods are examined for determining the instantaneous values of the parameters of aircraft by means of inertial navigation systems, based on the independent measurements of absolute acceleration of movement with subsequent consideration for the gravitational field in which the flight is being accomplished.

The authors limited their investigation to problems of inertial navigation in the vicinity of the Earth.

*Numbers in the margin indiate pagination in the foreign text.

Inertial navigation systems (INS) are quite promising at the present time. The basic advantage of these systems consists of the fact that they permit the completely independent (with no type of artifically connected Earth link) determination of instantaneous values required for the automatic control of kinematic motion parameters of various objects with respect to the surface of the Earth: components of acceleration in velocity vectors of the motion of the center of mass of an object with respect to the surface of the Earth, its coordinates, and also the deviation angles of an object relative to and in the plane of the horizon. It should be noted that not one of all the other known navigation systems is as capable as the inertial navigation system of determining the instantaneous values of such a complex of kinematic motion parameters.

We must also note that of all existing navigation systems, only the inertial may accomplish navigation of objects over minimum distances on the surface of the Earth without additional trajectory calculations, i.e., on geodetic lines in any region of the Earth.

In the majority of published works navigation systems with gyrostabilized platforms, which carry accelerometers and continuously follow the plane of the horizon, are examined. In such systems Schuler's pendulum is modeled /4 physically. The accuracy of the simulation is limited by the accuracy of the elements of the physical model.

Investigations directed toward the clarification of the possibilities of mathematically modeling Schuler's pendulum and the fundamental possibilities of three-component INS's present a great deal of theoretical and practical interest. The basic results of work in this field are set forth in this book.

In Chapter I we examine the important question for inertial navigation of the shape of the Earth and the acceleration of gravity.

In the process of these investigations the complete kinematic equations for a three-component INS of the I class were derived and analyzed. The basic results of theoretical investigations of the kinematics of INS's of this class are shown in Chapter II.

Chapter III includes several results obtained by the authors in the theoretical investigations of INS kinematics of the II class, and Chapter IV includes results of investigations of INS kinematics of the III class.

The results of calculations of systematic and instrumental INS errors are shown in the last paragraphs of each Chapter. The calculations are accomplished in accordance with a proposed method as applied to one hypothetical short-range object.

The problems of the technical realization of INS's of various classes are not examined in this book since this problem has been discussed in appropriate literature and in descriptions of specific INS installations.

The authors express their deep gratitude to their collaborators of the computer center of the T.G. Shevchenko Kiev State University Yu. N. Zdorenko and T.M. Shan'ko, who took upon themselves the labor of checking the mathematical material of the manuscript.

All criticisms and comments with respect to the book may be directed to the address: Moscow, K-51, Petrovka, 24, "Mashino stroyeniye" Press.

INTRODUCTION /5

INS operation is based on the measurement of the vector components of absolute acceleration in the motion of a point which interests us (for example, in the center of a mass) by means of accelerometers and with the subsequent calculation of the vector components of acceleration and velocity, and also of the coordinates of the movement of a point with respect to the surface of the Earth.

In the process of motion the INS must continuously calculate the vector components of acceleration and velocity with respect to the axis of a corresponding trihedron.

It is apparent that in order to calculate the vector components of acceleration and velocity of an object with respect to the Earth, it is necessary to choose a system of coordinates linked to the surface of the Earth; the indicated parameters will be determined along the coordinate lines of the system. Geographic and arbitrary coordinate systems [5], [6] are employed in navigational practice for various objects. Therefore it is advantageous to analyze INS operation in these coordinate systems. Coordinate trihedrons are introduced in this examination, the axes of which are oriented along corresponding coordinate lines. In the geographical system of coordinates the trihedron will be termed geographic, and in the arbitrary system, arbitrary. Initial data for system operation consists of absolute acceleration components along the axes of the corresponding coordinate trihedron, for the measurement of which the system accelerometers are intended.

The measurement axes of the accelerometers may be stabilized in various coordinate systems. INS's, the sensitive accelerometer axes of which are stabilized along the axes of the coordinate trihedrons, linked with the surface the Earth, with inertial space and with an object, will be referred to respectively as I, II and III classes.

In other words, class I pertains to an INS with a platform stabilized in the Earth coordinate system, class II with a platform immovable in inertial space, and class III to a system without a gyrostabilized platform.

The fundamental INS possibilities are explained during an analysis of the /6 kinematics of the three-component variations of these systems. Kinematics is construed to be that part of INS theory in which we study the equations which are subject to being solved in on-board computer installations of the systems. From the point of view of the passage of useful signal, INS elements

are considered to be without inertia, and the sensitive system accelerometer masses are considered to be distributed in one point, the motion parameters of which must be determined.

Such a statement of the problem permits us to reduce the investigation of INS kinematics to an analysis of the kinematics of trihedron motion, made up of the measurement accelerometer axes, with respect to the surface of the Earth.

CHAPTER I

SOME INFORMATION CONCERNING THE FIGURE OF THE EARTH AND THE ACCELERATION OF GRAVITY

§1.1. The Figure of the Earth

It is quite important for inertial navigation to know the shape of the physical surface of the Earth at mean sea level (the figure of the Earth), since it determines the Earth's gravitational field and the surface with respect to which the navigation of various objects is accomplished. /7

The figure of a liquid body with mass distribution and rotation corresponding to that of Earth is called a geoid [32]. The surface of the geoid is determined by the mean sea level and is an equipotential surface of the gravitational field. The surface of the geoid is rather complex, therefore in practice in many cases the figure of the Earth is approximated by simpler surfaces: by a triaxial ellipsoid, a spheroid or sphere - depending on navigation accuracy requirements.

Krasovskiy's ellipsoid (Figure 1.1) is a sufficiently accurate approximation of the figure of the Earth. The semiaxes are [32]:

$$a = 6{,}378.245 \text{ km},$$

$$b = 6{,}356.863 \text{ km}.$$

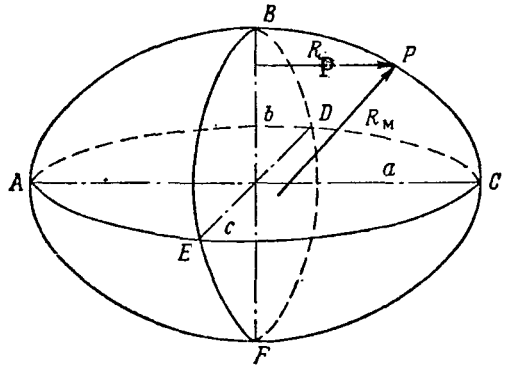

Figure 1.1. Krasovskiy's Ellipsoid

The polar flattening of the ellipsoid is /8

$$\alpha = \frac{1}{298.3},$$

The largest meridian ellipse is turned with respect to the Greenwich meridian by 15° east longitude.

The perimeter of the largest meridian ellipse ABCF is equal to 40,008.488 km, the perimeter of the smallest meridian ellipse DBEF is 40,008.535 km, and the perimeter of the equatorial ellipse is 40,075.945 km.

A less accurate approximation to the figure of the Earth is a spheroid with semiaxes

$$a = 6{,}378.245 \text{ km};$$
$$b = 6{,}356.863 \text{ km}$$

and the flattening

$$\alpha = \frac{1}{298.3}.$$

Finally, a sphere with the radius

$$R = a\left(1 - \frac{\alpha}{3}\right).$$

may serve as a first approximation to the figure of the Earth. Having established the numerical values a and α, we obtain

$$R = 6{,}371.116 \text{ km}.$$

The length of a great circle of the Earth's sphere is

$$\alpha_{gc} = 2\pi R = 40{,}030.913 \text{ km}.$$

It is interesting to note that the difference between the length of the great circle and the perimeter of the meridian ellipse is 22.069 km, and the difference between the semiaxes of the ellipsoid is

$$a - b = 21.382 \text{ km}.$$

§1.2. The Relationship between the Coordinates of Earth Coordinate Systems

In studying the figure of the Earth and in deriving the kinematic equations for inertial systems of Earth navigation we employ the following coordinate systems:

- The inertial coordinate system, which is associated with fixed stars. We note that for Earth navigation it is possible to assume that a coordinate system with the origin in the center of the Earth, but not rotating with the Earth, is also inertial. In this connection we disregard the acceleration of the center of the Earth due to the attraction of the Sun and the other planets; /9

- Earth coordinate systems linked with the Earth and in motion with the Earth with respect to inertial space. These include geographical, geocentric, corrected, arbitrary and rectangular coordinate systems.

In the geographic coordinate system the location of points on the surface of the Earth is determined by geographic latitude ϕ and longitude λ.

The angle formed by a perpendicular line on the normal to the surface of the geoid at the given point and the plane of the Earth equator is called the geographic latitude of the given point on the Earth's surface [32].

We shall call the angle between a projection of the normal to the surface of the geoid at this point to the plane of the equator and of the line of intersection of the plane of the Greenwich Meridian with the equatorial plane (OX) the geographic longitude λ of a given point.

The geographic meridians and parallels are the coordinate lines in the geographic coordinate system.

In the geocentric coordinate system the position of point P on the surface of the Earth is determined by the geocentric latitude ϕ' and by longitude λ' (Fig. 1.2).

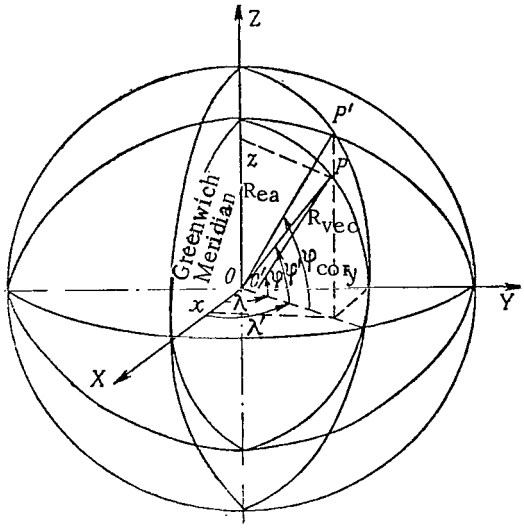

Figure 1.2. The Corrected and Rectangular Coordinate Systems

The latitude ϕ' is determined by the angle between the radius-vector R_{vec}, which is drawn from the geometric center 0 of the geoid to given point P, and the equatorial plane.

Longitude λ' is determined on the sphere and on the spheroid in the same way as for geographical longitude, and therefore for these approximations of the figure of the Earth $\lambda' = \lambda$.

In addition to the geocentric coordinate system we shall introduce the so-called corrected coordinate system ϕ_{cor} and λ_{cor}. The corrected latitude ϕ_{cor} is determined by the angle between the radius R_{ea}, extended to a given point, and the equatorial plane. In turn, $\lambda_{cor} = \lambda'$.

In the rectangular system of XYZ coordinates, the X and Y axes lie in the equatorial plane, but the Z axis is directed along the axis of Earth rotation. The position of the points on the surface of the geoid is determined by the coordinates X, Y, Z. /10

In the arbitrary coordinate system the location of the points on the surface of the Earth is determined by arbitrary longitude λ_{arb} and by arbitrary latitude ϕ_{arb} (Figure 1.3). In Figure 1.3 the surface of the Earth is presented for clarity in the form of a sphere, and the perpendicular lines are directed towards the geometric center of the sphere.

Arbitrary longitude λ_{arb} is a composite angle which is described by a perpendicular line during the motion of the point along the coordinate geodetic line from the origin to a given moment of time **t**:

$$\lambda_{arb} = \int_0^t \dot{\lambda}_{arb}\, dt,$$

where λ_{arb} is a component of the angular velocity of the rotation of a perpendicular line with respect to the initial position as a result of motion of the point along the arbitrary equator

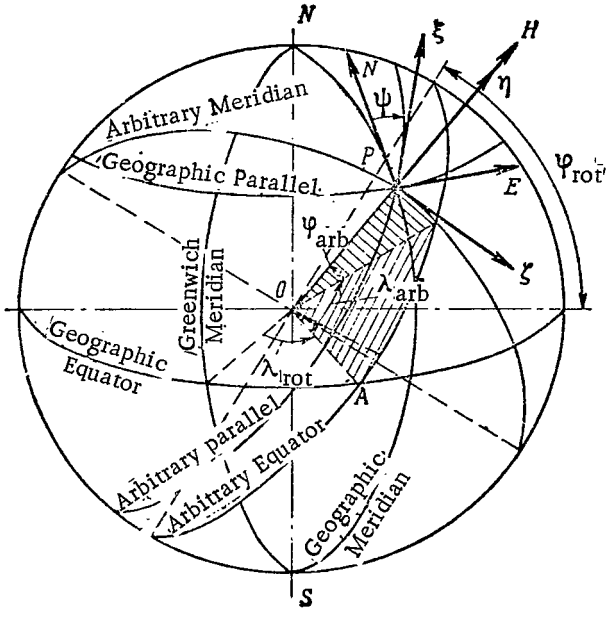

Figure 1.3. The Arbitrary Coordinate System.

The coordinate geodetic line joins two previously selected points on the surface of the Earth, for example, the start and target points. On the surface of the Earth this geodetic line is an orthodrome, and therefore the arbitrary coordinate system on a sphere may be called an orthodromic system. /11

Arbitrary latitude ϕ_{arb} is a composite angle, which is described by a perpendicular line during the motion of a point away from the arbitrary equator along a geodetic line (the arbitrary meridian) orthogonal to the equator, which passes through the point considered at the given moment in time t:

$$\phi_{arb} = \int_0^t \dot{\varphi}_{arb}\, dt,$$

where $\dot{\phi}_{arb}$ is a component of the angular velocity of the rotation of tne perpendicular line as a result of motion along the arbitrary meridian.

It should be noted that the arbitrary coordinate system may be obtained by a dual rotation of the geographic coordinate system (with corresponding deformation of the coordinate lines in the figure of the Earth); the first rotation is around the rotational axis Z of the Earth by the angle λ_{rot}, and the second is relative to the line OA by angle ϕ_{rot}.

The relationship between the coordinates of a given point on the surface in the rectangular (X, Y, Z) and the geographic (ϕ, λ) coordinate systems is defined [32], [33] by the following equations:

for a spheroid

$$\left.\begin{aligned} X &= \frac{a^2 \cos\varphi \cos\lambda}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}; \\ Y &= \frac{a^2 \cos\varphi \sin\lambda}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}; \\ Z &= \frac{b^2 \sin\varphi}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}; \end{aligned}\right\} \qquad (1.1)$$

for a triaxial ellipsoid

$$\left.\begin{aligned} X &= \frac{a^2 \cos\varphi \cos\lambda}{\sqrt{(a^2\cos^2\lambda + c^2\sin^2\lambda)\cos^2\varphi + b^2\sin^2\varphi}}; \\ Y &= \frac{a^2 \cos\varphi \sin\lambda}{\sqrt{(a^2\cos^2\lambda + c^2\sin^2\lambda)\cos^2\varphi + b^2\sin^2\varphi}}; \\ Z &= \frac{b^2 \sin\varphi}{\sqrt{(a^2\cos^2\lambda + c^2\sin^2\lambda)\cos^2\varphi + b^2\sin^2\varphi}}, \end{aligned}\right\} \qquad (1.2)$$

where a, b, and c are the corresponding semiaxes of the ellipsoid.

The rectangular and corrected coordinate systems are linked by the relationships:

for a spheroid

$$\left.\begin{aligned} X &= a\cos\phi_{cor}\cos\lambda_{cor} \\ Y &= a\cos\phi_{cor}\sin\lambda_{cor} \\ Z &= b\sin\phi_{cor} \end{aligned}\right\} \qquad (1.3)$$

for an ellipsoid /12

$$\left.\begin{aligned} X &= a\cos\phi_{cor}\cos\lambda_{cor} \\ Y &= c\cos\phi_{cor}\sin\lambda_{cor} \\ Z &= b\sin\phi_{cor} \end{aligned}\right\} \qquad (1.4)$$

The transition from the rectangular XYZ to the geocentric coordinate system and the reverse is defined by equations:

for a spheroid

$$\left.\begin{aligned} X &= \frac{ab\cos\varphi'\cos\lambda'}{\sqrt{a^2\sin^2\varphi' + b^2\cos^2\varphi'}}; \\ Y &= \frac{ab\cos\varphi'\sin\lambda'}{\sqrt{a^2\sin^2\varphi' + b^2\cos^2\varphi'}}; \\ Z &= \frac{ab\sin\varphi'}{\sqrt{a^2\sin^2\varphi' + b^2\cos^2\varphi'}}; \end{aligned}\right\} \qquad (1.5)$$

for an ellipsoid

$$\left.\begin{aligned} X &= \frac{abc\cos\varphi'\cos\lambda'}{\sqrt{b^2(c^2\cos^2\lambda' + a^2\sin^2\lambda')\cos^2\varphi' + a^2c^2\sin^2\varphi'}}; \\ Y &= \frac{abc\cos\varphi'\sin\lambda'}{\sqrt{b^2(c^2\cos^2\lambda' + a^2\sin^2\lambda')\cos^2\varphi' + a^2c^2\sin^2\varphi'}}; \\ Z &= \frac{abc\sin\varphi'}{\sqrt{b^2(c^2\cos^2\lambda' + a^2\sin^2\lambda')\cos^2\varphi' + a^2c^2\sin^2\varphi'}}. \end{aligned}\right\} \quad (1.6)$$

The geographic and geocentric coordinate systems are connected by the relationships

for a spheroid

$$\left.\begin{aligned} \tan\varphi' &= \frac{b^2}{a^2}\tan\varphi; \\ \lambda' &= \lambda. \end{aligned}\right\} \quad (1.7)$$

for an ellipsoid

$$\left.\begin{aligned} \tan\varphi' &= \frac{b^2}{\sqrt{a^4\cos^2\lambda + c^4\sin^2\lambda}}\cdot\tan\varphi; \\ \tan\lambda' &= \frac{c^2}{a^2}\tan\lambda. \end{aligned}\right\} \quad (1.8)$$

In turn, the following relationships exist between coordinates in the geographic and corrected systems:

for a spheroid

$$\left.\begin{aligned} \tan\varphi &= \frac{a}{b}\tan\varphi_{cor} \\ \lambda &= \lambda_{cor} \end{aligned}\right\} \quad (1.9)$$

for an ellipsoid /13

$$\left.\begin{aligned} \tan\varphi &= \frac{1}{b\sqrt{\frac{\sin^2\lambda_{cor}}{c^2} + \frac{\cos^2\lambda_{cor}}{a^2}}}\tan\varphi_{cor}; \\ \tan\lambda &= \frac{a}{c}\tan\lambda_{cor}. \end{aligned}\right\} \quad (1.10)$$

The connection between the geocentric and corrected coordinate systems is defined by the relationships:

for a spheroid

$$\left.\begin{aligned} \tan\varphi' &= \frac{b}{a}\tan\varphi_{cor}; \\ \lambda' &= \lambda_{cor}; \end{aligned}\right\} \qquad (1.11)$$

for an ellipsoid

$$\left.\begin{aligned} \tan\varphi' &= \frac{b}{\sqrt{a^2\cos^2\lambda_{cor}+c^2\sin^2\lambda_{cor}}}\tan\varphi_{cor}; \\ \tan\lambda' &= \frac{c}{a}\tan\lambda_{cor} \end{aligned}\right\} \qquad (1.12)$$

We shall find the relationships between the arbitrary and the geographic coordinates. First of all we shall introduce into the examination the NHE and $\xi\eta\zeta$ coordinate trihedrons which are connected with the geographic and arbitrary coordinate systems, respectively (Figure 1.4).

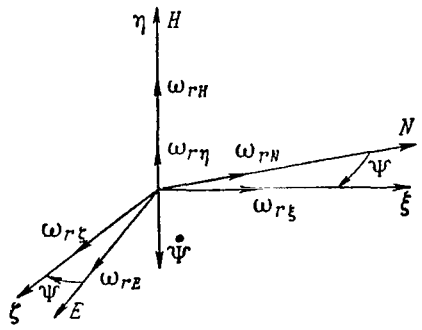

Figure 1.4. The Geographic and Arbitrary Coordinate Trihedrons

The geographic NHE coordinate trihedron is oriented with respect to the surface of the Earth so that N and E axes lie in the plane of the true horizon and directed toward the North and toward the East, respectively, while H axis is directed upward along the normal to the surface of the geoid.

The arbitrary $\xi\eta\zeta$ coordinate trihedron is oriented so that the ξ and ζ axes are directed along the tangents to the arbitrary parallel and meridian, and the η axis is directed along the normal to the surface of the geoid at Point P (λ_{arb}, ϕ_{arb}). With point P in motion along the surface of the Earth, the NHE and $\xi\eta\zeta$ coordinate trihedrons rotate with respect to the surface of the Earth, and it is easy to see that /14

$$\left.\begin{aligned} \omega_{rN} &= \omega_{r\xi}\cos\Psi - \omega_{r\zeta}\sin\Psi; \\ \omega_{rH} &= \omega_{r\eta} - \dot{\Psi}; \\ \omega_{rE} &= \omega_{r\xi}\sin\Psi + \omega_{r\zeta}\cos\Psi. \end{aligned}\right\} \qquad (1.13)$$

where ω_{rN}, ω_{rH}, ω_{rE} are vector projections of the angular velocity of rotation of the geographic trihedron with respect to the surface of the Earth on the NHE axes; $\omega_{r\xi}$, $\omega_{r\eta}$, $\omega_{r\zeta}$ are vector projections of the angular velocity of the rotation of the arbitrary coordinate trihedron on the $\xi\eta\zeta$ axes;

Ψ is the angle between the northern direction of the geographic meridian and the arbitrary equator.

From the other side (Figure 1.5),

$$\left.\begin{aligned}\omega_{rN}&=\dot{\lambda}\cos\varphi;\\ \omega_{rH}&=\dot{\lambda}\sin\varphi;\\ \omega_{rE}&=-\dot{\varphi}.\end{aligned}\right\} \tag{1.14}$$

similarly (Figure 1.6)

$$\left.\begin{aligned}\omega_{r\xi}&=-\dot{\varphi}_{arb}\\ \omega_{r\eta}&=\dot{\lambda}_{arb}\sin\phi_{arb}\\ \omega_{r\zeta}&=-\dot{\lambda}_{arb}\cos\phi_{arb}\end{aligned}\right\} \tag{1.15}$$

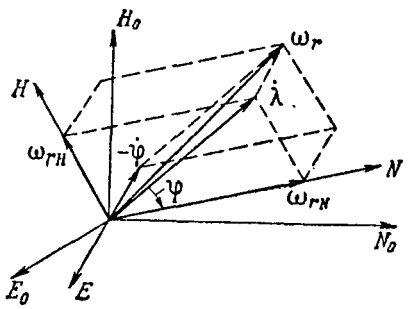

Figure 1.5

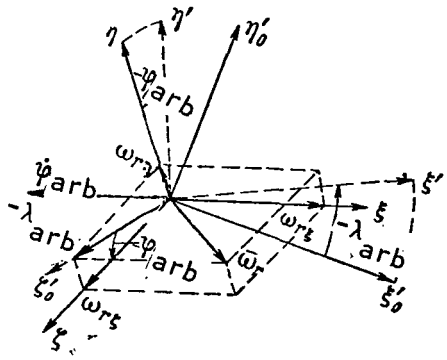

Figure 1.6

Having examined jointly the equations (1.14) and (1.15), after some simple transformations we obtain the following the relationships between the geographic and arbitrary coordinates: /15

$$\left.\begin{aligned}\dot{\lambda}&=\frac{1}{\cos}(\dot{\lambda}_{arb}\cos\phi_{arb}\sin\Psi-\dot{\phi}_{arb}\cos\Psi);\\ \dot{\Psi}&=\dot{\lambda}_{arb}(\sin\phi_{arb}-\cos\phi_{arb}\sin\Psi\tan\phi)+\dot{\phi}_{arb}\cos\Psi\tan\phi;\\ \dot{\phi}&=\dot{\phi}_{arb}\sin\Psi+\dot{\lambda}_{arb}\cos\phi_{arb}\cos\Psi.\end{aligned}\right\} \tag{1.16}$$

§1.3. The Radii of the Curvature of Coordinate Lines on the Surface of the Earth

In a derivation of the kinematic equations of the inertial navigation system with consideration for the nonsphericity of the surface of the Earth, it is necessary to know the dependences of the radii of the curves of the coordinate lines of the geographic and arbitrary systems on the coordinates of the instantaneous locations of an object.

The radii of the curvature of the geographic meridians R_{ms} and of the parallels R_{ps} of a spheroid at point P (λ, ϕ) are determined [32], [33] as

$$\left.\begin{aligned} R_{ms} &= \frac{a(1-l^2)}{(1-l^2\sin^2\varphi)^{3/2}}; \\ R_{ps} &= \frac{a\cos\Psi}{\sqrt{1-l^2\sin^2\varphi}}; \end{aligned}\right\} \tag{1.17}$$

where

$$l^2 = \frac{a^2-b^2}{a^2}. \tag{1.18}$$

Correspondingly, the radii of the curvature on an ellipsoid are as follows:

$$\left.\begin{aligned} R_{me} = {} & \frac{1}{abc}\left\{\left[(a^2\cos^2\lambda_{cor}+c^2\sin^2\lambda_{cor})\sin^2\varphi_{cor}+b^2\cos^2\varphi_{cor}+\right.\right. \\ & +(a^2\sin^2\lambda_{cor}+c^2\cos^2\lambda_{cor})+[[(a^2\cos^2\lambda_{cor}+ \\ & +c^2\sin^2\lambda_{cor})\sin^2\varphi_{cor}+b^2\cos^2\varphi_{cor}+a^2\sin^2\lambda_{cor}+ \\ & +c^2\cos^2\lambda_{cor}]^2-4[a^2c^2\cos^2\varphi_{cor}\sin^2\varphi_{cor}+ \\ & \left.+b^2(a^2\sin^2\lambda_{cor}+c^2\cos^2\lambda_{cor})\cos^2\varphi_{cor}]^{\frac{1}{2}}\right\}^{-1}2[a^2c^2\sin^2\varphi_{cor}+ \\ & \left.+b^2(a^2\sin^2\lambda_{cor}+c^2\cos^2\lambda_{cor})\cos^2\varphi_{cor}\right]^{\frac{3}{2}}\Big\}; \\ R_{pe} = {} & \frac{1}{2abc}[a^2c^2\sin^2\varphi_{cor}+b^2(a^2\sin^2\lambda_{cor}+ \\ & +c^2\cos^2\lambda_{cor})\cos^2\varphi_{cor}]^{\frac{1}{2}}\left\{[(a^2\cos^2\lambda_{cor}+c^2\sin^2\lambda_{cor})\sin^2\varphi_{cor}+\right. \\ & +b^2\cos^2\varphi_{cor}]+(a^2\sin^2\lambda_{cor}+c^2\cos^2\lambda_{cor})+ \\ & +\left\{\left[(a^2\cos^2\lambda_{cor}+c^2\sin^2\lambda_{cor})\sin^2\varphi_{cor}+b^2\cos^2\varphi_{cor}+\right.\right. \\ & +a^2\sin^2\lambda_{cor}+c^2\cos^2\lambda_{cor}]^2-4[a^2c^2\sin^2\varphi_{cor}+ \\ & \left.\left.\left.+b^2(a^2\sin^2\lambda_{cor}+c^2\cos^2\lambda_{cor})\cos^2\varphi_{cor}\right]^{\frac{1}{2}}\right\}. \end{aligned}\right\} \tag{1.19}$$

/16

Here it is necessary to consider the relationships which link the corrected and the geographic coordinate system (1.10).

For the arbitrary coordinate system the radii of the curvature of the coordinate lines passing through a given point P (λ_{arb}, ϕ_{arb}) may be determined (see [6]) from the following equations:

$$\left.\begin{aligned} R_{arb,p} &= \frac{R_1R_2}{R_1\cos^2\Psi+R_2\sin^2\Psi}; \\ R_{arb,m} &= \frac{R_1R_2}{R_1\sin^2\Psi+R_2\cos^2\Psi}, \end{aligned}\right\} \tag{1.20}$$

where for a spheroid

$$\left.\begin{aligned} R_1 &= \frac{a}{\sqrt{1-l^2\sin^2\varphi}}; \\ R_2 &= \frac{a(1-l^2)}{(1-l^2\sin^2\varphi)^{3/2}}, \end{aligned}\right\} \qquad (1.21)$$

and for an ellipsoid

$$R_1 = R_{me};$$
$$R_2 = R_{pe}.$$

In this connection it is necessary to consider the (1.10) relationships and the (1.16) system.

§1.4. The Acceleration of Gravity at Sea Level

First of all we shall assume that objects which are in motion along trajectories established with respect to the surface of the Earth participate in the rotation of the Earth.

The force of gravity, acting on a certain mass m, included in such objects, is determined by the vector sum of the force of Newtonian attraction of the Earth and of centrifugal force, caused by the rotation mass m with the Earth in inertial space. In a general case both gravity components are functions of the coordinates of the mass distribution point m with respect to the surface of the Earth. The dependence of the vector modulus /17 of the acceleration of gravity on the point coordinates on the surface of the Earth for several approximations of the figure of the Earth may be presented analytically [34], [35].

For example, on the surface of an Earth triaxial ellipsoid

$$g = 9{,}78045\,[1 + 0{,}0053033\sin^2\varphi - 0{,}0000059\sin^2 2\varphi + \\ + 0{,}0000167\cos^2\varphi\cos 2(\lambda - 15^\circ)]; \qquad (1.22)$$

and correspondingly on the surface of a spheroid

$$g = 9{,}78045\,(1 + 0{,}0053033\sin^2\varphi - 0{,}0000059\sin^2 2\varphi). \qquad (1.23)$$

and finally on the surface of the Earth's sphere

$$g = 9{,}78045\,(1 + 0.0053033\sin^2\varphi). \qquad (1.24)$$

§1.5. Projections of the Acceleration of Gravity on the Axes of Coordinate Trihedrons at Altitude H

With a change in altitude H (Figure 1.7), the gravity vector G(H) of the mass m, located at point P, changes in both value and direction, if we consider that point P rotates with the Earth with angular velocity Ω.

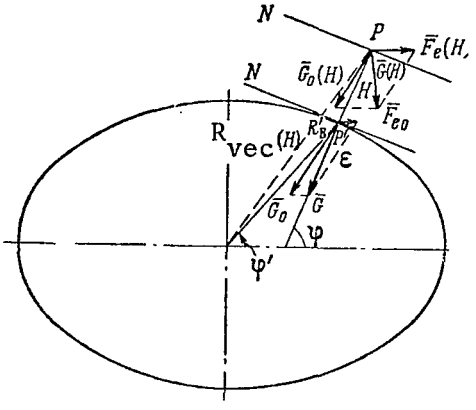

Figure 1.7

At sea level the gravity vector is

$$\bar{G}=\bar{G}_0+\bar{F}_{e0}, \tag{1.25}$$

where $\vec{G}_0$ is the vector of the force of Newtonian attraction at the surface of the geoid; F_{e0} is the centrifugal force on the surface of the geoid.

At altitude H above sea level /18

$$\bar{G}(H)=\bar{G}_0(H)+\bar{F}_e(H), \tag{1.26}$$

where $\bar{G}_0(H)$, $\bar{F}_e(H)$ are vectors of the force of Newtonian attraction and centrifugal force, respectively, at altitude H.

The vector moduli of the forces of Newtonian attraction $G_0(H)$ and of centrifugal force $F_e(H)$, acting on mass m at altitude H above sea level equal

$$\left.\begin{aligned} G_0(H)&=G_0+\Delta G_0(H);\\ F_e(H)&=F_{e0}+\Delta F_e(H), \end{aligned}\right\} \tag{1.27}$$

where $\Delta G_0(H)$, $\Delta F_e(H)$ are increments of corresponding forces, caused by a change in altitude.

It is possible to show that the vector projections of the acceleration of gravity of mass m, located at point P (ϕ, λ, H) on the NHE axes of the geographic trihedron, the origin of which is a central projection of point P to P' on the surface of the spheroid, equal

$$\begin{aligned} g_N=&\frac{1}{2}g+\frac{g_0^2}{2g}\left[\frac{2R'^2_{vec}}{R^2_{vec}(H)}-1\right]+\\ &+\frac{\Omega^4 R'_{vec}\cos\varphi'}{2g}[R'_{vec}\cos\varphi'+2H\cos\varphi]- \end{aligned} \tag{1.28}$$

$$-\frac{\Omega^2(g\cos\varphi+\Omega^2R_{vec}\cos\varphi')}{g}\frac{R'^2_{vec}}{R^2_{vec}(H)}[R'_{vec}\cos\varphi'+H\cos\varphi];$$

$$g_H=\left[g_0^2\frac{R'^2_{vec}}{R^2_{vec}(H)}+\Omega^4(R'_{vec}\cos\varphi'+H\cos\varphi)^2-\right.$$

$$-2\Omega^2\frac{R'^2_{vec}}{R^2_{vec}(H)}(R'_{vec}\cos\varphi'+H\cos\varphi)(g\cos\varphi+ \qquad (1.28)$$

$$\left.+\Omega^2R'_{vec}\cos\varphi')-g_0^2\right]^{\frac{1}{2}};$$

$$g_E=0.$$

Equations (1.28) are derived under the assumption that where $0 \leqslant H \leqslant 100$ km $\ll R_{ea}$, the variation in the direction of the forces of Newtonian attraction with a change in altitude may be disregarded.

After additional transformations, the equations (1.28) may be presented in the following form: /19

$$g_N=\Omega^2\sin\varphi\left[H\cos\varphi+R'_{vec}\cos\varphi'\left(1-\frac{R'^2_{vec}}{R^2_{vec}(H)}\right)\right];$$

$$g_H=\frac{R'^2_{vec}}{R^2_{vec}(H)}g-\Omega^2\cos\varphi\left[H\cos\varphi+R'_{vec}\cos\varphi'\times\right. \qquad (1.28a)$$

$$\left.\times\left(1-\frac{R'^2_{vec}}{R^2_{vec}(H)}\right)\right].$$

In considering (1.28a), we may show that

$$g_H=\frac{R'^2_{vec}}{R^2_{vec}(H)}g-g_N\cot\varphi, \qquad (1.28b)$$

where g is the vector modulus of the acceleration of gravity at point P; g_0 is the vector modulus of the forces of Newtonian attraction of the Earth; R'_{vec} is the radius-vector modulus, extended from the geometric center of the spheroid to point P'; $R_{vec}(H)$ is the modulus of the radius-vector, extended to point P.

In addition,

$$R'_{vec}=\frac{a}{\sqrt{\frac{b^2}{a^2}\tan\varphi+1}};$$

$$R^2_{vec}(H)=R'^2_{vec}+H^2+2R'_{vec}H\cos(\varphi-\varphi');$$

$$g_0=g\frac{\sin\varphi}{\sin(\varphi-\varepsilon)}; \qquad (1.29)$$

$$\varepsilon=\arcsin\left[\sin\varphi\frac{\Omega^2R'_{vec}\cos\varphi'\sin\varphi}{\sqrt{g^2+\Omega^4R'^2_{vec}\cos^2\varphi'+2g\Omega^2R'_{vec}\cos\varphi'\cos\varphi}}\right],$$

where ε is the angle which characterizes the deviation of the vector of the forces of Newtonian attraction at point P under consideration from the normal to the surface of the Earth at this point.

Next, for all points located at altitudes $H \ll R_{ea}$ above the surface of the triaxial ellipsoid, /20

$$g_H=\frac{[(a+H)\gamma_{(a+H)}\cos^2(\lambda-15°)+(c+H)\gamma_{(c+H)}\sin^2(\lambda-15°)]\cos^2\varphi+}{\sqrt{[(a+H)^2\cos^2(\lambda-15°)+(c+H)^2\sin^2(\lambda-15°)]\cos^2\varphi\times}}$$
$$\frac{+(b+H)\gamma_{(c+H)}\sin^2\varphi}{\times(b+H)^2\sin^2\varphi}, \qquad (1.30)$$

where $\gamma_{(a+H)}$, $\gamma_{(c+H)}$, $\gamma_{(b+H)}$ are values of the acceleration of gravity at points located at altitude H along each ellipsoid semiaxis:

$$\gamma_{(a+H)}=\frac{a^2}{(a+H)^2}(\gamma_a+\Omega^2a)-\Omega^2(a+H);$$

$$\gamma_{(c+H)}=\frac{c^2}{(c+H)^2}(\gamma_c+\Omega^2c)-\Omega^2(c+H); \qquad (1.31)$$

$$\gamma_{(b+H)}=\frac{b^2}{(b+H)^2}\gamma_b;$$

γ_a, γ_b, γ_c are values of the acceleration of Earth gravity at the ends of the ellipsoid semiaxes.

Projections of the acceleration of gravity on the E axis of the geographic trihedron will have maximum value at a point situated in the plane of the equator of the triaxial ellipsoid at altitude H.

It may be shown that the value of this component has the form

$$g_E=\left[g_0\frac{R'^4_{vec}}{R^4_{vec}(H)}+\Omega^4(R'_{vec}+H)^2+2\Omega^2g_0\frac{R'^2_{vec}}{(R_{ea}+H)^2}(R'_{vec}+H)-g_0^2\right]^{\frac{1}{2}} \qquad (1.32)$$

In analyzing equation (1.32), it is not difficult to verify that where altitude $H \leqslant 100$ km, the values of this projection are comparatively small.

It should be noted that at points located at altitude H above the Earth sphere:

$$g_N = \frac{1}{2} g + \frac{g_0^2}{2g}\left[\frac{2R_{ea}^2}{(R_{ea}+H)^2} - 1\right] + \frac{\Omega^4 R_{ea}\cos^2\varphi}{2g}(R_{ea}+2H) - \frac{\Omega^2\cos^2\varphi\,(g+\Omega^2 R_{ea})}{g}\left[\frac{R_{ea}^3}{(R_{ea}+H)^2}+H\right];$$

$$g_H = \left[g_0^2\frac{R_{ea}^4}{(R_{ea}+H)^4} + \Omega^4\cos^2\varphi(R_{ea}+H)^2 - 2\Omega^2\frac{R_{ea}^2\cos\varphi}{(R_{ea}+H)^2}(g+\Omega^2 R_{ea}) - g_0^2\right]^{\frac{1}{2}}; \quad (1.33)$$

or /21

$$g_N = \Omega^2\sin\varphi\left[H + R_{ea}\cos\varphi\left(1 - \frac{R_{ea}^2}{(R_{ea}+H)^2}\right)\right];$$

$$g_H = \frac{R_{ea}^2}{(R_{ea}+H)^2}\,g - \Omega^2\cos^2\varphi\left[H + R_{ea}\left(1 - \frac{R_{ea}^2}{(R_{ea}+H)^2}\right)\right]. \quad (1.34)$$

It is apprent for Figure 1.8 that the vector projections of the acceleration of gravity on the $\xi\eta\zeta$ axes of the trihedron are:

$$\left.\begin{aligned} g_\xi &= g_N\cos\Psi + g_E\sin\Psi;\\ g_\eta &= g_H;\\ g_\zeta &= g_E\cos\Psi - g_N\sin\Psi, \end{aligned}\right\} \quad (1.35)$$

where g_N, g_H, g_E are determined in accordance with previously derived equations, depending on the approximation accepted for the figure of the Earth.

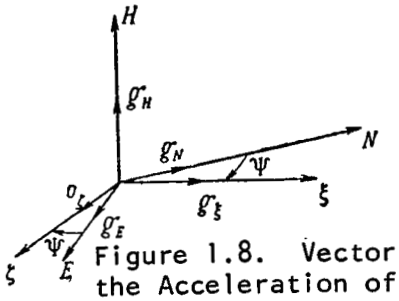

Figure 1.8. Vector Projection of the Acceleration of Gravity

§1.6. Gravity Anomalies

A deviation from the true vector of the acceleration of gravity from a certain standard established in accordance with theoretical considerations [34], [35] are termed gravity anomalies. In practice,

the influence of anomalies is reduced to a deviation in the gravity vector from the direction of the normal to the geoid. Anomalies are caused by the influence of local relief and nonuniformity in the mass of the Earth above the point at which the force of gravity is being determined. Anomalies caused by the influence of local relief and water basins are usually termed topographical. In their evaluation, it is necessary to consider the opinion of specialists in gravimetry [34], [35]: "...the existing Earth in its internal composition is much closer to a uniform plan of an ideal Earth than to a plan corresponding to topographical reduction."

In practice, the calculation of a topographic anomaly is not always possible since this requires the expenditure of a great deal of labor and the availability of cartographic material.

In gravimetry we turn to a simplified method of calculation by means of the Bouguer reduction. The Bouguer anomaly more or less repeats the profile of the base of the Earth crust and differs little from the topographic anomaly. The value of the acceleration of gravity at the surface of the Earth, considering the Bouguer anomaly (g_a) must be calculated in accordance with the formula /22

$$g_a = g + \delta_b, \tag{1.36}$$

where δ_b are Bouguer anomalies, determined in accordance with a gravimetric area map; g is the acceleration of gravity determined in accordance with equations (1.22), (1.23) or (1.24), depending on the approximation of the figure of the Earth accepted.

It is apparent from gravimetric maps of various areas of the Earth that the field of weak negative and positive anomalies encompasses a large part of the lowlands and low plains of Europe, Asia and America.

The maximum value of the anomaly occurs in mountainous areas, especially in areas of Tibet and Pamir.

CHAPTER II

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THE KINEMATICS OF INERTIAL NAVIGATION SYSTEMS WITH A PLATFORM STABILIZED IN THE EARTH COORDINATE SYSTEM

§2.1 The Kinematics of the Single-component INS

We should make the following assumptions in order to simplify the problem of deriving the kinematic equations of a single-component INS.

- We shall consider the surface of the Earth to be a sphere with radius R;

- The center of mass of the Earth is located at the geometric center of the Earth sphere;

- We shall not consider the rotation of the Earth;

- Object flight is accomplished in the plane of the geographic meridian (Figure 2.1).

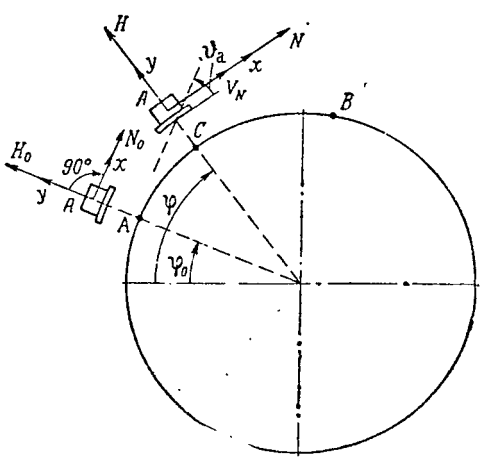

Figure 2.1

In order to determine the instantaneous value of the coordinate (ϕ) of the center of mass of the object with respect to the surface of the Earth Figure 2.1) by means of inertial navigation methods, it is necessary:

1) To have the coordinate initial value ϕ_0 at point A;

2) To know the instantaneous values V_N or $\dot{V}_N/R$ of the acceleration of the center of mass of the object with respect to the surface of the Earth;

3) To integrate with time the values $\dot{V}_N$ and $\dot{V}_N/R$ in order to calculate velocity V_N and coordinate ϕ.

We shall introduce into this examination the NH coordinate dihedron, the N axis of which is parallel to the instantaneous position of the tangent to the geographic meridian, the H axis is perpendicular to the N axis and is directed upward, and the origin is located at starting point A. We shall designate the initial position of the axes of the dihedron as N_0H_0.

We shall take the simplest INS. We shall assume that an accelerometer is employed as the primary measurement device, the responsive axis of which is stabilized along the N axis of the dihedron; for this purpose the instrument indicated is installed on a special inclined platform. /24

The kinematic equation of such a system - the roll equation of the plane with respect to the Earth sphere along the meridian is

$$\ddot{\varphi}=\frac{\dot{V}_N}{R}, \tag{2.1}$$

where $\ddot{\phi}$ is the second derivative of the geographic latitude with time.

In order to stabilize the responsive axis of the accelerometer along the N axis it is necessary that the platform in the process of motion be automatically inclined with respect to the N_0H_0 axes of the dihedron by the angle

$$\vartheta_a=-(\varphi-\varphi_0) \tag{2.2}$$

with angular velocity

$$\dot{\vartheta}_a=-\dot{\varphi}. \tag{2.3}$$

Here the output signal of the system with respect to $\ddot{\phi}$ is conveniently applied to the input of devices for the inclination of the accelerometer platform.

It is apparent from the block diagram shown in Figure 2.2 that the system is closed with respect to the $\dot{\phi}$ parameter.

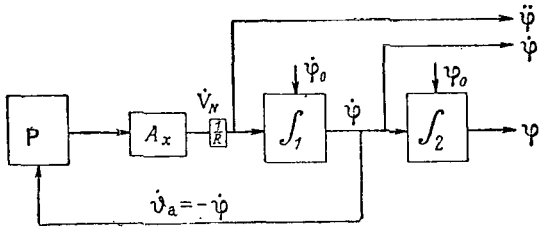

Figure 2.2

When errors are present in system elements, mistakes will occur in the determination of the coordinate ($\Delta\phi$) and in platform stabilization ($\Delta\vartheta_a$),, so that instrumental values of these parameters will assume the form

$$\begin{aligned}\varphi^*&=\varphi+\Delta\varphi;\\ \vartheta^*&=\vartheta_a+\Delta\vartheta_a.\end{aligned} \tag{2.4}$$

When the platform is inclined by angle $\Delta\vartheta_a$ with respect to the plane of the horizon accelerometer indications are determined as

$$\dot{V}_N^*=\dot{V}_N\cos\Delta\vartheta_a+g_0\sin\Delta\vartheta_a, \tag{2.5}$$

where g_0 is the acceleration of Earth gravity.

If angle $\Delta\vartheta_a$ is sufficiently smaller than /25

$$\dot{V}_N^*\cong\dot{V}_N+g_0\Delta\vartheta_a. \tag{2.6}$$

We shall evaluate in a first approximation the error in a single-component INS in the determination of the ϕ coordinate.

When accelerometer error Δa is present, instrument acceleration values are determined as

$$\dot{V}_N^* \cong \dot{V}_N + g_0 \Delta\vartheta_a + \Delta a. \tag{2.7}$$

Substituting the value of the parameter $\dot{V}^*_N$ in equation (2.1) and taking into consideration that $\Delta\vartheta_a = -\Delta\phi$, we obtain the following error equation:

$$\Delta\varphi + \frac{g_0}{R}\Delta\varphi \cong \frac{\Delta a}{R}. \tag{2.8}$$

If we assume that $\Delta a = \Delta a_0 = \text{const}$, then the solution of equation (2.8) will be as follows:

$$\Delta\varphi \cong \frac{\Delta a}{g_0}\left(1 - \cos\sqrt{\frac{g_0}{R}}\cdot t\right). \tag{2.9}$$

The graph of $\Delta\phi(t)$ is shown in Figure 2.3a.

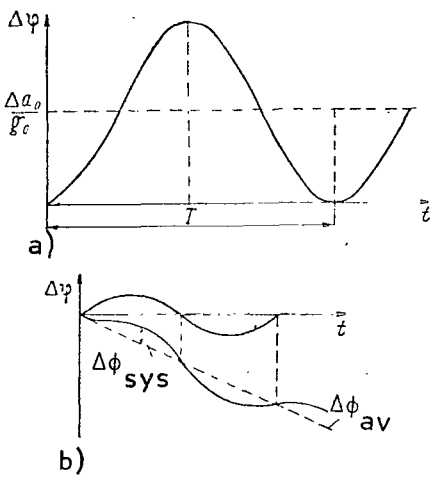

Figure 2.3

It follows from equation (2.9) that system error in the determination of the ϕ coordinate, caused by constant error in the measurement of acceleration, changes with time in accordance with the harmonic principle with a period of

$$T = 2\pi\sqrt{\frac{R}{g_0}} \cong 84.4 \text{ min}$$

equal to the period of oscillations of the Schuler mathematical pendulum, and with amplitude

$$\Delta\phi_{sys} = \frac{\Delta a}{g_0}$$

and the approximate average value is

$$\Delta\phi_{av} \doteq \frac{\Delta a}{g_0}. \tag{2.10}$$

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In a similar fashion we may determine the error in a single-component INS, caused by constant error $\Delta\vartheta_0$ in the platform stabilization devices

$$\Delta\varphi = \Delta\vartheta_0\left(1 - \cos\sqrt{\frac{g_0}{R}}\cdot t\right). \tag{2.11}$$

The graph of this error $\Delta\phi(t)$ is similar to the graph in 2.3a.

It is easy to see that error in a single-component INS as a result of constant error in platform stabilization has the same oscillation period, but the average value is $\Delta\phi_{av} = \Delta\vartheta_0$.

In turn, it is possible to show that when systematic platform drift is present in inertial space with an angular velocity of $\Delta\dot{\vartheta} = \Delta\dot{\vartheta}_0 = \text{const}$, error in coordinate measurement is determined by the following equation as a first approximation:

$$\Delta\varphi = \Delta\dot{\vartheta}_0\left(\sqrt{\frac{R}{g_0}}\sin\sqrt{\frac{g_0}{R}}\cdot t - t\right). \qquad (2.12)$$

It is apparent from equation (2.12) that this error changes (Fig. 2.3b) also in accordance with the harmonic principle with a period of $T \cong 84.4$ and with an amplitude of $\Delta\phi_m \cong \Delta\vartheta_0\sqrt{\frac{R}{g_0}}$, but the average value increases with time linearly:

$$\Delta\phi_{av} = \Delta\dot{\vartheta}_0 t.$$

It is easy to show that with a platform orientation in the orthodromic coordinate system (if the measurement axis of the accelerometer is directed along the tangent to the coordinate orthodrome), the nature of the change in instrumental error in a single-component INS will, in a first approximation, be the same.

If the INS were constructed in a closed design, than error in coordinate determination would change with time in accordance with laws differing from those cited above.

For example, when constant error Δa_0 is present in an accelerometer installed on an ideally stabilized platform for an INS of closed design, error in coordinate determination would increase with time in accordance with the square law principle

$$\Delta\varphi = \frac{\Delta a_0 t^2}{2}. \qquad (2.13)$$

INS error in coordinate determination caused by constant error in platform stabilization would have a similar nature, but constant systematic platform drift in inertial space would lead to error $\Delta\phi$, which would show a cubic increase with time. /27

It is apparent from these considerations that closed INS's may furnish higher accuracy in the determination of object coordinates with respect to the surface of the Earth than open systems, especially with an extended control time of object motion.

The analysis shown of the kinematics of a single-component INS reveals that this system is a physical model of the Schuler pendulum.

§2.2. The Kinematic Equations of a Three-Component INS in the Geographic Coordinate System

Let us assume that three INS accelerometers A_x, A_y, A_z with mutually perpendicular measurement x, y, z axes (Fig. 2.4) are installed in the center of mass of an object (point O), which is in motion with respect to the surface of the Earth at altitude H from point A to point B.

In a general case the altitude of an object in motion is a function of time H(t).

We shall determine object coordinates in the geographic system. Therefore the measurement axes of the accelerometers, which form the xyz accelerometer trihedron are conveniently oriented in parallel to the corresponding NHE coordinate trihedron axes, which are linked with the geographic coordinate system (see §1.2).

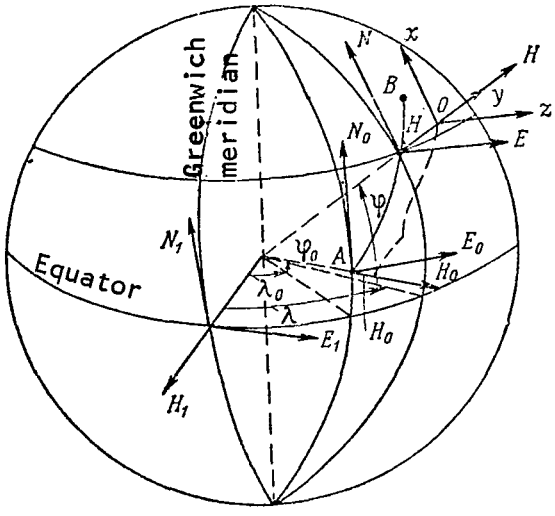

Figure 2.4

The H axis of the geographic trihedron is oriented along a perpendicular line (along the normal to the surface of the geoid), but the N and E axes are parallel to the tangents of the meridian and parallel, respectively at the point of location of the object on the surface of the Earth. /28

We shall designate the position of the NHE coordinate trihedron at the point corresponding to the origin of motion (point A) as $N_0H_0E_0$.

The instantaneous values of the kinematic parameters of object motion with respect to the surface of the Earth may be determined from the known values of the kinematic parameters of the motion of point O of the xyz and NHE coordinate trihedrons; therefore we shall first of all derive the kinematic equations of the motion of these trihedrons with respect to the surface of the Earth.

We shall introduce into the discussion the $N_1H_1E_1$ coordinate trihedron, which is linked with inertial space, with the origin at the point of the intersection of the Greenwich meridian with the equator ($\phi = 0$, $\lambda = 0$).

The position of the axes of this trihedron with respect to the surface of the Earth at the starting point is such that the H_1 axis is directed along

the perpendicular, but the N_1 and E_1 axes are tangent to the Greenwich meridian and to the equator, respectively.

The system accelerometer will measure acceleration caused by the difference in forces acting on the housing and on its responsive mass. Therefore the accelerometer will measure the acceleration of known gravitational forces, since the gravitational forces act identically on the housing and on the sensitive mass of the instrument [6], [36]:

$$\overline{A}=\left(\frac{d^2\overline{R}}{dt^2}\right)_a-\overline{g}_0(\overline{R}), \tag{2.14}$$

where $\overline{R}$ is the vector of the object position in the inertial coordinate system;

$(\frac{d^2\overline{R}}{dt^2})$ is absolute acceleration in the inertial coordinate system;

$\overline{g}_0(\overline{R})$ is the acceleration of the gravitational field of the Earth.

In differentiating the position vector in a system which is linked with the rotation of the Earth, we obtain the relative velocity

$$\overline{V}=\left(\frac{d\overline{R}}{dt}\right)_{ea}. \tag{2.15}$$

Here the velocity in the inertial coordinate system is

$$\left(\frac{d\overline{R}}{dt}\right)_a=\overline{V}_r+\overline{\Omega}\times\overline{R}, \tag{2.16}$$

where $\vec{\Omega}$ is the vector of the angular velocity of the Earth.
Correspondingly, the absolute acceleration is

$$\left(\frac{d^2\overline{R}}{dt^2}\right)_a=\left(\frac{d\overline{V}_r}{dt}\right)_a+\overline{\Omega}\times\left(\frac{d\overline{R}}{dt}\right)_a. \tag{2.17}$$

Keeping in mind that /29

$$\left(\frac{d\overline{V}_r}{dt}\right)_a=\left(\frac{d\overline{V}_r}{dt}\right)_{ea}+\overline{\Omega}\times\overline{V}_r \tag{2.18}$$

and taking (2.14), (2.17), into consideration, we obtain the vector of the acceleration of nongravitational forces

$$\overline{A}=\left(\frac{d\overline{V}_r}{dt}\right)_{ea}+2\overline{\Omega}\times\overline{V}_r+\overline{\Omega}\times(\overline{\Omega}\times\overline{R})-\overline{g}_0(\overline{R}) \tag{2.19}$$

or

$$\overline{A}+\overline{g}_0(\overline{R})=\left(\frac{d\overline{V}_r}{dt}\right)_{ea}+\overline{\Omega}\times(\overline{\Omega}\times\overline{R})+2\overline{\Omega}\times\overline{V}_r. \tag{2.19a}$$

The vector sum in the left side of the equation represents absolute acceleration and in the right we have the so-called relative, translational and Coriolis acceleration. For simplicity in notation we shall introduce the designations:

$$\left.\begin{aligned}\left(\frac{d\overline{V}_r}{dt}\right)_{ea}&=\overline{a}_r;\\ \overline{\Omega}\times(\overline{\Omega}\times\overline{R})&=\overline{a}_e;\\ 2\overline{\Omega}\times\overline{V}_r&=\overline{a}_c.\end{aligned}\right\} \tag{2.20}$$

Usually translational acceleration is combined vectorially with the acceleration of the forces of Earth attraction, thus forming the acceleration of gravity

$$\overline{g}=\overline{g}_0(\overline{R})-\overline{\Omega}\times(\overline{\Omega}\times\overline{R}). \tag{2.21}$$

Considering (2.20) and (2.21), we may rewrite equation (2.19a) in the form

$$\overline{A}=\overline{a}_r+\overline{a}_c-\overline{g}. \tag{2.22}$$

The velocity vector $\overline{V}_r$ of the motion of point O of the xyz coordinate trihedron with respect to the surface of the Earth is equal to (Figure 2.5)

$$\overline{V}_r = \overline{i}V_N + \overline{j}V_H + \overline{k}V_E, \tag{2.23}$$

where $\overline{i}$, $\overline{j}$, $\overline{k}$ are unit vectors of the corresponding NHE axes of the coordinate trihedron;
V_N, V_H, V_E are vector projections of $\overline{V}_r$ on the NHE axes of the trihedron.
Relative object acceleration

$$\overline{a}_r = \frac{d\overline{V}_r}{dt} \tag{2.24}$$

is determined by the equation /30

$$\frac{d\overline{V}_r}{dt} = \overline{i}\frac{dV_N}{dt} + \overline{j}\frac{dV_H}{dt} + \overline{k}\frac{dV_E}{dt} + V_N\frac{d\overline{i}}{dt} + V_H\frac{d\overline{j}}{dt} + V_E\frac{d\overline{k}}{dt}. \tag{2.25}$$

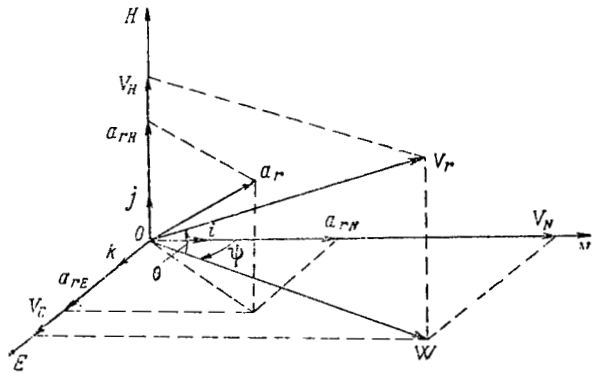

Figure 2.5

In the process of object motion, the xyz and NHE coordinate trihedrons will rotate with respect to the $N_0H_0E_0$ trihedron corresponding to the starting point with angular velocity $\overline{\omega}_r$, the projections of which on the NHE axes (Figure 2.6) will have the form

$$\left.\begin{aligned} \omega_{rN} &= \dot{\lambda}\cos\varphi; \\ \omega_{rH} &= \dot{\lambda}\sin\varphi; \\ \omega_{rE} &= -\dot{\varphi}, \end{aligned}\right\} \tag{2.26}$$

where ϕ is the geographical latitude of the point under consideration; $\dot{\lambda}$, $\dot{\phi}$ are velocities of the change in the geographic longitude and latitude of the point under consideration.

In turn,

$$\left.\begin{aligned}\dot{\lambda}&=\frac{V_E}{R_p+H\cos\varphi};\\ \dot{\varphi}&=\frac{V_N}{R_m+H},\end{aligned}\right\} \qquad (2.27)$$

where R_p and R_m are the radii of the curvature of the corresponding coordinate lines;
H is the altitude above sea level.

It should be noted that the analytical equation for the $R_m(\phi, \lambda)$ and $R_p(\phi, \lambda)$ dependencies are determined by the approximation taken as the figure of the Earth. /31

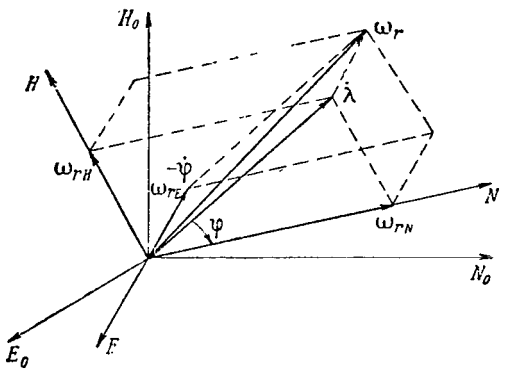

Figure 2.6

Derivatives of the unity vectors, which are included in the equation (2.25), will be determined in the following manner:

$$\left.\begin{aligned}\frac{d\bar{i}}{dt}&=-\bar{j}\frac{V_N}{R_m+H}-\bar{k}\frac{V_E\cdot\sin\varphi}{R_p+H\cos\varphi};\\ \frac{d\bar{j}}{dt}&=\bar{i}\frac{V_N}{R_m+H}+\bar{k}\frac{V_E\cos\varphi}{R_p+H\cos\varphi};\\ \frac{d\bar{k}}{dt}&=\bar{i}\frac{V_E\sin\varphi}{R_m+H\cos\varphi}-\bar{j}\frac{V_E\cos\varphi}{R_p+H\cos\varphi}.\end{aligned}\right\} \qquad (2.29)$$

After substituting derivative values from equations (2.29) in (2.25), we obtained

$$\left.\begin{aligned}a_{rN}&=\dot{V}_N+\frac{V_E^2\sin\varphi}{R_p+H\cos\varphi}+\frac{V_NV_H}{R_m+H};\\ a_{rH}&=\dot{V}_H-\frac{V_N^2}{R_m+H}-\frac{V_E^2\cos\varphi}{R_p+H\cos\varphi};\\ a_{rE}&=\dot{V}_E-\frac{V_EV_N\sin\varphi}{R_p+H\cos\varphi}-\frac{V_HV_E\cos\varphi}{R_m+H\cos\varphi},\end{aligned}\right\} \qquad (2.30)$$

where a_{rN}, a_{rH}, a_{rE} are vector projections of relative acceleration $\bar{a}_r$ on the NHE axes of the trihedron;
$\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$ are accelerations which characterize the motion of the xyz trihedron with respect to the surface of the Earth

It is apparent in equations (2.25) that the vector projection a_{rN}, a_{rH}, a_{rE} of relative acceleration comprise both the vector projections of accelerations $\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$, which characterize the motion of point O of the xyz coordinate trihedron with respect to the surface of the Earth, as well as additional accelerations caused by the rotation of this trihedron in the process of motion. Additional acceleration values are determined by the velocity vector components V_N, V_H, V_E, the geographic latitude ϕ, the radii R_p, R_m of the curvature of the coordinate lines of the surface of the Earth and by flight altitude H. /32

The vector projections of Coriolis acceleration on the NHE axes of the coordinate trihedron are determined by the relationships

$$\left.\begin{aligned} a_{cN} &= 2(\Omega_H V_E - \Omega_E V_H); \\ a_{cH} &= 2(\Omega_E V_N - \Omega_N V_E); \\ a_{cE} &= 2(\Omega_N V_H - \Omega_H V_N), \end{aligned}\right\} \qquad (2.31)$$

where Ω_N, Ω_H, Ω_E are vector projections of the angular velocity of Earth rotation on the corresponding NHE axes of the trihedron.

It is apparent from Figure 2.7 that

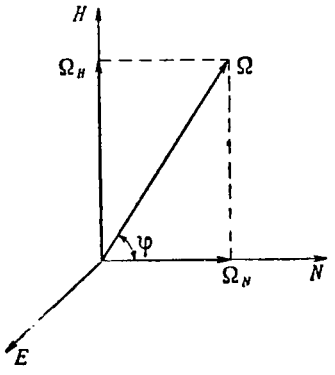

Figure 2.7

$$\left.\begin{aligned} \Omega_N &= \Omega \cos\varphi; \\ \Omega_H &= \Omega \sin\varphi; \\ \Omega_E &= 0. \end{aligned}\right\} \qquad (2.32)$$

Substituting these projection values Ω_N, Ω_H, Ω_E in equations (2.31), we obtain

$$\left.\begin{aligned} a_{cN} &= 2\Omega V_E \sin\varphi; \\ a_{cH} &= -2\Omega V_E \cos\varphi; \\ a_{cE} &= 2\Omega (V_H \cos\varphi - V_N \sin\varphi). \end{aligned}\right\} \qquad (2.33)$$

It is apparent from equations (2.33) that the vector projections a_{cN}, a_{cH}, a_{cE} of Coriolis acceleration depend on the velocity components of the motion of the NHE trihedron with respect to the surface of the Earth and on the geographic latitude.

In projecting the vector equality (2.22) on the NHE axes of the trihedron, we obtain

$$
\begin{aligned}
a_N &= \dot{V}_N + \frac{V_E^2 \sin\varphi}{R_p + H\cos\varphi} + \frac{V_N V_H}{R_m + H} + \\
&+ 2\Omega V_E \sin\varphi - g_N(\varphi, \lambda, H); \\
a_H &= \dot{V}_H - \frac{V_N^2}{R_m + H} - \frac{V_E^2 \cos\varphi}{R_p + H\cos\varphi} - \\
&- 2\Omega V_E \cos\varphi - g_H(\varphi, \lambda, H) \\
a_E &= \dot{V}_E - \frac{V_N V_E \sin\varphi}{R_p + H\cos\varphi} + \frac{V_H V_E \cos\varphi}{R_p + H\cos\varphi} + \\
&+ 2\Omega(V_H \cos\varphi - V_N \sin\varphi) - g_E(\lambda, \varphi, H).
\end{aligned}
\qquad (2.34)
$$

/33

In the process of object motion the NHE coordinate trihedron rotates in inertial space (with respect to the $N_1H_1E_1$ coordinate trihedron) with angular velocity $\overline{\omega}_r$, caused by Earth rotation Ω and by relative object motion $\overline{\omega}_r$. Therefore,

$$
\left.
\begin{aligned}
\omega_{aN} &= \omega_{rN} + \Omega_N; \\
\omega_{aH} &= \omega_{rH} + \Omega_H; \\
\omega_{aE} &= \omega_{rE} + \Omega_E.
\end{aligned}
\right\} \qquad (2.35)
$$

Taking into consideration the previously derived relationships (2.26) and (2.32), we obtain

$$
\left.
\begin{aligned}
\omega_{aN} &= (\dot{\lambda} + \Omega)\cos\varphi; \\
\omega_{aH} &= (\dot{\lambda} + \Omega)\sin\varphi; \\
\omega_{aE} &= -\dot{\varphi}.
\end{aligned}
\right\} \qquad (2.36)
$$

It is apparent from equations (2.36) that the ω_{aN}, ω_{aH}, ω_{aE} components are functions of geographic latitude, angular velocity of Earth rotation and of derivatives of the geographic coordinates.

In order to stabilize the xyz measurement axes of the accelerometers along the NHE axes of the trihedron, it is necessary in the process of object motion that the xyz trihedron continuously rotate with respect to inertial space with an angular velocity the projection of which is determined by equations (2.36).

In the design of a platform which carries accelerometers for the inclination of the accelerometer measurement axes it is necessary to provide for devices which materialize the inertial coordinate trihedron in space, and for inclination devices.

Gyroscopic devices are employed for the former, which permit fixing the position of the inertial coordinate trihedron with accuracy limited only by inherent drift.

In order to incline the platform carrying the accelerometers in an INS, special tracking systems or devices for the correction of gyroscopic instruments have found application in the form of moment sensors.

At the present time the following kinematic design versions are employed in INS's of this class:

- The version with a fixed platform P_1 (Figure 2.8) in inertial space, carrying [10], [11] a supplementary Cardan joint for the platform which supports accelerometers A_x, A_y, A_z. In addition, the system contains a calculator (VM) for the determination of the instantaneous values of the true parameters of object motion and for the formulation of correction signals /34 which are applied to the input of tracking systems $CC_{\gamma a}$, $CC_{\vartheta a}$, $CC_{\psi a}$ for the inclination of the accelerometer platform;

- The version with three-way stabilized platform (Fig. 2.9) which supports the accelerometers. Platform inclination is accomplished by the application of corrected moments from the corresponding sensors $DM_{\psi a}$, $DM_{\vartheta a}$, $DM_{\gamma a}$. The values of these moments must correspond to the angular velocity components $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$ of platform inclination in inertial space.

We note that the system accelerometers, regardless of the design version, are in a Cardan joint, and therefore the correction signals for both design versions must correspond to the angular velocities $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$ of the inclination of the gimbals with respect to the corresponding axes.

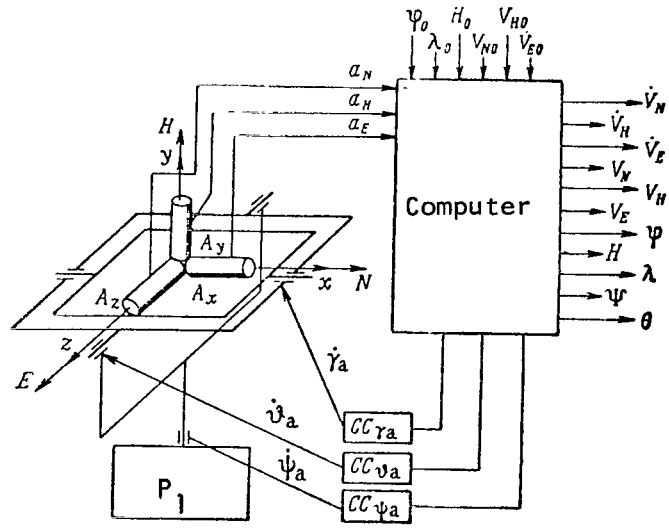

Figure 2.8. INS Design with Three Cardan Rings.

/35

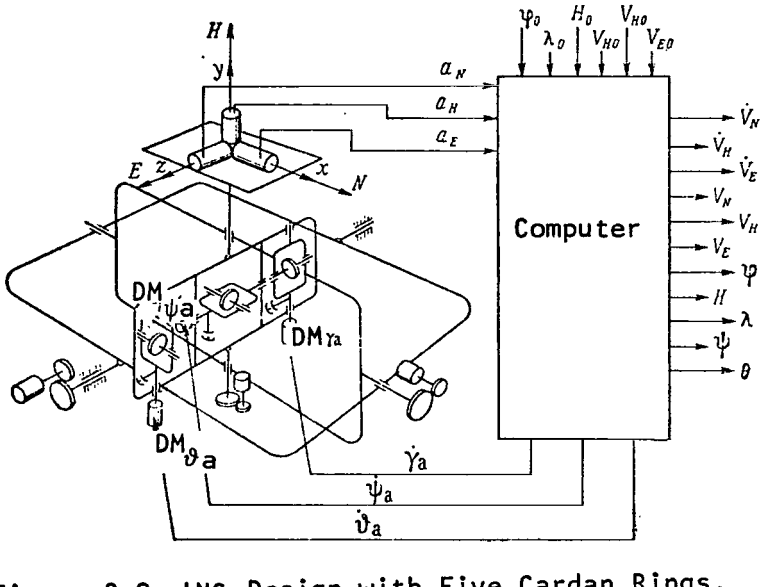

Figure 2.9. INS Design with Five Cardan Rings.

These velocities may be determined from the values ω_{aN}, ω_{aH}, ω_{aE} (Figure 2.10) by means of solving the well-known system of differential equations

$$\left.\begin{aligned}\dot{\psi}_a &= \frac{1}{\cos\vartheta_a}(\omega_{aN}\cos\gamma_a - \omega_{aE}\sin\gamma_a);\\ \dot{\vartheta}_a &= \omega_{aH}\sin\gamma_a + \omega_{aE}\cos\gamma_a;\\ \dot{\gamma}_a &= \omega_{aN} - (\omega_{aH}\cos\gamma_a - \omega_{aE}\sin\gamma_a)\tan\vartheta_a.\end{aligned}\right\} \qquad (2.37)$$

The system of INS kinematic equations for both design versions indicated will contain equations (2.34), (2.27), (2.36), (2.37) /36

$$\left.\begin{aligned}\dot{V}_N &= a_N - \frac{V_E^2\sin\varphi}{R_p + H\cos\varphi} - \frac{V_N V_H}{R_m + H} - \\ &\quad - 2\Omega V_E\sin\varphi + g_N(\varphi, \lambda, H);\\ \dot{V}_H &= a_H + \frac{V_N^2}{R_m + H} + \frac{V_E^2\cos\varphi}{R_p + H\cos\varphi} + \\ &\quad + 2\Omega V_E\cos\varphi + g_H(\varphi, \lambda, H);\\ \dot{V}_E &= a_E + \frac{V_N V_E\sin\varphi}{R_p + H\cos\varphi} - \frac{V_H V_E\cos\varphi}{R_p + H\cos\varphi} - \\ &\quad - 2\Omega(V_H\cos\varphi - V_N\sin\varphi) + g_E(\varphi, \lambda, H);\end{aligned}\right\} \qquad (2.38)$$

$$
\begin{aligned}
&\dot{\varphi} = \frac{V_N}{R_m + H};\\
&\dot{H} = V_H;\\
&\dot{\lambda} = \frac{V_E}{R_p + H\cos\varphi};\\
&\dot{\psi}_a = \frac{1}{\cos\vartheta_a}[(\dot{\lambda}+\Omega)\cos\varphi\cos\gamma_a + \dot{\varphi}\sin\gamma_a];\\
&\dot{\vartheta}_a = (\dot{\lambda}+\Omega)\sin\varphi\sin\gamma_a - \dot{\varphi}\cos\gamma_a;\\
&\dot{\gamma}_a = (\dot{\lambda}+\Omega)\cos\varphi - [(\dot{\lambda}+\Omega)\sin\varphi\cos\gamma_a - \dot{\varphi}\sin\gamma_a]\tan\vartheta_a.
\end{aligned}
\tag{2.38}
$$

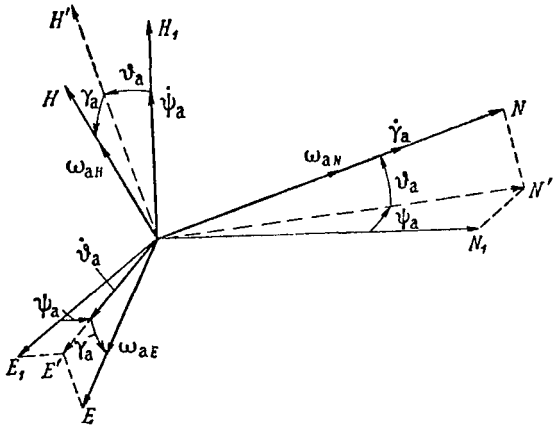

Figure 2.10

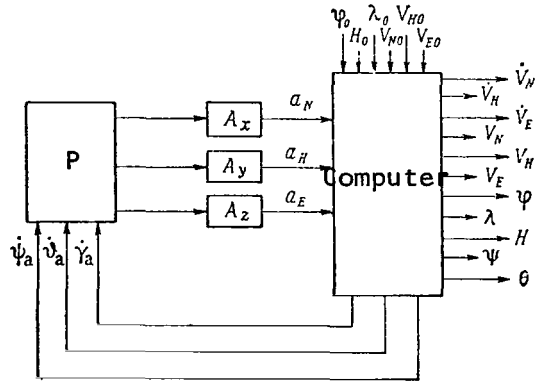

Figure 2.11. Block Diagram of a Three-Component INS.

It is easy to see that this system of nonlinear differential equations has crossover connections along all three velocity projections V_N, V_H, V_E of the motion of point O of the xyz accelerometer trihedron on the NHE axes of the geographic coordinate trihedron, and also along latitude ϕ and altitude H. Therefore in a general case the INS must be a three-component system. The block diagram of the INS under consideration is shown in Figure 2.11.

The system of kinematic equations (2.38) is correct for any approximation of the shape of the surface of the Earth. Instantaneous values of the vector projections of the acceleration of gravity $g_N(\phi, \lambda, H)$, $g_H(\phi, \lambda, H)$ $g_E(\phi, \lambda, H)$ are determined depending on how the figure of the Earth is approximated (see Chapter I), with consideration for gravity anomalies according to the area gravimetric map.

The analytical relationships of the radii of the curvature of the parallel $R_p(\phi, \lambda)$ and the meridian $R_m(\phi, \lambda)$ are determined in a similar fashion.

Solution of the (2.38) system by means of an on-board computer permits a determination to be made of the instantaneous values of the velocity vector projections V_N, V_H, V_E, of the geographic coordinates ϕ, λ, of the altitude H of the object in motion above the surface of the Earth, and also of $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$ data for platform correction. Accelerometer indications a_N, a_H, a_E and initial values V_{N0}, V_{H0}, V_{E0}, ϕ_0, λ_0, H_0 of the true kinematic parameters serve as basic data for the solution of system (2.38). /37

In addition, the instantaneous values of course angle Ψ and trajectory angle θ (Figure 2.12) may be determined from known values V_N, V_H and V_E.

$$\Psi = \arcsin \frac{V_E}{W}; \qquad \theta = \arcsin \frac{V_H}{V_r}, \tag{2.39}$$

where

$$W = \sqrt{V_N^2 + V_E^2};$$
$$V_r = \sqrt{V_N^2 + V_H^2 + V_E^2}.$$

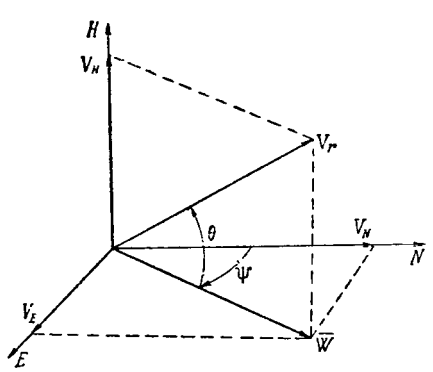

Figure 2.12

The value W is called the modulus of the ground speed vector. In analyzing the system (2.38), it is not difficult to see that at high latitudes the denominator of the λ expression approaches 0 and causes difficulty in the determination of longitude. In addition, at high latitudes and for the usual object velocities, moments for platform inclination become very large. This limits the application of the geographic coordinate system at high latitudes for INS.

Having chosen the arbitrary coordinate system in place of the geographic system, we may eliminate the objections noted and at the same time provide for the employment of the INS at high latitudes. /38

§2.3. Kinematic INS Equations in the Arbitrary Coordinate System

Let us assume that a certain object is in motion with respect to the surface of the Earth from point A to point B (Figure 2.13).

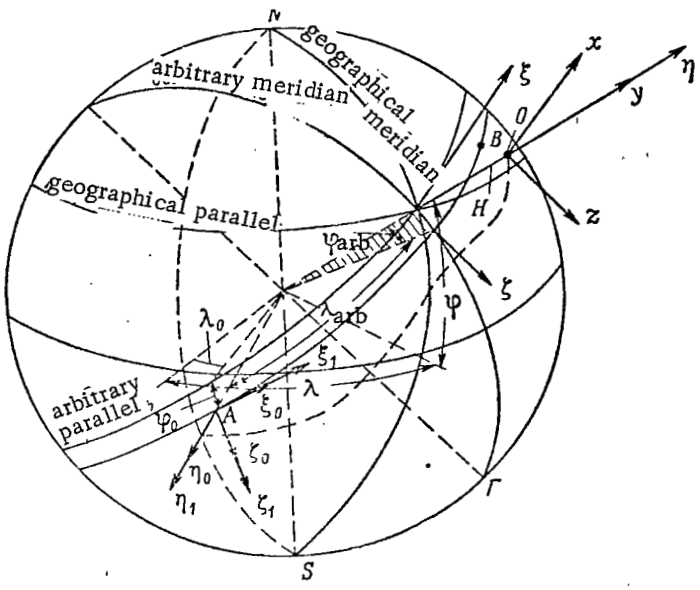

Figure 2.13

As in §2.2, we shall introduce into the examination the xyz accelerometer trihedron, the origin of which is located at the center of mass of the object.

We must derive the equation of motion of the origin (point O) of the accelerometer trihedron with respect to the surface of the Earth.

We shall determine the location of point O with respect to the surface of the Earth in the arbitrary coordinate system.

In addition to the accelerometer trihedron, we shall in the future also employ the following coordinate trihedrons (see Fig. 2.13):

the $\xi_1\eta_1\zeta_1$ inertial, and the $\xi\eta\zeta$ arbitrary coordinate trihedrons (in analogy with §1.2).

We shall designate the position of the arbitrary trihedron at a point taken as the origin (for example, point A) as ξ_0, η_0, ζ_0.

The $\xi_1\eta_1\zeta_1$ axes of the inertial trihedron are oriented along the corresponding ξ_0, η_0, ζ_0 axes of the trihedron at the starting point and subsequently the position of the ξ_1, η_1, ζ_1 axes remains fixed in inertial space, whereas the ξ_0, η_0, ζ_0 coordinate trihedron rotates in inertial space with the angular velocity of the Earth's rotation. /39

In the process of object motion, the xyz axes of the trihedron are stabilized in parallel with the corresponding axes of the arbitrary trihedron. The derivation of the kinematic equations is similar to that set forth in the preceding chapter. However in this case the vector equality (2.22) must be projected on the axes of the arbitrary trihedron.

The relative velocity vector of the motion of the origin of the accelerometer trihedron (Figure 2.14) is determined by the equation

$$\bar{V}_r = \bar{i}V_\xi + \bar{j}V_\eta + \bar{k}V_\zeta,$$

where $\bar{i}$, $\bar{j}$, $\bar{k}$ are unit vectors of the arbitrary trihedron axes; V_ξ, V_η, V_ζ are vector projections of $\bar{V}_r$ on the corresponding axes of the arbitrary trihedron.

Relative acceleration is written in the form

$$\frac{d\bar{V}_r}{dt} = \bar{i}\frac{dV_\xi}{dt} + \bar{j}\frac{dV_\eta}{dt} + \bar{k}\frac{dV_\zeta}{dt} + V_\xi\frac{d\bar{i}}{dt} + V_\eta\frac{d\bar{j}}{dt} + V_\zeta\frac{d\bar{k}}{dt}. \qquad (2.40)$$

Since in the process of motion the $\xi\eta\zeta$ coordinate trihedron continuously rotates with respect to its original position $\xi_0\eta_0\zeta_0$ with angular velocity ω_r, angular velocity projections on the axes of the arbitrary trihedron (see Figure 2.14) correspondingly equal /40

$$\left.\begin{aligned} \omega_{r\xi} &= -\dot{\varphi}_{arb} \\ \omega_{r\eta} &= \dot{\lambda}_{arb}\sin\varphi_{arb} \\ \omega_{r\zeta} &= -\dot{\lambda}_{arb}\cos\varphi_{arb} \end{aligned}\right\} \qquad (2.41)$$

where

$$\left.\begin{aligned} \dot{\lambda}_{arb} &= \frac{V_\xi}{R_{arb,p} + H\cos\varphi_{arb}}; \\ \dot{H} &= V_\eta; \\ \dot{\varphi}_{arb} &= -\frac{V_\zeta}{R_{arb,m} + H}; \end{aligned}\right\} \qquad (2.42)$$

$R_{arb,p}$ and $R_{arb,m}$ are the radii of the curvature of the arbitrary coordinate lines and the derivatives of the unit vectors with respect to time are determined by the equations

$$\left.\begin{aligned} \frac{d\bar{i}}{dt} &= -\bar{j}\frac{V_\xi\cos\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} - \bar{k}\frac{V_\xi\sin\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}}; \\ \frac{d\bar{j}}{dt} &= \bar{i}\frac{V_\xi\cos\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} + \bar{k}\frac{V_\zeta}{R_{arb,m} + H}; \\ \frac{d\bar{k}}{dt} &= \bar{i}\frac{V_\xi\sin\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} - \bar{j}\frac{V_\zeta}{R_{arb,m} + H}. \end{aligned}\right\} \qquad (2.43)$$

By substituting the values of the unit vector derivatives from system (2.43) in equation (2.40) and by grouping the terms under identical unit vectors, we obtain the equations for the vector projections of relative acceleration

$$\left.\begin{aligned} a_{r\xi} &= \dot{V}_\xi + \frac{V_\xi V_\zeta \sin\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} + \frac{V_\xi V_\eta \cos\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}};\\ a_{r\eta} &= \dot{V}_\eta - \frac{V_\xi^2 \cos\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} - \frac{V_\xi^2}{R_{arb,m} + H};\\ a_{r\zeta} &= \dot{V}_\zeta - \frac{V_\xi^2 \sin\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} + \frac{V_\xi V_\zeta}{R_{arb,m} + H}. \end{aligned}\right\} \qquad (2.44)$$

It is apparent from the system (2.44) that the vector projections of relative acceleration, in addition to accelerations V_ξ, V_η, V_ζ, which characterize the motion of the accelerometer trihedron with respect to the surface of the Earth, include additional accelerations, caused by the rotation of the arbitrary trihedron with respect to its initial position $\xi_0\eta_0\zeta_0$.

The vector projections of Coriolis acceleration on the axes of the $\xi\eta\zeta$ coordinate system are determined by the following relationships /41

$$\left.\begin{aligned} a_{c\xi} &= 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta);\\ a_{c\eta} &= 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta);\\ a_{c\zeta} &= 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi), \end{aligned}\right\} \qquad (2.45)$$

where Ω_ξ, Ω_η, Ω_ζ are vector projections of the angular velocity of the rotation of the Earth (Figure 2.15) on the corresponding axes of the arbitrary trihedron.

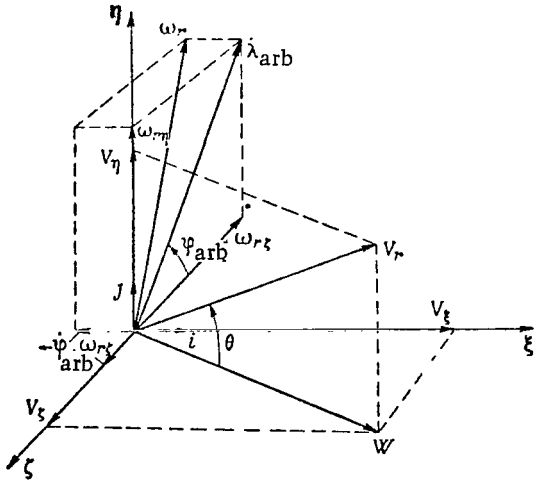

Figure 2.14

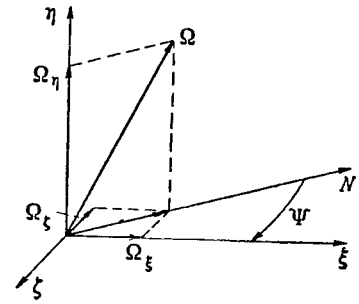

Figure 2.15

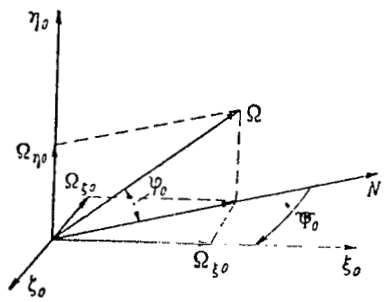

Figure 2.16

The values Ω_ξ, Ω_η, Ω_ζ are uniquely determined by their values at the point of reference (Figure 2.16)

$$\left.\begin{aligned}\Omega_{\xi 0} &= \Omega\cos\varphi_0\cos\Psi_0;\\ \Omega_{\eta 0} &= \Omega\sin\varphi_0;\\ \Omega_{\zeta 0} &= -\Omega\cos\varphi_0\sin\Psi_0\end{aligned}\right\} \qquad (2.46)$$

and by the instantaneous values of the arbitrary coordinates (Figure 2.17):

$$\left.\begin{aligned}\Omega_\xi &= \Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0}\sin\lambda_{arb}\\ \Omega_\eta &= (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\cos\varphi_{arb} - \Omega_{\zeta 0}\sin\varphi_{arb}\\ \Omega_\zeta &= (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\sin\varphi_{arb} + \Omega_{\zeta 0}\cos\varphi_{arb}\end{aligned}\right\} \qquad (2.47)$$

where ϕ_0, ψ_0 represent the geographic latitude and the course angle of the arbitrary equator at the point of reference.

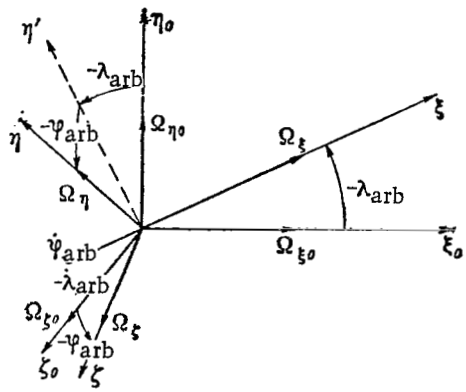

Figure 2.17

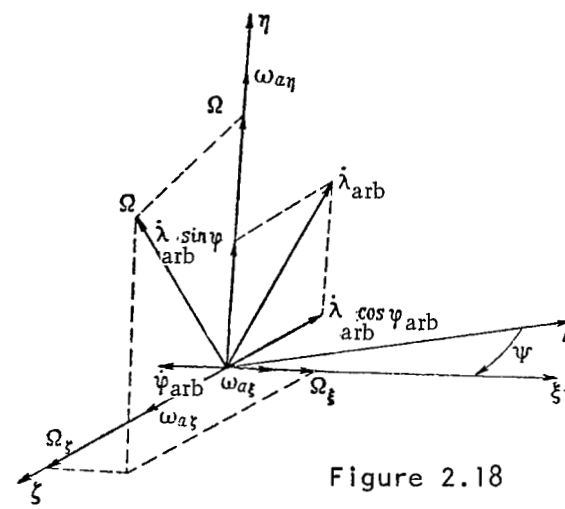

Figure 2.18

/42

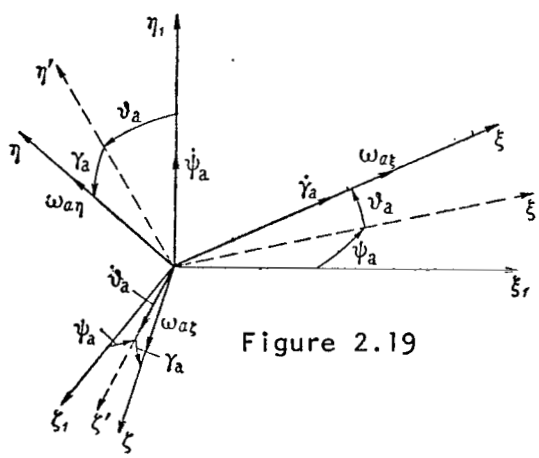

Figure 2.19

/43

For the stabilization of the accelerometer trihedron along the axes of the arbitrary coordinate trihedron, it is necessary that the measurement axes of the accelerometers in the process of object motion be inclined in inertial space with an absolute angular velocity, the projections of which on the axes of the arbitrary trihedron (Figure 2.18) equal, respectively

$$\left.\begin{aligned}\omega_{a\xi} &= \Omega_\xi - \dot{\varphi}_{arb}\\ \omega_{a\eta} &= \Omega_\eta + \dot{\lambda}_{arb}\sin\varphi_{arb}\\ \omega_{a\zeta} &= \Omega_\zeta - \dot{\lambda}_{arb}\cos\varphi_{arb}\end{aligned}\right\} \qquad (2.48)$$

The moving coordinates λ_{arb}, H, ϕ_{arb} of the arbitrary trihedron may be found from equation (2.42) with known initial parameter values $\lambda_{arb\ 0}$, H_0, $\phi_{arb\ 0}$.

Therefore the design formulation of the INS may be the same as previously specified, but the measurement axes of the accelerometers must be oriented not in the geographic coordinate system, but in the arbitrary system.

In this connection inclination devices must rotate the accelerometer platform (Figure 2.19) with respect to the Cardan joint axes with angular velocities:

$$\left.\begin{aligned}\dot{\psi}_a &= \frac{1}{\cos\vartheta_a}(\omega_{a\eta}\cos\gamma_a - \omega_{a\zeta}\sin\gamma_a);\\ \dot{\vartheta}_a &= \omega_{a\eta}\sin\gamma_a + \omega_{a\zeta}\cos\gamma_a;\\ \dot{\gamma}_a &= \omega_{a\xi} - (\omega_{a\eta}\cos\gamma_a - \omega_{a\zeta}\sin\gamma_a)\tan\vartheta_a.\end{aligned}\right\} \qquad (2.49)$$

The system of kinematic INS equations in the arbitrary coordinate system /44 is similar to the system (2.38):

$$\left.\begin{aligned}\dot{V}_\xi &= a_\xi - \frac{V_\xi V_\zeta \sin\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} - \frac{V_\xi V_\eta \cos\varphi_{arb}}{R_{arb,m} + H\cos\varphi_{arb}} - \\ &\quad - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta) + g_\xi(\varphi_{arb}, \lambda_{arb}, H);\\ \dot{V}_\eta &= a_\eta + \frac{V_\xi^2\cos\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} + \frac{V_\zeta^2}{R_{arb,m} + H} - \\ &\quad - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta(\varphi_{arb}, \lambda_{arb}, H);\\ \dot{V}_\zeta &= a_\zeta + \frac{V_\xi^2\sin\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} - \frac{V_\eta V_\zeta}{R_{arb,m} + H} - \\ &\quad - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi) + g_\zeta(\varphi_{arb}, \lambda_{arb}, H);\\ \dot{H} &= V_\eta;\\ \dot{\lambda}_{arb} &= \frac{V_\xi}{R_{arb,p} + H\cos\varphi_{arb}};\\ \dot{\varphi}_{arb} &= -\frac{V_\zeta}{R_{arb,m} + H};\end{aligned}\right\} \qquad (2.50)$$

$$
\begin{aligned}
\dot{\psi}_a &= \frac{1}{\cos\vartheta_a}\left[\Omega_\eta\cos\gamma_a - \Omega_\zeta\sin\gamma_a + \dot{\lambda}_{arb}\sin(\varphi_{arb} + \gamma_a)\right];\\
\dot{\vartheta}_a &= (\Omega_\eta + \dot{\lambda}_{arb}\sin\varphi_{arb})\sin\gamma_a + (\Omega_\zeta - \dot{\lambda}_{arb}\cos\varphi_{arb})\cos\gamma_a;\\
\dot{\gamma}_a &= \Omega_\xi - \dot{\varphi}_{arb} - \left[\Omega_\eta\cos\gamma_a - \Omega_\zeta\sin\gamma_a + \dot{\lambda}_{arb}\sin(\varphi_{arb} + \gamma_a)\right]\tan\vartheta_a;\\
\Omega_\xi &= \Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0}\sin\lambda_{arb}\\
\Omega_\eta &= (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\cos\varphi_{arb} - \Omega_{\zeta 0}\sin\varphi_{arb}\\
\Omega_\zeta &= (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\sin\varphi_{arb} + \Omega_{\zeta 0}\cos\varphi_{arb}\\
\Omega_{\xi 0} &= \Omega\cos\varphi_0\cos\Psi_0;\\
\Omega_{\eta 0} &= \Omega\sin\varphi_0;\\
\Omega_{\zeta 0} &= -\Omega\cos\varphi_0\sin\Psi_0.
\end{aligned}
\qquad (2.50)
$$

It is apparent that the system (2.50) of kinematic INS equations is nonlinear and that it has cross links with the velocity vector projections V_ξ, V_η, V_ζ, with the arbitrary coordinates and with the projections of the angular velocity of rotation Ω_ξ, Ω_η, Ω_ζ of the arbitrary trihedron in inertial space, and therefore the INS must be constructed in accordance with the three-component design.

Analytical expressions for the radii of the curvature of the arbitrary coordinate lines are determined depending on the approximation of the figure of the Earth taken (see Chapter I). Projections of the acceleration of gravity g_ξ (λ, ϕ, H); g_η (λ, ϕ, H); g_ζ (λ, ϕ, H) on the axes of the arbitrary trihedron may be determined in a similar fashion.

The solution of the system of INS kinematic equations according to a_ξ, a_η, a_ζ data obtained from the accelerometers and with known initial true parameter values $V_{\xi 0}$, $V_{\eta 0}$, $V_{\zeta 0}$; $\lambda_{arb\ 0}$, $\phi_{arb\ 0}$, H_0 permits a determination to be made of the following parameters of object motion with respect to the surface of the Earth: velocity vector components V_ξ, V_η, V_ζ, arbitrary coordinates ϕ_{arb}, λ_{arb} and altitude H. The employment of a platform stabilized in the plane of the horizon and in azimuth along the arbitrary equator makes it possible to obtain also yaw angle ψ, pitch angle ϑ, and angle of bank γ. The block diagram of the INS under consideration as shown in Figure 2.20. /45

In addition, whenever required it is possible to determine instantaneous values of the course angle Ψ and trajectory angle θ, which determine the angular position of the speed vector for object motion with respect to the surface of the Earth. For this purpose it is necessary to solve the following additional system of equations:

$$\left.\begin{aligned}
\dot{\Psi} &= \dot{\lambda}_{arb}(\sin\varphi_{arb} - \cos\varphi_{arb}\sin\Psi\tan\varphi) + \dot{\varphi}_{arb}\cos\Psi\tan\varphi;\\
\dot{\varphi} &= \dot{\lambda}_{arb}(\sin\varphi_{arb} - \cos\varphi_{arb}\cos\Psi) + \dot{\varphi}_{arb}\sin\Psi;\\
\dot{\lambda} &= \frac{1}{\cos\varphi}(\dot{\lambda}_{arb}\cos\varphi_{arb}\sin\Psi - \dot{\varphi}_{arb}\cos\Psi);\\
\theta &= \arcsin\frac{V_\eta}{\sqrt{V_\xi^2 + V_\eta^2 + V_\zeta^2}}.
\end{aligned}\right\} \qquad (2.51)$$

It should be noted that through an appropriate choice of course angle Ψ_0, it is possible to provide sufficiently small values ϕ_{arb}, which, in turn, permit the employment of the INS for object navigation in any areas of the Earth, including the areas of the North and South poles. /46

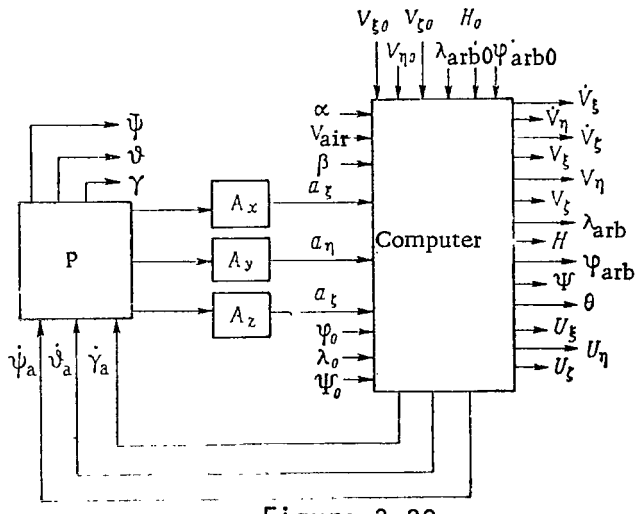

Figure 2.20

The inertial navigation system permits determination to be made not only of the motion parameters of an object with respect to the surface of the Earth, but also the motion parameters of the medium, for example, the wind. For this purpose it is necessary to introduce into the computer of the system instantaneous values of object speed with respect to the medium (for example, true air speed V_{air}) and instantaneous values of the angular position of the object with respect to the medium (for example, angles of attack α and slip β). The vector projections of the speed of the medium with respect to the surface of the Earth (of the wind U_ξ, U_η, U_ζ) on the axes of the arbitrary trihedron equal, respectively,

$$\left.\begin{aligned}
U_\xi &= V_\xi - V_{air\,\xi};\\
U_\eta &= V_\eta - V_{air\,\eta};\\
U_\zeta &= V_\zeta - V_{air\,\zeta},
\end{aligned}\right\} \qquad (2.52)$$

where $V_{air\ \xi}$, $V_{air\ \eta}$, $V_{air\ \zeta}$ are vector projections of object true air speed.

The values V_ξ, V_η, V_ζ are determined by the inertial system, but the values of the true air speed projections may be determined from the following equations (Figure 2.21):

$$\left.\begin{aligned}
V_{\text{air}\xi} &= V_{\text{air}\,x}\cos\psi\cos\vartheta + V_{\text{air}\,y}(\sin\psi\sin\gamma - \\
&\quad -\cos\psi\sin\vartheta\cos\gamma) + V_{\text{air}z}\sin\psi\cos\gamma; \\
V_{\text{air}\eta} &= V_{\text{air}x}\sin\vartheta + V_{\text{air}y}\cos\vartheta\cos\gamma - \\
&\quad -V_{\text{air}\,z}\cos\vartheta\sin\gamma; \\
V_{\text{air}\zeta} &= -V_{\text{air}x}\sin\psi\cos\vartheta + V_{\text{air}\,y}(\cos\psi\sin\gamma + \\
&\quad +\sin\psi\sin\vartheta\cos\gamma) + V_{\text{air}z}\cos\psi\cos\gamma.
\end{aligned}\right\} \qquad (2.53)$$

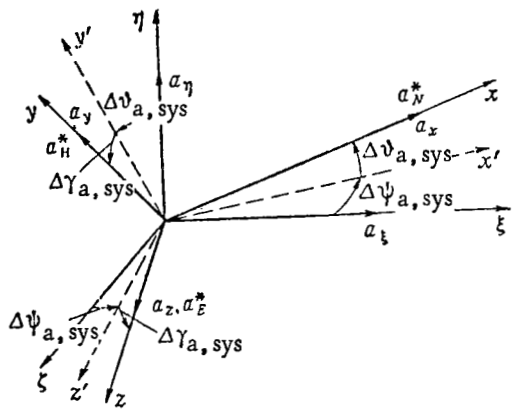

Figure 2.21

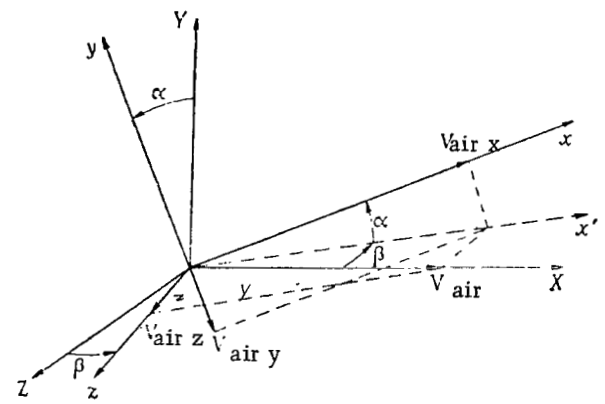

Figure 2.22

In turn, it is apparent from Fig. 2.22 that

$$\left.\begin{aligned}
V_{\text{air}^x} &= V_{\text{air}}\cos\beta\cos\alpha; \\
V_{\text{air}^y} &= -V_{\text{air}}\cos\beta\sin\alpha; \\
V_{\text{air}z} &= V_{\text{air}}\sin\beta.
\end{aligned}\right\} \qquad (2.54)$$

If during flight on the basic portion of the object trajectory the values of the angles α, β, ψ, ϑ, γ are sufficiently small (units of degrees), then equations (2.53) and (2.54) may be simplified to the form

$$\left.\begin{aligned}
V_{\text{air}\xi} &\cong V_{\text{air}}(1+\alpha\vartheta+\beta\psi); \\
V_{\text{air}^\eta} &\cong V_{\text{air}}(\vartheta-\alpha-\beta\gamma); \\
V_{\text{air}^\zeta} &\cong V_{\text{air}}(-\psi-\alpha\gamma+\beta).
\end{aligned}\right\} \qquad (2.55)$$

Under conditions where the angles ψ, ϑ, γ are small, the derivatives within the parentheses are relatively small and the system (2.55) may be additionally simplified to the form /48

$$\left.\begin{aligned} V_{\text{air}\xi} &\cong V_{\text{air}} \\ V_{\text{air}\eta} &\cong V_{\text{air}}(\vartheta - \alpha); \\ V_{\text{air}\zeta} &\cong V_{\text{air}}(\beta - \psi). \end{aligned}\right\} \tag{2.56}$$

Then the values of the vector projections of the speed of the medium with respect to the surface of the Earth on the axes of the arbitrary trihedron may, in a first approximation, be determined from the equations

$$\left.\begin{aligned} U_\xi &\cong V_\xi - V_{\text{air}} \\ U_\eta &\cong V_\eta - V_{\text{air}}(\vartheta - \alpha); \\ U_\zeta &\cong V_\zeta - V_{\text{air}}(\beta - \psi), \end{aligned}\right\} \tag{2.57}$$

which when required may be solved by the computer of the INS.

§2.4. Simplification of the INS Kinematic Equations

The kinematic equations (2.38), (2.50) for the INS are rather complex and may be simplified as applied to specific objects, but this leads to the appearance of systematic errors.

It is advantageous to simplify the kinematic equations for the INS after the basic elements of the INS (accelerometers, gyroscopes, computer elements) have been selected. Selection of ISN elements may be accomplished after an approximate error calculation for the simplest case of a single-component ISN in accordance with the equations derived in §2.1.

The simplification of INS equations is advantageous until systematic error caused by these simplifications coincides with dead zones or with the zero drift of the appropriate elements [15].

In fact, the construction of an INS in accordance with the complete system of INS kinematic equations (which would provide minimal values of systematic error) cannot be considered as advantageous if the dead zones of the system elements or zero drift exceeds the values of systematic error.

. .For.example, to calculate the vector components of relative acceleration V_ξ, V_η, V_ζ with an accuracy to 10^{-6}g has no meaning if the INS accelerometer dead zones lie within the limits 10^{-4}g - 10^{-5}g, or to calculate angular velocities ψ_a, ϑ_a, $\dot{\gamma}_a$ of the inclination of the accelerometer measurement axes with an accuracy to 10^{-7} 1/sec has no meaning if inherent drift of the gyroscopic elements of the INS platform amounts to 10^{-6} 1/sec. /49

The simplification technique is reduced to the following. Each of the equations which is included in the system of INS kinematic equations is simplified. The removal of the simplification of any of the terms included in the right sides of these equations causes corresponding systematic error,

the value of which is determined by the difference between the right sides of the simplified and the full equations.

Further, we shall compare the values of systematic errors in the determination of the parameters on the left side of the INS equations with the values of the dead zones or the zero drift zones of corresponding system elements (accelerometers, gyroscopes, etc.). In this manner we shall establish the permissible simplification limits.

We shall employ the technique indicated for a case of an INS with a platform stabilized in the geographic coordinate system.

For objects in motion along a trajectory known in advance and established by the values $\phi_{pro}(t)$, $H_{pro}(t)$, $\lambda_{pro}(t)$, and with a control system which stabilizes motion along the trajectory with sufficient accuracy, the projections of the acceleration of gravity $g_{Npro}(\phi_{pro}, H_{pro}, \lambda_{pro})$, $g_{Hpro}(\phi_{pro}, H_{pro}, \lambda_{pro})$ and the radii of the curvature of the coordinate lines $R_{m\,pro}(\phi_{pro}, \lambda_{pro})$, $R_{p\,pro}(\phi_{pro}, \lambda_{pro})$ may be calculated in advance in accordance with the corresponding equations of Chapter I. With deviation of the true trajectory from the programmed trajectory, partial systematic errors will occur:

$$
\left.
\begin{aligned}
\delta_{sys}\dot{V}_N &= -\frac{V_E^2 \sin\varphi \cdot \Delta R_p}{(R_{p\,pro} + H\cos\varphi)(R_p + H\cos\varphi)} - \\
&\quad - \frac{V_N V_H \Delta R_m}{(R_{m\,pro} + H)(R_m + H)} + \Delta g_{N\,pro} \\
\delta_{sys}\dot{V}_H &= \frac{V_N^2 \cdot \Delta R_m}{(R_{p.\,pro} + H)(R_m + H)} + \\
&\quad + \frac{V_E^2 \cos\varphi \Delta R_m}{(R_{p.\,pro} + H\cos\varphi)(R_m + H\cos\varphi)} + \Delta g_{H\,pro} \\
\delta_{sys}\dot{V}_E &= \frac{V_N V_E \sin\varphi \Delta R_p}{(R_{p\,pro} + H\cos\varphi)(R_p + H\cos\varphi)} - \\
&\quad - \frac{V_H V_E \cos\varphi \Delta R_p}{(R_{p.\,pro} + H\cos\varphi)(R_p + H\cos\varphi)} + \Delta g_{E\,pro} \\
\delta_{sys}\dot{\varphi} &= \frac{V_N}{(R_{m\,pro} + H)(R_m + H)} \Delta R_m \\
\delta_{sys}\dot{\lambda} &= \frac{V_E}{(R_{p.\,pro} + H\cos\varphi)(R_p + H\cos\varphi)} \Delta R_p
\end{aligned}
\right\} \qquad (2.58)
$$

where

/50

$$\left.\begin{aligned}
\Delta g_{N_{\text{pro}}} &= g_{N_{\text{pro}}}(\varphi_{\text{pro}}, H_{\text{pro}}, \lambda_{\text{pro}}) - g_N(\varphi, H, \lambda);\\
\Delta g_{H_{\text{pro}}} &= g_{H_{\text{pro}}}(\varphi_{\text{pro}}, H_{\text{pro}}, \lambda_{\text{pro}}) - g_H(\varphi, H, \lambda);\\
\Delta g_{E_{\text{pro}}} &= g_{E_{\text{pro}}}(\varphi_{\text{pro}}, H_{\text{pro}}, \lambda_{\text{pro}}) - g_E(\varphi, H, \lambda);\\
\Delta R_{\text{M}} &= R_{\text{m. pro}}(\varphi_{\text{pro}}, \lambda_{\text{pro}}) - R_{\text{M}}(\varphi, \lambda);\\
\Delta R_{\text{pro}} &= R_{\text{pro}}(\varphi_{\text{pro}}, \lambda_{\text{pro}}) - R_{\text{p}}(\varphi, \lambda).
\end{aligned}\right\} \qquad (2.59)$$

We note that the partial systematic errors are determined as the difference between the corresponding full and simplified kinematic equations.

With sufficiently small ΔR_p and ΔR_m, we may consider that

$$\left.\begin{aligned}
(R_{\text{mpro}} + H)(R_{\text{m}} + H) &\cong (R_{\text{mpro}} + H)^2;\\
(R_{\text{mpro}} + H\cos\varphi)(R_{\text{p}} + H\cos\varphi) &\cong (R_{\text{ppro}} + H\cos\varphi)^2.
\end{aligned}\right\} \qquad (2.60)$$

Then we obtain

$$\left.\begin{aligned}
\delta_{\text{sys}}\dot{V}_N &\cong -\frac{V_E^2 \sin\varphi}{(R_{\text{ppro}} + H\cos\varphi)^2}\Delta R_{\text{p}} - \frac{V_N V_H}{(R_{\text{m pro}} + H)^2}\Delta R_{\text{m}} + \Delta g_{N\text{ pro}}\\
\delta_{\text{sys}}\dot{V}_H &\cong -\frac{V_N^2}{(R_{\text{m. pro}} + H)^2}\Delta R_{\text{m}} - \frac{V_E^2 \cos\varphi}{(R_{\text{ppro}} + H\cos\varphi)^2}\Delta R_{\text{p}} + \Delta g_{H\text{pro}}\\
\delta_{\text{sys}}\dot{V}_E &\cong -\frac{V_N V_E \sin\varphi}{(R_{\text{ppro}} + H\cos\varphi)^2}\Delta R_{\text{p}} - \frac{V_H V_E \cos\varphi}{(R_{\text{ppro}} + H\cos\varphi)^2}\Delta R_{\text{p}} + \Delta g_{E\text{pro}}\\
\delta_{\text{sys}}\dot{\varphi} &\cong \frac{V_N}{(R_{\text{mpro}} + H)^2}\Delta R_{\text{m}}\\
\delta_{\text{sys}}\dot{\lambda} &\cong \frac{V_E}{(R_{\text{ppro}} + H\cos\varphi)^2}\Delta R_{\text{p}}
\end{aligned}\right\} \qquad (2.61)$$

where

$$\left.\begin{aligned}
\Delta R_{\text{p}} &\cong \frac{\partial R_{\text{p}}}{\partial\varphi}\Delta\varphi + \frac{\partial R_{\text{p}}}{\partial\lambda}\Delta\lambda;\\
\Delta R_{\text{m}} &\cong \frac{\partial R_{\text{m}}}{\partial\varphi}\Delta\varphi + \frac{\partial R_{\text{m}}}{\partial\lambda}\Delta\lambda.
\end{aligned}\right\} \qquad (2.62)$$

A numerical analysis of the partial systematic errors in accordance with the equations obtained above reveals that if we employ the previously calculated values $R_{m\ pro}(\phi, \lambda)$, $R_{p\ pro}(\phi, \lambda)$, $g_{N\ pro}(\phi, \lambda)$, $g_{H\ pro}(\phi, \lambda)$ for the programmed trajectory, then permissible deviations of the factual coordinates (ϕ, λ) from the programmed coordinates $(\phi_{pro}, \lambda_{pro})$ are:

- For $V < 500$ m/sec, degree units (hundreds of kilometers in an area);

- For $500 \text{ m/sec} \leqslant V \leqslant 1500$ m/sec, respectively, tenths of a degree (tens of kilometers);

- For $1500 \text{ m/sec} < V < 5000$ m/sec, units of angular minutes (kikometer units). /51

Here the partial systematic errors in the determination of relative accelerations will not exceed $10^{-5}g$, and in the determination of rate of change of the geographic coordinates, 10^{-7} 1/sec.

Therefore, for objects in motion in which the factual coordinates deviate from the programmed coordinates within the indicated limits, it is advantageous to calculate in advance the projections of the acceleration of gravity and the radii of the curvature of the coordinate lines, after which they are introduced into the flight in the form of a program.

In turn, analysis reveals that the permissible deviations in factual altitude from the programmed altitude (from the point of view of permissible errors in the determination of the projection of the gravity vector) amounts to hundreds of meters.

Simplifications of INS Equations in the Geographic Coordinate System With Different Approximations for the Figure of the Earth

If in INS equations the radii of the curvature of the coordinate lines are calculated in accordance with formulas derived for a spheroid, i.e., if the ellipticity of the equator is not taken into consideration, then partial systematic errors will occur in the determination of accelerations $\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$:

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_N &= -V_E^2 \sin\varphi \frac{R_{pe}-R_{ps}}{(R_{ps}+H\cos\varphi)(R_{pe}+H\cos\varphi)} - \\
&\quad - V_N V_H \frac{R_{me}-R_{ms}}{(R_{ms}+H)(R_{me}+H)}; \\
\delta_{sys}\dot{V}_H &= V_N^2 \frac{R_{me}-R_{ms}}{(R_{ms}+H)(R_{ms}+H)} + \\
&\quad + V_E^2 \cos\varphi \frac{R_{pe}-R_{ps}}{(R_{pe}+H\cos\varphi)(R_{ps}+H\cos\varphi)}; \\
\delta_{sys}\dot{V}_E &= V_N V_E \sin\varphi \frac{R_{pe}-R_{ps}}{(R_{pe}+H\cos\varphi)(R_{ps}+H\cos\varphi)} - \\
&\quad - V_H V_E \cos\varphi \frac{R_{pe}-R_{ps}}{(R_{pe}+H\cos\varphi)(R_{ps}+H\cos\varphi)},
\end{aligned}\right\} \qquad (2.63)$$

which are determined as the difference between the right sides of the equations for accelerations $\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$ with consideration given to the radii of the curvature of the coordinate lines correct for an ellipsoid and for a spheroid, respectively.

It is apparent from equations (2.63) that the partial systematic errors caused by disregarding the ellipticity of the equator are proportional to the differences between the radii of the curvature of the corresponding coordinate lines of the triaxial ellipsoid and the spheroid at a point corresponding to the location of the object on the surface of the Earth, and depend on the relative velocity projections (V_N, V_H, V_E), on the geographic latitude and on the altitude of the object in motion. /52

If in the denominators of the equations we consider that

$$\left.\begin{aligned}&(R_{ps}+H\cos\varphi)(R_{pe}+H\cos\varphi)\cong(R_{ea}+H)^2\cos^2\varphi;\\&(R_{me}+H)(R_{me}+H)\cong(R_{ea}+H)^2,\end{aligned}\right\}\qquad(2.64)$$

and considering

$$\left.\begin{aligned}&V_N=V\cos\Psi\cos\theta;\\&V_H=V\sin\theta;\\&V_E=V\sin\Psi\cos\theta\end{aligned}\right\}\qquad(2.65)$$

we obtain the values of the partial systematic errors according to the modulus

$$\left.\begin{aligned}&|\delta_{sys}\dot{V}_N|\cong\left|\frac{V^2}{(R_{ea}+H)^2}\left[(R_{pe}-R_{ps})\frac{\tan\varphi}{\cos\varphi}\sin\Psi\cos\theta+\right.\right.\\&\qquad\left.\left.+(R_{me}-R_{ms})\cos\varphi\sin\theta\right]\cos\theta\right|;\\&|\delta_{sys}\dot{V}_H|\cong\left|\frac{V^2}{(R_{ea}+H)^2}[(R_{me}-R_{м.с})\cos^2\Psi+\right.\\&\qquad\left.+\frac{R_{pe}-R_{ps}}{\cos\varphi}\sin^2\Psi\right]\cos^2\theta\Bigg|;\\&|\delta_{sys}\dot{V}_E|\cong\left|\frac{V^2(R_{pe}-R_{ps})}{(R_{ea}+H)^2\cos\varphi}(\cos\Psi\cos\theta\tan\varphi-\sin\theta)\cos\theta\sin\Psi\right|\end{aligned}\right\}\qquad(2.66)$$

Thus the limiting values of the errors indicated are proportional to the square of the relative velocity of object motion and to the differences of the radii of the curvature of the corresponding coordinate lines on the triaxial ellipsoid and on the spheroid, and also depend on the direction of motion in space (Ψ, θ) and on the geographic latitude of the area.

For objects in motion near the plane of the instantaneous horizon ($\theta \cong 0$):

$$\left.\begin{aligned}
|\delta_{sys}\dot{V}_N| &\cong \left|\frac{V^2}{(R_{ea}+H)^2}\left(R_{pe}-R_{ps}\right)\frac{\tan\varphi}{\cos\varphi}\sin^2\Psi\right|;\\
|\delta_{sys}\dot{V}_H| &\cong \left|\frac{V^2}{(R_{ea}+H)^2}\left[R_{me}-R_{ms})\cos^2\Psi+\frac{R_{pe}-R_{ps}}{\cos\varphi}\sin^2\Psi\right]\right|;\\
|\delta_{sys}\dot{V}_E| &\cong \left|\frac{V^2\,(R_{pe}-R_{ps})}{(R_{ea}+H)^2\cos\varphi}\cos\Psi\tan\varphi\sin\Psi\right|.
\end{aligned}\right\} \qquad (2.67)$$

An evaluation of the errors introduced according to the equations given for objects with $V < 5\cdot10^3$ m/sec and $H > 1000$km reveals that the values of the partial systematic errors under consideration at medium latitude ($\phi \leqslant 60°$) do not exceed 10^5g. It is apparent that henceforth it will be advantageous to examine object navigation as applied to the Earth spheroid. Here the maximum values of the errors under consideration are determined by the equations /53

$$\left.\begin{aligned}
(\delta_{sys}\dot{V}_N)_{max} &= \frac{V^2}{(R_{ea}+H)^2}\left(R_{pe}-R_{ps}\right)\frac{\tan\varphi}{\cos\varphi};\\
(\delta_{sys}\dot{V}_H)_{max} &= \frac{V^2}{(R_{ea}+H)^2}\left[R_{me}-R_{ms}-\frac{R_{pe}-R_{ps}}{\cos\varphi}\right];\\
(\delta_{sys}\dot{V}_E)_{max} &= \frac{V^2\,(R_{pe}-R_{ps})}{(R_{ea}+H)^2\cos\varphi}\tan\varphi.
\end{aligned}\right\} \qquad (2.68)$$

It is apparent from (2.68) that the limiting values of errors in the determination of the horizontal components with respect to acceleration ($\dot{V}_N$, $\dot{V}_E$) are proportional to tan ϕ, which is an important shortcoming of INS's, operating in the geographic coordinate system.

In equations for $\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$ by replacing formulas g_N (ϕ, λ, H), g_H (ϕ, λ, H), g_E (ϕ, λ, H) for a triaxial ellipsoid with formulas g_N (ϕ, H), g_H (ϕ, H), g_E (ϕ, H) for a spheroid, we cause the following systematic errors to occur:

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_N &= g_N(\varphi, H)-g_N(\varphi, \lambda, H);\\
\delta_{sys}\dot{V}_H &= g_H(\varphi, H)-g_H(\varphi, \lambda, H);\\
\delta_E\dot{V}_E &= g_E(\varphi, H)-g_E(\varphi, \lambda, H).
\end{aligned}\right\} \qquad (2.69)$$

In analyzing these equations we may verify that where $H \leqslant 100$ km, the partial systematic errors indicated do not exceed the dead zones of contemporary accelerometers ($\sim10^{-5}$g).

We may note in passing that the expressions for g_H and g_N (1.28), (1.28a) may be simplified under the assumption that

$$\varphi=\varphi';$$

$$R_{\text{air}}(H)=R'_{\text{air}}+H.$$

Then we obtain

$$\left.\begin{aligned} g_N&=\Omega^2\sin\varphi\cos\varphi\left[H+R'_{\text{air}}\left(1-\frac{R'^2_{\text{air}}}{(R'_{\text{air}}+H)^2}\right)\right];\\ g_H&=\frac{R'^2_{\text{ea}}}{(R'_{\text{air}}+H)^2}\,g-\Omega^2\cos^2\varphi\left[H+R'_{\text{air}}\left(1-\frac{R'^2_{\text{air}}}{(R'_{\text{air}}+H)^2}\right)\right].\end{aligned}\right\} \tag{2.70}$$

Here the following systematic errors occur: /54

$$\left.\begin{aligned} \Delta g_N&=\Omega^2\sin\varphi\left\{R'_{\text{air}}(\cos\varphi-\cos\varphi)\left[1-R'^2_{\text{air}}\frac{R^2_{\text{air}}(H)-(R'_{\text{air}}+H)^2}{R^2_{\text{air}}(H)\,(R'_{\text{air}}+H)^2}\right]\right\};\\ \Delta g_H&=R'^2_{\text{air}}\frac{R^2_{\text{air}}(H)-(R'_{\text{air}}+H)^2}{R^2_{\text{air}}(H)\,(R'_{\text{air}}+H)^2}-\Omega^2\cos\varphi\left\{R_{\text{air}}(\cos\varphi-\right.\\ &\quad\left.-\cos\varphi')\left[1-R_{\text{air}}\frac{R^2_{\text{air}}(H)-(R'_{\text{air}}+H)^2}{R^2_{\text{air}}(H)\,(R'_{\text{air}}+H)^2}\right]\right\}.\end{aligned}\right\} \tag{2.71}$$

In analyzing these equations for $H\leqslant 100$ km, it is possible to verify the permissibility of the simplifications indicated.

Let us examine the systematic errors which arise during the transition from an ellipsoid to a spheroid for systems with platforms stablized in the arbitrary coordinate system.

By replacing in equations pertaining to $\dot{V}_\xi$, $\dot{V}_\eta$, $\dot{V}_\zeta$ the radii of the curvature of the coordinate lines of the triaxial ellipsoid with the corresponding radii of the curvature of a spheroid we have partial systematic errors:

$$\left.\begin{aligned} \delta_{\text{sys}}\dot{V}_\xi&=-V_\xi V_\zeta\sin\varphi_{\text{arb}}\frac{\Delta R'_{\text{arb,p}}}{(R_{\text{arb,pe}}+H\cos\varphi_{\text{arb}})(R_{\text{arb,ps}}+H\cos\varphi_{\text{arb}})}-\\ &-V_\xi V_\eta\cos\varphi_{\text{arb}}\frac{\Delta R'_{\text{arb,p}}}{(R_{\text{arb,pe}}+H\cos\varphi_{\text{arb}})(R_{\text{arb,ps}}+H\cos\varphi_{\text{arb}})}+\Delta g'_\xi;\end{aligned}\right. \tag{2.72}$$

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\eta &= V_\xi^2 \cos\varphi_{arb}\frac{\Delta R'_{arb,p}}{(R_{arb,pe}+H\cos\varphi_{arb})(R_{arb,ps}+H\cos\varphi_{arb})}+\\
&+V_\zeta^2\frac{\Delta R'_{arb,m}}{(R_{arb,me}+H)(R_{arb,ms}+H)}+\Delta g'_\eta;\\
\delta_{sys}\dot{V}_\zeta &= V_\xi^2\sin\varphi_{arb}\frac{\Delta R'_{arb,p}}{(R_{arb,pe}+H\cos\varphi_{arb})(R_{arb,ps}+H\cos\varphi_{arb})}-\\
&-V_\eta V_\zeta\frac{\Delta R'_{arb,m}}{(R_{arb,me}+H)(R_{arb,ms}+H)}+\Delta g'_\zeta.
\end{aligned}\right\} \qquad (2.73)$$

In the equations (2.72)

$$\left.\begin{aligned}
\Delta R'_{arb,p} &= R_{arb,pe}-R_{arb,ps};\\
\Delta R'_{arb,m} &= R_{arb,me}-R_{arb,ms};\\
\Delta g'_\xi &= g_{\xi s}-g_{\xi e};\\
\Delta g'_\eta &= g_{\eta s}-g_{\eta e};\\
\Delta g'_\zeta &= g_{\zeta s}-g_{\zeta e},
\end{aligned}\right\} \qquad (2.74)$$

where $g_{\xi s}$, $g_{\eta s}$, $g_{\zeta s}$ are vector projections of the acceleration of gravity for a spheroid; /55

$g_{\xi e}$, $g_{\eta e}$, $g_{\zeta e}$ are corresponding projections for an ellipsoid.

If we consider that

$$\left.\begin{aligned}
V_\xi &= V\cos\theta\cos\Delta\Psi_{av};\\
V_\eta &= V\sin\theta;\\
V_\zeta &= V\cos\theta\sin\Delta\Psi_{av},
\end{aligned}\right\} \qquad (2.75)$$

where $\Delta\Psi_{av} = \frac{1}{T}\int_0^t \Delta\Psi\,(t)\,dt$ ($\Delta\Psi$ is the angle of inclination of the ground speed vector from the direction of the arbitrary parallel), and we accept that

$$\left.\begin{aligned}
(R_{arb,pe}+H\cos\varphi_{arb})(R_{arb,ps}+H\cos\varphi_{arb}) &\cong (R_{ea}+H)^2\cos^2\varphi_{arb};\\
(R_{arb,me}+H)(R_{arb,ms}+H) &\cong (R_{ea}+H)^2,
\end{aligned}\right\} \qquad (2.76)$$

then with ϕ_{arb} and $\Delta\Psi_{av}$, we obtain

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\xi &\cong -V^2\frac{\cos\theta\,(\varphi_{arb}\Delta\Psi_{av}\cos\theta+\sin\theta)\,\Delta R'_{arb,p}}{(R_{ea}+H)^2}+\Delta g'_\xi;\\
\delta_{sys}\dot{V}_\eta &\cong V^2\cos^2\theta\frac{\Delta R'_{arb,p}+\Delta R'_{arb,m}\Delta\Psi^2_{av}}{(R_{ea}+H)^2}+\Delta g'_\eta;\\
\delta_{sys}\dot{V}_\zeta &\cong V^2\cos\theta\frac{\Delta R'_{arb,p}\varphi_{arb}\cos\theta-\Delta R'_{arb,m}\Delta\Psi_{av}\sin\theta}{(R_{ea}+H)^2}+\Delta g'_\zeta.
\end{aligned}\right\}\qquad(2.77)$$

For objects which are in horizontal motion,

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\xi &\cong -V^2\frac{\Delta R'_{arb,p}\Delta\Psi_{av}\varphi_{arb}}{(R_{ea}+H)^2}+\Delta g'_\xi;\\
\delta_{sys}\dot{V}_\eta &\cong V^2\frac{\Delta R'_{arb,p}+\Delta R'_{arb,m}\Delta\Psi^2_{av}}{(R_{ea}+H)^2}+\Delta g'_\eta;\\
\delta_{sys}\dot{V}_\zeta &\cong V^2\frac{\Delta R'_{arb,p}\varphi_{arb}}{(R_{ea}+H)^2}+\Delta g'_\zeta.
\end{aligned}\right\}\qquad(2.78)$$

Thus, the errors considered are proportional to the square of the velocity and depend on the differences of the corresponding radii of the curvature.

An evaluation of the errors cited in accordance with equations (2.78) where $V < 10^{-4}$ m/sec reveals that the values of the errors under consideration do not exceed 10^{-5}g and therefore for the majority of INS's of various objects, the replacement indicated may be accomplished.

We shall examine a simplification of the INS equations as a result of the replacement of the spheroid by a sphere.

If we disregard the nonsphericity of the surface of the Earth in INS equations pertaining to $\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$, and consider that /56

$$R_m = R_{ea}$$
$$R_p = R_{ea}\cos\varphi,$$

here the following systematic errors will occur

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_N = &-V_E^2\tan\varphi\frac{R_p-R_{ea}\cos\varphi}{(R_{ea}+H)(R_p+H\cos\varphi)}-\\
&-V_NV_H\frac{R_m-R_{ea}}{(R_{ea}+H)(R_m+H)};
\end{aligned}\right\}\qquad(2.79)$$

$$\left.\begin{aligned}
\delta_{\text{sys}}\dot{V}_H &= V_N^2\frac{R_m-R_{ea}}{(R_{ea}+H)(R_m+H)}-\\
&\quad -V_E^2\frac{R_p-R_{ea}\cos\varphi}{(R_{ea}+H)(R_p+H\cos\varphi)};\\
\delta_{\text{sys}}\dot{V}_E &= \frac{V_E(R_p-R_{ea}\cos\varphi)}{(R_{ea}+H)(R_p+H\cos\varphi)}(V_N\tan\varphi-V_H).
\end{aligned}\right\} \qquad (2.79)$$

For an approximate evaluation (2.79) we may assume that

$$\left.\begin{aligned}
R_p+H\cos\varphi &\cong (R_{ea}+H)\cos\varphi;\\
R_m+H &\cong R_{ea}+H.
\end{aligned}\right\} \qquad (2.80)$$

Then, considering (2.65), we obtain

$$\left.\begin{aligned}
\delta_{\text{sys}}\dot{V}_N &\cong \frac{V^2\cos\theta}{(R_{ea}+H)^2}\left[\frac{R_p-R_{ea}\cos\varphi}{\cos\varphi}\sin^2\Psi\cos\theta\tan\varphi+\right.\\
&\quad \left.+(R_m-R_{ea})\sin\theta\cos\Psi\right];\\
\delta_{\text{sys}}\dot{V}_H &\cong \frac{V^2}{(R_{ea}+H)^2}\left[(R_m-R_{ea})\cos^2\Psi\cdot\theta+\right.\\
&\quad \left.+\frac{R_p-R_{ea}\cos\varphi}{\cos\varphi}\sin^2\Psi\right]\cos^2\theta;\\
\delta_{\text{sys}}\dot{V}_E &\cong \frac{V^2\sin\Psi\cos\theta(R_p-R_{ea}\cos\varphi)}{(R_{ea}+H)^2\cos\varphi}(\cos\Psi\cos\theta\tan\varphi-\sin\theta).
\end{aligned}\right\} \qquad (2.81)$$

It is apparent from equation (2.81) that systematic errors due to disregarding the nonsphericity of the Earth are proportional to the square of the relative velocity and to the differences between the radii of the coordinate lines and to the radii of the sphere, and also depend on the geographic latitude of the area, the direction Ψ and altitude H of motion in space and on trajectory angle θ.

It is easy to see that /57

$$\left.\begin{aligned}
|\delta_{\text{sys}}\dot{V}_N| &< \left|\frac{V^2}{(R_{ea}+H)^2}\left(\frac{R_p-R_{ea}\cos\varphi}{\cos\varphi}\tan\varphi+R_m-R_{ea}\right)\right|;\\
|\delta_{\text{sys}}\dot{V}_H| &< \left|\frac{V^2}{(R_{ea}+H)^2}\left(R_m-R_{ea}+\frac{R_p-R_{ea}\cos\varphi}{\cos\varphi}\right)\right|;\\
|\delta_{\text{sys}}\dot{V}_E| &< \left|\frac{V^2(R_p-R_{ea}\cos\varphi)}{(R_{ea}+H)^2\cos\varphi}(\tan\varphi-1)\right|.
\end{aligned}\right\} \qquad (2.82)$$

In utilizing (2.82), it is not difficult to verify that for objects in motion with velocity $V \leqslant 500$ m/sec at medium latitudes, the systematic errors indicated will not exceed 10^{-5}g.

Similarly, for INS's which employ the arbitrary coordinate system, by replacing the radii of the curvature of the coordinate lines by the radii of the Earth sphere we obtain

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\xi &= -V_\xi V_\zeta \frac{\Delta R''_{arb,m}\sin\varphi_{arb}}{(R_{arb,ps}+H\cos\varphi_{arb})(R_{ea}+H)\cos\varphi_{arb}} - \\
&\quad - V_\xi V_\eta \frac{\Delta R''_{arb,p}\cos\varphi_{arb}}{(R_{arb,ps}+H\cos\varphi_{arb})(R_{ea}+H)\cos\varphi_{arb}} + \Delta g''_\xi; \\
\delta_{sys}\dot{V}_\eta &= V_\xi^2 \frac{\Delta R''_{arb,p}\cos\varphi_{arb}}{(R_{arb,ps}+H\cos\varphi_{arb})(R_{ea}+H)\cos\varphi_{arb}} + \\
&\quad + V_\zeta^2 \frac{\Delta R''_{arb,m}}{(R_{arb,ms}+H)(R_{ea}+H)} + \Delta g''_\eta; \\
\delta_{sys}\dot{V}_\zeta &= V_\xi^2 \frac{\Delta R''_{arb,p}\sin\varphi_{arb}}{(R_{arb,ps}+H\cos\varphi_{arb})(R_{ea}+H)\cos\varphi_{arb}} - \\
&\quad - V_\eta V_\zeta \frac{\Delta R''_{arb,m}}{(R_{arb,ms}+H)(R_{ea}+H)} + \Delta g''_\zeta,
\end{aligned}\right\} \qquad (2.83)$$

where

$$\left.\begin{aligned}
\Delta R''_{arb,p} &= R_{arb,ps} - R_{ea}; \\
\Delta R''_{arb,m} &= R_{arb,ms} - R_{ea}; \\
\Delta g''_\xi &= g_{\xi ea} - g_{\xi s}; \\
\Delta g''_\eta &= g_{\eta ea} - g_{\eta s}; \\
\Delta g''_\zeta &= g_{\zeta ea} - g_{\zeta s}
\end{aligned}\right\} \qquad (2.84)$$

Having made the same assumptions as for the derivation of (2.77), we have /58

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\xi &\cong -V^2 \frac{\cos\theta\left(\varphi_{arb}\Delta\Psi_{av}\cos\theta\Delta R''_{arb,m} + \sin\theta\cdot\Delta R''_{arb,p}\right)}{(R_{ea}+H)^2} + \Delta g''_\xi; \\
\delta_{sys}\dot{V}_\eta &\cong V^2 \frac{\cos^2\theta\Delta R''_{arb,p} + \Delta R''_{arb,m}\Delta\psi^2_{av}\cos^2\theta}{(R_{ea}+H)^2} + \Delta g''_\eta; \\
\delta_{sys}\dot{V}_\zeta &\cong V^2 \frac{\Delta R''_{arb,p}\varphi_{arb}\cos^2\theta - \Delta R''_{arb,m}\sin\theta\cos\theta\Delta\Psi_{av}}{(R_{ea}+H)^2} + \Delta g''_\zeta.
\end{aligned}\right\} \qquad (2.85)$$

For object horizontal motion,

$$\left.\begin{aligned}
\delta_{\text{sys}}\dot{V}_\xi &\cong -V^2\frac{\varphi_{\text{arb}}\Delta\Psi_{\text{av}}}{(R_{\text{ea}}+H)^2}\Delta R''_{\text{arb,m}}+\Delta g''_\xi;\\
\delta_{\text{sys}}\dot{V}_\eta &\cong V^2\frac{\Delta R''_{\text{arb,p}}+\Delta R''_{\text{arb,m}}\Delta\Psi^2_{\text{av}}}{(R_{\text{ea}}+H)^2}+\Delta g''_\eta;\\
\delta_{\text{sys}}\dot{V}_\zeta &\cong V^2\frac{\Delta R''_{\text{arb,p}}\varphi_{\text{arb}}}{(R_{\text{ea}}+H)^2}+\Delta g''_\zeta.
\end{aligned}\right\} \qquad (2.86)$$

An approximate evaluation of the errors introduced by equations (2.86) reveals that where $V \leqslant 500$ m/sec and $\phi_{\text{arb}} \leqslant 1°$, the values of the systematic errors under consideration will not exceed 10^{-5}g.

We shall examine the kinematic equation simplifications for the rates of change of the geographic and arbitrary coordinates for various approximations of the figure of the Earth.

These equations may be simplified by replacing the radii of the curvature of the triaxial ellipsoid coordinate lines by the radii of the curvature of the spheroid coordinate lines.

Here the following systematic errors occur:

$$\left.\begin{aligned}
\delta_{\text{sys}}\dot{\varphi} &= V_N\frac{R_{\text{me}}-R_{\text{ms}}}{(R_{\text{ms}}+H)(R_{\text{me}}+H)};\\
\delta_{\text{sys}}\dot{\lambda} &= V_E\frac{R_{\text{pe}}-R_{\text{ps}}}{(R_{\text{ps}}+H\cos\varphi)(R_{\text{pe}}+H\cos\varphi)},
\end{aligned}\right\} \qquad (2.87)$$

or approximately

$$\left.\begin{aligned}
\delta_{\text{sys}}\dot{\varphi} &\cong V\frac{R_{\text{me}}-R_{\text{ms}}}{(R_{\text{ea}}+H)^2};\\
\delta_{\text{m}}\dot{\lambda} &\cong V\frac{R_{\text{me}}-R_{\text{ms}}}{(R_{\text{ea}}+H)^2\cos^2\varphi}.
\end{aligned}\right\} \qquad (2.88)$$

It is not difficult to verify that where $V \leqslant 500$ m/sec, the values of these errors will not exceed the drift rates of contemporary gyroscopes ($5\cdot10^{-7}$ 1/sec), and therefore it is advantageous to introduce such a simplification in INS equations.

It should be noted that errors in the determination of the rates of change of the geographic coordinates lead to the following platform inclination errors in inertial space: /59

$$\left.\begin{aligned}
\delta_{\text{sys}}\omega_{aN} &= \delta_{\text{sys}}\dot{\lambda}\cos\varphi;\\
\delta_{\text{sys}}\omega_{aH} &= \delta_{\text{sys}}\dot{\lambda}\sin\varphi;\\
\delta_{\text{sys}}\omega_{aE} &= -\delta_{\text{sys}}\dot{\varphi}.
\end{aligned}\right\} \qquad (2.89)$$

It is apparent that

$$\left.\begin{aligned} |\delta_{sys}\omega_{aN}| &< |\delta_{sys}\dot{\lambda}|; \\ |\delta_{sys}\omega_{aH}| &< |\delta_{sys}\dot{\lambda}|; \\ |\delta_{sys}\omega_{aE}| &< |\delta_{sys}\dot{\varphi}|. \end{aligned}\right\} \tag{2.90}$$

As a first approximation, errors in the determination of the geographic coordinates are

$$\left.\begin{aligned} |\Delta\lambda_{sys}| &< \left|\delta_{sys}\dot{\lambda}\left(\cos\sqrt{\frac{g}{R}}\cdot t - t\right)\right|; \\ |\Delta\varphi_{sys}| &< \left|\delta_{sys}\dot{\varphi}\left(\cos\sqrt{\frac{g}{R}}\cdot t - t\right)\right|. \end{aligned}\right\} \tag{2.91}$$

For example, where $V \leq 500$ m/sec, $\Delta\dot{\lambda} \leq 5\cdot10^{-8}$ 1/sec and for time $t = 10^4$ sec

$$\Delta\lambda_{sys} \leqslant 1{,}5';$$
$$\Delta\varphi_{sys} \leqslant 1{,}5',$$

therefore for the navigation of the majority of contemporary objects, the figure of the Earth must be approximated by a spheroid.

If the radii of the coordinate lines are replaced by the radius of the Earth sphere, then systematic errors of the following form appear:

$$\left.\begin{aligned} \delta_{sys}\dot{\varphi} &= V_N \frac{R_m - R_{ea}}{(R_{ea}+H)(R_m+H)}; \\ \delta_{sys}\dot{\lambda} &= V_E \frac{R_p - R_{ea}\cos\varphi}{(R_{ea}+H)\cos\varphi\,(R_p + H\cos\varphi)}. \end{aligned}\right\} \tag{2.92}$$

Considering (2.65), we obtain

$$\left.\begin{aligned} |\delta_{sys}\dot{\varphi}| &\cong \left|V\cos\Psi\cos\theta\,\frac{R_m - R_{ea}}{(R_{ea}+H)^2}\right|; \\ |\delta_{sys}\dot{\lambda}| &\cong V\sin\Psi\cos\theta\,\frac{R_p - R_{ea}\cos\varphi}{(R_{ea}+H)^2\cos^2\varphi}. \end{aligned}\right\} \tag{2.93}$$

It is easy to see that /60

$$\left.\begin{aligned} |\delta_{sys}\dot{\varphi}| &< \left|V\,\frac{R_m - R_{ea}}{(R_{ea}+H)^2}\right|; \\ |\delta_{sys}\dot{\lambda}| &< \left|V\,\frac{R_p - R_{ea}\cos\varphi}{(R_{ea}+H)^2\cos^2\varphi}\right|. \end{aligned}\right\} \tag{2.94}$$

An evaluation of equations (2.94) reveals that for $V \leqslant 500$ m/sec, errors in the determination of the rate of change of the geographic coordinates will not exceed 10^{-7} 1/sec.

Where $V > 500$ m/sec in equations for $\dot{\lambda}$ and $\dot{\phi}$ it is necessary to consider the nonsphericity of a spheroidal surface. We note that in simplifying equations for $\dot{\lambda}$ and $\dot{\phi}$ it is necessary to take the time of controlled flight into consideration.

We shall analyze the possibility of simplifying the equations for the rate of change of the arbitrary coordinates.

In replacing the radii of the curvature of the triaxial ellipsoid coordinate lines with the radii of the curvature of the spheroid, we obtain

$$\left.\begin{aligned} \delta_{\text{sys arb}}\dot{\lambda} &\cong V_\xi \frac{R_{\text{arb,pe}} - R_{\text{arb,ps}}}{(R_{\text{arb,pe}} + H\cos\varphi_{\text{arb}})(R_{\text{arb,ps}} + H\cos\varphi_{\text{arb}})}; \\ \delta_{\text{sys arb}}\dot{\varphi} &= -V_\zeta \frac{R_{\text{arb,pe}} - R_{\text{arb,ps}}}{(R_{\text{arb,me}} + H)(R_{\text{arb,ms}} + H)}. \end{aligned}\right\} \tag{2.95}$$

Considering (2.76) in employing the designations

$$\left.\begin{aligned} R_{\text{arb,pe}} - R_{\text{arb,ps}} &= \Delta R_{\text{arb,p}} \\ R_{\text{arb,me}} - R_{\text{arb,ms}} &= \Delta R_{\text{arb,m}} \end{aligned}\right\} \tag{2.96}$$

we obtain

$$\left.\begin{aligned} \delta_{\text{sys arb}}\dot{\lambda} &\cong V_\xi \frac{\Delta R_{\text{arb,p}}}{(R_{\text{ea}} + H)^2\cos^2\varphi_{\text{arb}}}; \\ \delta_{\text{sys arb}}\dot{\varphi} &\cong -V_\zeta \frac{\Delta R_{\text{arb,m}}}{(R_{\text{ea}} + H)^2} \end{aligned}\right\} \tag{2.97}$$

With small ϕ_{arb} and $\Delta\Psi_{\text{av}}$, we have

$$\left.\begin{aligned} \delta_{\text{sys arb}}\dot{\lambda} &\cong V \frac{\Delta R_{\text{arb,p}}}{(R_{\text{ea}} + H)^2}\cos\theta; \\ \delta_{\text{sys arb}}\dot{\varphi} &\cong -V \frac{\Delta R_{\text{arb,m}}}{(R_{\text{ea}} + H)^2}\Delta\Psi_{\text{av}}\cos\theta. \end{aligned}\right\} \tag{2.98}$$

For objects in a basically horizontal trajectory we have

$$\left.\begin{aligned} \left|\delta_{\text{sys arb}}\dot{\lambda}\right| &\cong \left|V \frac{\Delta R_{\text{arb,p}}}{(R_{\text{ea}} + H)^2}\right|; \\ \left|\delta_{\text{sys arb}}\dot{\varphi}\right| &\leqslant \left|V \frac{\Delta R_{\text{arb,m}}}{(R_{\text{ea}} + H)^2}\Delta\Psi_{\text{av}}\right|. \end{aligned}\right\} \tag{2.99}$$

It is possible to show through (2.99) that for objects in motion close to the arbitrary equator at altitudes $H \leqslant 100$ km with velocities $V \leqslant 500$ m/sec, the values of the partial systematic errors examined do not exceed the drift rates of contemporary gyroscopes. /61

If we replace in INS equations pertaining to $\dot{\lambda}_{arb}$ and $\dot{\phi}_{arb}$ the values of the radii of the curvature of the coordinate lines with the radius of the Earth sphere, the following systematic errors arise

$$\left.\begin{aligned} \delta\dot{\lambda}_{\text{sys arb}} &= V_\xi \frac{R_{\text{arb,pe}} - R_{ea}}{\cos\varphi_{arb}(R_{ea}+H)(R_{\text{arb,pe}}+H\cos\varphi_{arb})}; \\ \delta\dot{\varphi}_{\text{sys arb}} &= -V_\zeta \frac{R_{\text{arb,me}} - R_{ea}}{(R_{ea}+H)(R_{\text{arb,me}}+H)}. \end{aligned}\right\} \qquad (2.100)$$

With small ϕ_{arb} and $\Delta\psi_{av}$, we obtain

$$\left.\begin{aligned} \delta\dot{\lambda}_{\text{sys arb}} &\approx V \frac{\Delta R_{\text{arb,p}}}{(R_{ea}+H)^2}\cos\theta; \\ \delta\dot{\varphi}_{\text{sys arb}} &\approx -V \frac{\Delta R_{\text{arb,m}}}{(R_{ea}+H)^2}\Delta\psi_{av}\cos\theta; \end{aligned}\right\} \qquad (2.101)$$

where

$$\Delta R_{\text{arb,p}} = R_{\text{arb,pe}} - R_{ea}$$
$$\Delta R_{\text{arb,m}} = R_{\text{arb,me}} - R_{ea}$$

For objects in horizontal motion, we have

$$\left.\begin{aligned} |\delta\dot{\lambda}_{\text{sys arb}}| &\leqslant \left|V \frac{\Delta R_{\text{arb,p}}}{(R_{ea}+H)^2}\right|; \\ |\delta\dot{\varphi}_{\text{sys arb}}| &\leqslant \left|V \frac{\Delta R_{\text{arb,m}}}{(R_{ea}+H)^2}\Delta\psi_{av}\right|. \end{aligned}\right\} \qquad (2.102)$$

It is apparent from these equations that partial systematic INS errors in the determination of the rate of change of the arbitrary coordinates are proportional to the velocity of the object with respect to the surface of Earth and to the differences between the radii of the curvature of the coordinatelines of these figures. Apparently the maximum errors will occur in the determination of the rate of change of the longitudinal coordinate.

In analyzing equations (2.102) for objects with relative velocities $V \leqslant 500$ m/sec, it is not difficult to verify that here the systematic errors examined will not exceed the average drift rates of contemporary gyroscopes. For similar objects, therefore, it is advantageous to accomplish the simplifications indicated.

In order to simplify the problem in subsequent discussions, we shall employ as initial data the kinematic equations for a sphere as an approximation of the figure of the Earth.

The nonsphericity of the Earth's surface or errors arising from disregarding this factor may be calculated in accordance with the method outlined above. /62

Possible Equation Simplifications for Relative Accelerations In the Geographic Coordinate System

Equations for $\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$ may be simplified for objects in motion at comparatively low altitudes when it is possible to consider that

$$R_{ea}+H\cong R_{ea}$$

Then the equations for $\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$ will be as follows:

$$\dot{V}_N=a_N-\frac{V_E^2}{R_{ea}}\tan\varphi-\frac{V_NV_H}{R_{ea}}-2\Omega V_E\sin\varphi+g_N; \tag{2.103}$$

$$\left.\begin{aligned}\dot{V}_H&=a_H+\frac{V_N^2}{R_{ea}}+\frac{V_E^2}{R_{ea}}+2\Omega V_E\cos\varphi+g_H;\\ \dot{V}_E&=a_E+\frac{V_NV_E}{R_{ea}}\tan\varphi-\frac{V_HV_E}{R_{ea}}-\\ &\quad-2\Omega(V_H\cos\varphi-V_N\sin\varphi)+g_E,\end{aligned}\right\} \tag{2.104}$$

where

$$g_N=g_E=0. \tag{b}$$

Considering (2.65), we have the equations for partial systematic errors:

$$\left.\begin{aligned}\delta'_{sys}\dot{V}_N&=-\frac{V^2H}{R_{ea}^2}\cos\theta(\sin^2\Psi\cos^2\theta\tan\varphi+\sin\theta\cos\Psi);\\ \delta'_{sys}\dot{V}_H&=\frac{V^2H}{R_{ea}^2}\cos^2\theta;\\ \delta'_{sys}\dot{V}_E&=\frac{V^2H}{R_{ea}^2}\cos\theta\sin\Psi(\cos\theta\cos\Psi\tan\varphi+\sin\theta),\end{aligned}\right\} \tag{2.105}$$

which are obtained as the difference between the right sides of equations (2.10) and (2.38). It is apparent that for objects in horizontal motion ($\theta\cong 0$)

$$\left.\begin{aligned}|\delta'_{sys}\dot{V}_N|&<\left|\frac{V^2H}{R_{ea}^2}\tan\varphi\right|;\\ \delta'_{sys}\dot{V}_H&<\frac{V^2H}{R_{ea}^2};\\ |\delta'_{sys}\dot{V}_E|&<\left|\frac{V^2H}{R_{ea}^2}\tan\varphi\right|.\end{aligned}\right\} \tag{2.106}$$

At average latitudes ($\phi \leqslant 60°$): /63

$$\left.\begin{aligned} |\delta'_{sys}\dot{V}_N| &< 2\frac{V^2H}{R^2_{ea}};\\ \delta'_{sys}\dot{V}_H &< \frac{V^2H}{R^2_{ea}};\\ |\delta'_{sys}\dot{V}_E| &< 2\frac{V^2H}{R^2_{ea}}. \end{aligned}\right\} \qquad (2.107)$$

Calculations accomplished in accordance with equations (2.107) reveal that at average latitudes for objects with V < 500 m/sec, H < 30 km the systematic error values lie within the limits of dead zones of contemporary accelerometers (10^{-4}g - 10^{-5}g); therefore for such objects the flight altitude may be disregarded in the first three INS equations.

For objects having a basic portion of the trajectory in a horizontal direction ($V_H \cong 0$) and operating at altitude H < 30 km, terms containing V_H may be excluded from the INS equations pertaining $\dot{V}_N$ and $\dot{V}_E$.

$$\left.\begin{aligned} \dot{V}_N &= a_N - \frac{V_E^2}{R_{ea}}\tan\varphi - 2\Omega V_E \sin\varphi;\\ \dot{V}_E &= a_E + \frac{V_N V_E}{R_{ea}}\tan\varphi + 2\Omega V_N \sin\varphi. \end{aligned}\right\} \qquad (2.108)$$

Considering equations (2.65), we obtain error values

$$\left.\begin{aligned} \delta'_{sys}\dot{V}_N &= \frac{V^2}{R_{ea}}\cos\theta\sin\theta\cos\Psi;\\ \delta'_{sys}\dot{V}_E &= V\sin\theta\left(\frac{V}{R_{ea}}\cos\theta\sin\Psi + 2\Omega\cos\varphi\right), \end{aligned}\right\} \qquad (2.109)$$

which are determined as the difference between the right sides of equations (2.108) and the corresponding full equations. It is also apparent that

$$\left.\begin{aligned} \delta_{sys}\dot{V}_N &< \frac{V^2}{R_{ea}}\sin\theta;\\ \delta_{sys}\dot{V}_E &< \left(\frac{V^2}{R_{ea}} + 2\Omega V\right)\sin\theta. \end{aligned}\right\} \qquad (2.110)$$

Calculations accomplished in accordance with equations (2.109) and (2.110) reveal that for objects in motion with velocity V > 1000 m/sec with trajectory angle $\theta \geqslant 1°$, INS errors caused by the lack of terms containing V_H in the equations will exceed dead zones of contemporary accelerometers. /64
Therefore for such objects is is advantageous to consider the terms containing V_H.

On the basis of the INS objects discussed which are in motion with velocity V < 500 m/sec at altitude H < 30 km in a horizontal trajectory, it is possible to recommend the following equations:

$$\left.\begin{aligned}\dot{V}_N &= a_N - \frac{V_E^2}{R_{ea}}\tan\varphi - 2\Omega V_E \sin\varphi;\\ \dot{V}_E &= a_E + \frac{V_N V_E}{R_{ea}}\tan\varphi + 2\Omega V_N \sin\varphi.\end{aligned}\right\} \qquad (2.111)$$

It is apparent (2.111) that the INS may be constructed in accordance with the two-component design.

For objects in motion with velocity V > 500 m/sec at altitude H > 30 km, it is necessary to consider additional terms of equations, depending on the peculiarities of the kinematics of object motion. Simplifications may be accomplished in accordance with the technique discussed.

For example, for INS objects with distance 500 km < L < 1500 km accomplishing horizontal flight at altitudes H < 30 km with velocity 500 m/sec < V < 1500 m/sec, the equations pertaining to $\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$ assume the form

$$\left.\begin{aligned}\dot{V}_N &= a_N - \frac{V_E^2}{R_{ea}+H}\tan\varphi - 2\Omega V_E \sin\varphi;\\ \dot{V}_H &= a_H + \frac{V_N^2}{R_{ea}+H} + \frac{V_E^2}{R_{ea}+H} + 2\Omega V_E \cos\varphi + g_H;\\ \dot{V}_E &= a_E + \frac{V_N V_E}{R_{ea}+H}\tan\varphi + 2\Omega V_N \sin\varphi.\end{aligned}\right\} \qquad 2.112)$$

After simplifications it is advantageous to evaluate as first approximation the values of the full systematic errors in a determination of accelerations

$$\delta_{sys}\dot{V}_N = \sum_{i=1}^{n} \delta^i \dot{V}_{N\,sys};$$

$$\delta_{sys}\dot{V}_H = \sum_{i=1}^{n} \delta^i \dot{V}_{H\,sys};$$

$$\delta_{sys}\dot{V}_E = \sum_{i=1}^{n} \delta^i \dot{V}_{E\,sys},$$

and also in the determination of coordinates in accordance with equations (2.9), in which in place of Δa we insert the values $\delta_{sys}\dot{V}_N$, $\delta_{sys}\dot{V}_E$, considering these to be constants and equal to the average values of systematic errors during the time of object motion: /65

$$\left.\begin{aligned}\Delta\lambda_{sys} &\cong \frac{\sum_{i=1}^{n} \delta^i \dot{V}_E}{g_0}\left(1-\cos\sqrt{\frac{g_0}{R_{ea}}}\cdot t\right);\\ \Delta\varphi_{sys} &\cong \frac{\sum_{i=1}^{n} \delta^i \dot{V}_N}{g_0}\left(1-\cos\sqrt{\frac{g_0}{R_{ea}}}\cdot t\right)..\end{aligned}\right\} \quad (2.113)$$

The Simplification of Equations For the Rates of Change Of Geographic Coordinates

For objects in motion at sufficiently low altitudes, we may consider that

$$\left.\begin{aligned}\dot{\varphi} &= \frac{V_N}{R_{ea}};\\ \dot{\lambda} &= \frac{V_E}{R_{ea}\cos\varphi}.\end{aligned}\right\} \quad (2.114)$$

Here

$$\left.\begin{aligned}\Delta\dot{\varphi}_{sys} &= \frac{V_N H}{R^2_{ea}};\\ \Delta\dot{\lambda}_{sys} &= \frac{V_E H}{R^2_{ea}\cos\varphi}.\end{aligned}\right\} \quad (2.115)$$

If we take equations (2.65) into consideration, we obtain

$$\left.\begin{aligned}\Delta\dot{\varphi}_{sys} &= \frac{V}{R^2_{ea}} H\cos\theta\cos\Psi;\\ \Delta\dot{\lambda}_{sys} &= \frac{V}{R^2_{ea}\cos\varphi} H\cos\theta\sin\Psi.\end{aligned}\right\} \quad (2.116)$$

It is apparent that

$$\left.\begin{aligned}\Delta\dot{\varphi}_{sys} &< \frac{V}{R^2_{ea}} H;\\ \Delta\dot{\lambda}_{sys} &< \frac{V}{R^2_{ea}\cos\varphi} H.\end{aligned}\right\} \quad (2.117)$$

We may verify through the use of equations (2.116), (2.117) that for objects in motion at altitudes H < 30 km the systematic errors in the determination of coordinate rates of change do not exceed the drift rate of contemporary gyroscopic instruments.

The Simplification of Equations For Angular Velocities

For objects with a short flight distance the values of the platform inclination angles ψ_a, ϑ_a, γ_a are also correspondingly small, and therefore it is possible to consider that

$$\left.\begin{aligned}\dot{\psi}_a &= (\dot{\lambda}+\Omega)\cos\varphi+\dot{\varphi}\cdot\gamma_a;\\ \dot{\vartheta}_a &= (\dot{\lambda}+\Omega)\sin\varphi\gamma_a-\dot{\varphi};\\ \dot{\gamma}_a &= (\dot{\lambda}+\Omega)-[(\dot{\lambda}+\Omega)\sin\varphi-\dot{\varphi}\gamma_a]\,\vartheta_a.\end{aligned}\right\} \qquad (2.118)$$

This assumption leads to the appearance of errors

$$\left.\begin{aligned}\delta_{sys}\dot{\psi}_a &= (\dot{\lambda}+\Omega)\left(1-\frac{\cos\gamma_a}{\cos\vartheta_a}\right)+\dot{\varphi}\left(\gamma_a-\frac{\sin\gamma_a}{\cos\vartheta_a}\right);\\ \delta_{sys}\dot{\vartheta}_a &= (\dot{\lambda}+\Omega)(\varphi\gamma_a-\sin\gamma_a)-\dot{\varphi}(1-\cos\gamma_a);\\ \delta_{sys}\dot{\gamma}_a &= (\dot{\lambda}+\Omega)[(1-\cos\varphi)-\dot{\varphi}\vartheta_a+\cos\gamma_a\tan\vartheta_a]+\\ &\quad+\dot{\varphi}(\gamma_a\vartheta_a-\sin\gamma_a\tan\vartheta_a).\end{aligned}\right\} \qquad (2.119)$$

An evaluation of the errors in equations (2.119) for L < 500 km and V < 500 m/sec reveals that systematic INS errors in the determination of $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$ do not exceed the angular velocity values of the drift of contemporary gyroscopic instruments.

For objects of greater distances it is advantageous to employ equations for $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$ of the form (2.37):

We shall examine the simplified equations for an INS with a platform oriented in the geographic coordinate system.

On the basis of what has already been stated in this chapter, we may propose the following system of INS equations.

For short range objects (V < 500 m/sec, L < 500 km and H < 30 km) the INS kinematic equations may be simplified to the form

$$\left.\begin{aligned}\dot{V}_N &= a_N-\frac{V_E^2}{R_{ea}}\tan\varphi-2\Omega V_E\sin\varphi;\\ \dot{V}_E &= a_E+\frac{V_N V_E}{R_{ea}}\tan\varphi+2\Omega V_N\sin\varphi;\\ \dot{\varphi} &= \frac{V_N}{R_{ea}};\\ \dot{\lambda} &= \frac{V_E}{R_{ea}\cos\varphi};\end{aligned}\right\} \qquad (2.120)$$

$$\left.\begin{aligned}
\dot{\psi}_a &= (\dot{\lambda}+\Omega)\cos\varphi+\dot{\varphi}\gamma_a;\\
\dot{\vartheta}_a &= (\dot{\lambda}+\Omega)\gamma_a\sin\varphi-\dot{\varphi};\\
\dot{\gamma}_a &= (\dot{\lambda}+\Omega)-[(\dot{\lambda}+\Omega)\sin\varphi-\dot{\varphi}\gamma_a]\,\vartheta_a.
\end{aligned}\right\} \qquad (2.120)$$

It is obvious that for such objects the INS may be constructed in accordance with a two-component design, since the calculation of $\dot{V}_N$ and $\dot{V}_E$ does not require vertical velocity V_H and altitude H. /67

For medium-range objects in horizontal motion (500 km < L < 1500 km, 500 m/sec < V < 1500 m/sec, H > 30 km) the INS may be advantageously constructed in accordance with the following system of kinematic equations:

$$\left.\begin{aligned}
\dot{V}_N &= a_N-\frac{V_E}{R_{ea}+H}\tan\varphi-2\Omega V_E\sin\varphi;\\
\dot{V}_H &= a_H+\frac{V_N^2}{R_{ea}+H}+\frac{V_E^2}{R_{ea}+H}+2\Omega V_E\cos\varphi+g_H;\\
\dot{V}_E &= a_E+\frac{V_N V_E}{R_{ea}+H}\tan\varphi+2\Omega V_N\sin\varphi;\\
\dot{\varphi} &= \frac{V_N}{R_{ea}+H};\\
\dot{H} &= V_H;\\
\dot{\lambda} &= \frac{V_E}{(R_{ea}+H)\cos\varphi};\\
\dot{\psi}_a &= \frac{1}{\cos\vartheta_a}[(\dot{\lambda}+\Omega)\cos\varphi\cos\gamma_a+\dot{\varphi}\sin\gamma_a];\\
\dot{\vartheta}_a &= (\dot{\lambda}+\Omega)\sin\varphi\sin\gamma_a-\dot{\varphi}\cos\gamma_a;\\
\dot{\gamma}_a &= (\dot{\lambda}+\Omega)\cos\varphi-[(\dot{\lambda}+\Omega)\sin\varphi\cos\gamma_a-\dot{\varphi}\sin\gamma_a]\tan\vartheta_a.
\end{aligned}\right\} \qquad (2.121)$$

It is apparent that for such objects it is necessary to employ an INS of the three-component design, since it is necessary to consider H in the calculation of $\dot{V}_N$ and V_E.

For objects in motion with velocities V > 1500 m/sec, at distance L > 1500 km with a sufficiently extended climb H > 30 km or with rates of descent V_H > 1000 m/sec, it is advantageous to construct the INS in accordance with the full system of kinematic equations.

The Simplification of Acceleration Equations In the Arbitrary Coordinate System

For objects in flight near the arbitrary equator, the ϕ_{arb} coordinate is comparatively small; therefore in the first and third INS equations (2.50) it is possible to disregard the terms containing tan ϕ_{arb} /68

$$\left.\begin{aligned}
\dot{V}_\xi &= a_\xi - \frac{V_\xi V_\eta}{R_{ea}+H} - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta);\\
\dot{V}_\eta &= a_\eta + \frac{V_\xi^2}{R_{ea}+H} + \frac{V_\zeta^2}{R_{ea}+H} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta;\\
\dot{V}_\zeta &= a_\zeta - \frac{V_\eta V_\zeta}{R_{ea}+H} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi).
\end{aligned}\right\} \qquad (2.122)$$

Here the following systematic errors are introduced:

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\xi^I &= \frac{1}{R_{ea}+H} V_\xi V_\zeta \tan\varphi_y;\\
\delta_{sys}\dot{V}_\zeta^I &= \frac{1}{R_{ea}+H} V_\xi^2 \tan\varphi_y.
\end{aligned}\right\} \qquad (2.123)$$

For an approximate evaluation of these errors we shall consider that

$$\left.\begin{aligned}
R_{ea}+H &\cong R_{ea}\\
\tan\varphi_{arb} &\cong \varphi_{arb}\\
\varphi_{arb} &\cong -\frac{1}{R_{ea}}\int_0^t V_\zeta dt.
\end{aligned}\right\} \qquad (2.124)$$

In addition,

$$\left.\begin{aligned}
V_\xi &\cong V;\\
V_\zeta &\cong V\Delta\Psi,
\end{aligned}\right\} \qquad (2.125)$$

where $\Delta\Psi$ is the angle of deviation of the ground speed vector from the direction of the arbitrary equator.

Where V = const

$$\varphi_{arb} \cong \frac{V}{R_{ea}}\int_0^t \Delta\Psi dt \qquad (2.126)$$

or

$$\varphi_{arb} \cong -\frac{L}{R_{ea}}\Delta\Psi_{av} \qquad (2.127)$$

where

$$\left.\begin{aligned}
L &\cong Vt;\\
\Delta\Psi_{av} &= \frac{1}{t}\int_0^t \Delta\Psi dt.
\end{aligned}\right\} \qquad (2.128)$$

Based upon (2.124), (2.125) and (2.127), the equations (2.123) assume the following form /69

$$\left.\begin{aligned} \delta_{sys}\dot{V}_\xi^{I} &= -\frac{L}{R_{ea}^2}V^2\Delta\Psi_{av}^2; \\ \delta_{sys}\dot{V}_\zeta^{I} &= \frac{L}{R_{ea}^2}V^2\Delta\Psi_{av}. \end{aligned}\right\} \tag{2.129}$$

True systematic errors are proportional to the range of action of object L, to the square of the rate of motion V with respect to the surface of the Earth and depend on the average value of the angle of inclination of the ground speed vector with respect to the direction of the arbitrary equator from the starting time to the moment under consideration.

It is not difficult to verify through the use of equations (2.129) that where $\Delta\Psi_{av} = 0.5^\circ$ ($\Delta\xi_{av} = \pm\frac{1}{100}$ L), V ≤ 500 m/sec and L ≤ 10,000 km systematic error $\delta\dot{V}_\xi^{I}$ does not exceed the dead zone of contemporary accelerometers (10^{-4}g - 10^{-5}g), and therefore the simplification discussed is permissible.

Also, the value of systematic error $\delta\dot{V}_\zeta$ does not exceed the dead zones of contemporary accelerometers where V < 1500 m/sec and L < 1500 km.

For objects in motion at comparatively low altitudes H << R_{ea}, in the first three INS equations pertaining to $\dot{V}_\xi$, $\dot{V}_\eta$, $\dot{V}_\zeta$ it is possible to disregard the influence of H, and consider that

$$\left.\begin{aligned} \dot{V}_\xi &= a_\xi - \frac{V_\xi V_\zeta}{R_{ea}}\tan\varphi_{arb} - \frac{V_\xi V_\eta}{R_{ea}} - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta); \\ \dot{V}_\eta &= a_\eta + \frac{V_\xi^2}{R_{ea}} + \frac{V_\zeta^2}{R_{ea}} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta; \\ \dot{V}_\zeta &= a_\zeta + \frac{V_\xi^2}{R_{ea}}\tan\varphi_{arb} - \frac{V_\eta V_\zeta}{R_{ea}} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi). \end{aligned}\right\} \tag{2.130}$$

Here the following systematic errors in the determination of $\dot{V}_\xi$, $\dot{V}_\eta$, $\dot{V}_\zeta$ will occur:

$$\left.\begin{aligned} \delta_{sys}\dot{V}_\xi^{II} &= -V_\xi(V_\eta + V_\zeta\tan\varphi_{arb})\frac{H}{R_{ea}^2}; \\ \delta_{sys}\dot{V}_\eta^{II} &= (V_\xi^2 + V_\zeta^2)\frac{H}{R_{ea}^2}; \\ \delta_{sys}\dot{V}_\zeta^{II} &= (V_\xi^2\tan\varphi_{arb} - V_\eta V_\zeta)\frac{H}{R_{ea}^2}. \end{aligned}\right\} \tag{2.131}$$

We must consider that for objects in flight near the arbitrary equator, /70

$$\left.\begin{aligned} V_\xi &\cong V\cos\theta\cos\Delta\Psi_{av};\\ V_\eta &\cong V\sin\theta;\\ \dot{V}_\zeta &\cong V\cos\theta\sin\Delta\Psi_{av}. \end{aligned}\right\} \qquad (2.132)$$

In considering further that $\Delta\Psi_{av}$ is sufficiently small, we obtain

$$\left.\begin{aligned} V_\xi &\cong V\cos\theta;\\ V_\eta &\cong V\sin\theta;\\ V_\zeta &\cong V\cos\theta\Delta\Psi_{av} \end{aligned}\right\} \qquad (2.133)$$

Having substituted the values V_ξ, V_η, V_ζ from (2.133) in equations (2.130) and having taken the low value of Φ_{arb} into consideration, we have

$$\left.\begin{aligned} \delta_{sys}V_\xi^{II} &\cong -\frac{1}{2}\frac{V^2H}{R_{ea}^2}\sin 2\theta;\\ \delta_{sys}V_\eta^{II} &\cong \frac{V^2H}{R_{ea}^2}\cos^2\theta;\\ \delta_{sys}V_\zeta^{II} &\cong -\frac{1}{2}\frac{V^2H}{R_{ea}^2}\Delta\Psi_{av}\sin 2\theta. \end{aligned}\right\} \qquad (2.134)$$

It follows from equations (2.134) that INS systematic errors in the determination of accelerations $\dot{V}_\xi$, $\dot{V}_\eta$, $\dot{V}_\zeta$ are proportional to the square of the speed and the altitude of the moving object, and also depend on the trajectory angle.

An analysis of the equation (2.134) reveals that with horizontal flight ($\theta \leqslant 5°$, V < 1000 m/sec, H < 30 km), systematic error data does not exceed the dead zones of contemporary accelerometers ($10^{-4}g \div 10^{-5}g$), and therefore any consideration for H in the kinematic INS equations for such objects has no meaning.

It is possible to disregard the influence of terms containing V_η in the first and third equations (2.50) for objects with a basic trajectory segment close to the horizontal ($V_\eta \cong 0$):

$$\left.\begin{aligned} \dot{V}_\xi &= a_\xi - \frac{V_\xi V_\zeta}{R_{ea}+H}\tan\varphi_{arb} - 2\Omega_\eta V_\zeta;\\ \dot{V}_\zeta &= a_\eta + \frac{V_\xi^2}{R_{ea}+H}\tan\varphi_{arb} + 2\Omega_\eta V_\xi. \end{aligned}\right\} \qquad (2.135)$$

The values of systematic errors caused by this simplification are determined as the difference between equations (2.135) and equations (2.50) for $\dot{V}_\xi$ and $\dot{V}_\zeta$: /71

$$\left.\begin{aligned}\delta_{sys}\dot{V}_\xi^{II} &= \frac{1}{R_{ea}+H}V_\eta V_\xi + 2\Omega_\zeta V_\eta;\\ \delta_{sys}\dot{V}_\zeta^{II} &= \frac{1}{R_{ea}+H}(V_\eta V_\zeta + \Omega_\xi V_\eta).\end{aligned}\right\} \qquad (2.136)$$

For an approximate evaluation of the errors we may assume that

$$\left.\begin{aligned}\Omega_\zeta &\cong \Omega;\\ \Omega_\xi &\cong \Omega,\end{aligned}\right\} \qquad (2.137)$$

although in practice these equalities are not simultaneously feasible.

Then, with consideration for (2.124) and (2.133)

$$\left.\begin{aligned}\delta_{sys}\dot{V}_\xi^{II} &\cong \frac{V^2}{2R_{ea}^2}\sin 2\theta + 2\Omega V\sin\theta;\\ \delta_{sys}\dot{V}_\zeta^{II} &\cong \frac{V^2}{2R_{ea}^2}\sin 2\theta\Delta\Psi_{a\grave{v}} + 2\Omega V\sin\theta.\end{aligned}\right\} \qquad (2.138)$$

True systematic errors $\delta_{sys}\dot{V}_\xi^{II}$, $\delta_{sys}\dot{V}_\zeta^{II}$ contain two terms each: one of these is proportional to the square of object speed with respect to the surface of the Earth, and the second is proportional to the first stage of flight speed and to the sine of the trajectory angle. Therefore at low flight speeds it is feasible to consider the second term, and at high speeds, the first.

In evaluating errors in accordance with equations (2.138), it is possible to verify that for objects in flight with velocities V > 1000 m/sec for $\theta \leqslant 5°$, the true systematic error values exceed the limits of the dead zones of contemporary accelerometers. Here it is also necessary to take into consideration that for the majority of objects the period of time when accelerations $\dot{V}_\xi$, $\dot{V}_\zeta \neq 0$, is determined by the time of flight, climbing and descent.

In the determination of coordinates, INS errors caused by these systematic errors may be found in a first approximation from the equation (2.9), in which $\delta_{sys}\dot{V}_\xi^{II}$ and $\delta_{sys}\dot{V}_\zeta^{II}$ must be substituted in place of Δa_0

$$\left.\begin{aligned}\Delta\lambda_{arb}^{III} &= \frac{\delta_{sys}\dot{V}_\xi^{II}}{g_0}\left(1-\cos\sqrt{\frac{g_0}{R_{ea}}}\cdot t\right);\\ \Delta\varphi_{arb}^{III} &= \frac{\delta_{sys}\dot{V}_\zeta^{II}}{g_0}\left(1-\cos\sqrt{\frac{g_0}{R_{ea}}}\cdot t\right).\end{aligned}\right\} \qquad (2.139)$$

For objects in motion with velocities V < 500 m/sec and attaining programmed altitude H = 30 km in t < 100 sec, true systematic errors of the INS in the determination of angular coordinates λ_{arb} and ϕ_{arb} will be found within the limits of tenths of an angular minute. /72

For objects with V > 500 m/sec and t > 100 sec, the influence of V_η in INS equations must be considered.

There are terms in the equations for $\dot{V}_\xi$, $\dot{V}_\eta$, $\dot{V}_\zeta$, which contain Ω_ξ, Ω_η, Ω_ζ; these values for short-range objects may be assumed to be constant and equal to the values of these parameters at the point of reference: $\Omega_\xi = \Omega_{\xi 0}$, $\Omega_\eta = \Omega_{\eta 0}$, $\Omega_\zeta = \Omega_{\zeta 0}$. Here the following systematic errors will occur:

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\xi^{III} &= -2[V_\zeta(\Omega_{\eta 0}-\Omega_\eta)-V_\eta(\Omega_{\zeta 0}-\Omega_\zeta)];\\
\delta_{sys}\dot{V}_\eta^{III} &= -2[V_\xi(\Omega_{\zeta 0}-\Omega_\zeta)-V_\zeta(\Omega_{\xi 0}-\Omega_\xi)];\\
\delta_{sys}\dot{V}_\zeta^{III} &= -2[V_\eta(\Omega_{\xi 0}-\Omega_\xi)-V_\xi(\Omega_{\eta 0}-\Omega_\eta)].
\end{aligned}\right\} \qquad (2.140)$$

If we assume for short-range objects (when ϕ_{arb} and λ_{arb} are small) that

$$\left.\begin{aligned}
\Omega_\xi &\cong \Omega_{\xi 0}-\Omega_{\eta 0}\lambda_{arb}\\
\Omega_\eta &\cong \Omega_{\eta 0}+\Omega_{\xi 0}\lambda_{arb}\\
\Omega_\zeta &\cong \Omega_{\zeta 0},
\end{aligned}\right\} \qquad (2.141)$$

then after transformations, considering (2.133) and assuming that $\Omega_{\xi 0} \cong \Omega_{\eta 0} \cong \Omega_{\zeta 0} \cong \Omega$, we obtain

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\xi^{III} &\cong -2\Omega V(\varphi_{arb}\sin\theta-\lambda_{arb}\Delta\Psi_{av}\cos\theta);\\
\delta_{sys}\dot{V}_\eta^{III} &\cong 2\Omega V\cos\theta(\varphi_{arb}+\lambda_{arb}\Delta\Psi_{av});\\
\delta_{sys}\dot{V}_\zeta^{III} &\cong -2\Omega V\lambda_{arb}(\sin\theta+\cos\theta).
\end{aligned}\right\} \qquad (2.142)$$

For objects having the basic trajectory segment in the horizontal direction ($\theta \cong 0$),

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_\xi^{III} &\cong 2\Omega V\lambda_{arb}\Delta\Psi_{av};\\
\delta_{sys}\dot{V}_\eta^{III} &\cong 2\Omega V(\varphi_{arb}+\lambda_{arb}\Delta\Psi_{av});\\
\delta_{sys}\dot{V}_\zeta^{III} &\cong 2\Omega V\lambda_{arb}
\end{aligned}\right\} \qquad (2.143)$$

The calculations accomplished in accordance with equations (2.143) reveal that the systematic errors $\delta_{sys}\dot{V}^{III}_{\xi}$ and $\delta_{sys}\dot{V}^{III}_{\eta}$ for short-range objects lie within the limits of the dead zones of contemporary accelerometers, but $\delta_{sys}\dot{V}^{III}_{\zeta}$ exceeds these limits.

On the basis of what has already been stated for short-range objects accomplishing flights near the arbitrary equator, it is possible to recommend the following equations with respect to $\dot{V}_{\xi}$, $\dot{V}_{\eta}$, $\dot{V}_{\zeta}$: /73

$$\left.\begin{aligned}\dot{V}_{\xi}&=\alpha_{\xi}-2\Omega_{\eta 0}V_{\zeta};\\ \dot{V}_{\zeta}&=a_{\zeta}+2V_{\xi}(\Omega_{\eta 0}+\Omega_{\xi 0}\lambda_{arb});\end{aligned}\right\} \qquad (2.144)$$

for objects having a medium range of action

$$\left.\begin{aligned}\dot{V}_{\xi}&=a_{\xi}-\frac{\dot{V}_{\xi}V_{\eta}}{R_{ea}}-2(\Omega_{\eta}V_{\zeta}-\Omega_{\zeta}V_{\eta});\\ \dot{V}_{\eta}&=a_{\eta}+\frac{V_{\xi}^{2}}{R_{ea}}-2(\Omega_{\zeta}V_{\xi}-\Omega_{\xi}V_{\zeta})+g_{\eta};\\ \dot{V}_{\zeta}&=a_{\zeta}-\frac{V_{\eta}V_{\zeta}}{R_{ea}}-2(\Omega_{\xi}V_{\eta}-\Omega_{\eta}V_{\xi}).\end{aligned}\right\} \qquad (2.145)$$

After simplifications it is necessary to evaluate the full INS systematic errors

$$\left.\begin{aligned}\delta_{sys}\dot{V}_{\xi}&=\sum_{i=1}^{n}\delta^{i}_{sys}\dot{V}_{\xi};\\ \delta_{sys}\dot{V}_{\eta}&=\sum_{i=1}^{n}\delta^{i}_{sys}\dot{V}_{\eta};\\ \delta_{sys}\dot{V}_{\zeta}&=\sum_{i=1}^{n}\delta^{i}_{sys}\dot{V}_{\zeta}\end{aligned}\right\} \qquad (2.146)$$

and approximately

$$\left.\begin{aligned}\Delta\lambda_{arb,m}&\cong\frac{\delta_{sys}\dot{V}_{\xi}}{g_{0}}\left(1-\cos\sqrt{\frac{g_{0}}{R}}\cdot t\right);\\ \Delta\varphi_{arb,m}&\cong\frac{\delta_{sys}\dot{V}_{\zeta}}{g_{0}}\left(1-\cos\sqrt{\frac{g_{0}}{R}}\cdot t\right).\end{aligned}\right\} \qquad (2.147)$$

The latter are obtained from equations (2.9) by replacement of Δa by $\delta_{sys}\dot{V}_{\xi}$ and $\delta_{sys}\dot{V}_{\zeta}$

Systematic error $\delta_{sys}\dot{V}_{\eta}$ causes error in the determination of H, which may be calculated in accordance with the formula

$$\Delta H_{sys} \cong \frac{(\delta_{sys}\dot{V}_{\eta})^2 t}{2}. \tag{2.148}$$

We shall examine a simplification of the equations for the rate of change /74 of the arbitrary coordinates.

For objects in motion at comparatively low altitudes, we may consider that

$$\dot{\lambda}_{arb} = \frac{V_{\xi}}{R_{ea}\cos\varphi_{arb}}; \tag{2.149}$$

$$\dot{\varphi}_{arb} = \frac{V_{\zeta}}{R_{ea}}. \tag{2.150}$$

Here the following errors will occur:

$$\delta^{I}_{sys}\dot{\lambda}_{arb} = \frac{H}{(R_{ea}+H)R_{ea}\cos\varphi_{arb}} V_{\xi}; \tag{2.151}$$

$$\delta^{I}_{sys}\dot{\varphi}_{arb} = \frac{H}{(R_{ea}+H)R_{ea}} V_{\zeta}. \tag{2.152}$$

Considering equations (2.133) and the small value of ϕ_{arb}, and that $R_{ea} + H \cong R_{ea}$, we obtain

$$\left.\begin{aligned} \delta^{I}_{sys}\dot{\lambda}_{arb} &\cong \frac{HV}{R^2_{ea}}\cos\theta; \\ \delta^{I}_{sys}\dot{\varphi}_{arb} &\cong -\frac{HV}{R^2_{ea}}\cos\theta\Delta\Psi. \end{aligned}\right\} \tag{2.153}$$

For objects in horizontal flight,

$$\left.\begin{aligned} \delta^{I}_{sys\,arb}\dot{\lambda} &\cong \frac{V}{R^2_{ea}}H; \\ \delta^{I}_{sys\,arb}\dot{\varphi} &\cong -\frac{V}{R^2_{ea}}H\Delta\Psi. \end{aligned}\right\} \tag{2.154}$$

In an evaluation of systematic errors in the determination of the arbitrary coordinates the values $\delta^{I}_{sys}\dot{\lambda}_{arb}$ and $\delta^{I}_{sys}\dot{\phi}_{arb}$ must be integrated in time

$$\left.\begin{aligned}\delta_{sys}\lambda_{arb} &\cong \frac{1}{R_{ea}^2}\int_0^t VHdt;\\ \delta_{sys}\varphi_{arb} &\cong -\frac{1}{R_{ea}^2}\int_0^t VH\Delta\Psi dt.\end{aligned}\right\} \qquad (2.155)$$

If V = const, H = const, $\Delta\Psi = \Delta\Psi_{av}$ = const, then /75

$$\left.\begin{aligned}\Delta\lambda_{arb,m} &\cong \frac{H}{R_{ea}^2}L;\\ \Delta\varphi_{arb,m} &\cong -\frac{H}{R_{ea}^2}L\Delta\Psi_{av}.\end{aligned}\right\} \qquad (2.156)$$

It follows from equations (2.156) that INS errors in the determination of arbitrary coordinates, caused by the exclusion from the equations for $\dot{\lambda}_{arb}$ and $\dot{\phi}_{arb}$ of the flight altitude, are proportional to the product of the latter and the flight distance of the object (of the plane, enclosed between the surface of the Earth and the flight trajectory).

A numerical analysis of equations (2.156) reveals that for short-range objects accomplishing flights in the vicinity of the arbitrary equator, systematic INS errors in the determination of λ_{arb} do not exceed 1'. It is necessary to consider H in equations for $\dot{\lambda}_{arb}$ where L > 500 km H > 30 km.

It is also not difficult to verify that in the equations for $\dot{\phi}_{arb}$, the flight altitude must be taken into consideration only where L > 1500 km and H > 30 km. In addition, for objects in motion near the arbitrary equator, it is possible to assume that

$$\cos\varphi_{arb} \cong 1$$

and

$$\dot{\lambda}_{arb} = \frac{V_\xi}{R_{ea}}, \qquad (2.157)$$

which leads to the appearance of systematic error

$$\delta^{II}_{sys\,arb} \cong \frac{V_\xi}{R_{ea}}\left(1 - \frac{1}{\cos\varphi_{arb}}\right)$$

If we consider that $V_\xi \cong V = \text{const}$, then after integration where $\phi_{arb} = \text{const}$, we obtain

$$\delta^{II}_{sys\ arb} \lambda \cong \frac{L}{R_{ea}}\left(1 - \frac{1}{\varphi_{arb}}\right). \tag{2.158}$$

It is easy to see that systematic error in the determination of the λ_{arb} coordinate, caused by the simplification mentioned, is proportional to object range of action and during flight near the arbitrary equator has low values, which in practice may be disregarded for any distance which coincides with the dimensions of the Earth.

Based on data that has already been presented, it is possible to recommend the following INS equations with reference to $\dot{\lambda}_{arb}$ and ϕ_{arb} for objects in motion near the arbitrary equator at altitudes H < 30 km:

- At distance L < 500 km /76

$$\left.\begin{aligned} \dot{\lambda}_{arb} &= \frac{1}{R_{ea}} V_\xi; \\ \dot{\varphi}_{arb} &= -\frac{1}{R_{ea}} V_\zeta; \end{aligned}\right\} \tag{2.159}$$

- At distances 500 < L < 1500

$$\left.\begin{aligned} \dot{\lambda}_{arb} &= \frac{1}{R_{ea} + H} V_\xi; \\ \dot{\varphi}_{arb} &= -\frac{1}{R_{ea}} V_\zeta. \end{aligned}\right\} \tag{2.160}$$

After simplification it is feasible to evaluate full systematic error

$$\left.\begin{aligned} \delta_{sys\ arb} \dot{\lambda} &= \sum_{i=1}^{2} \delta^i_{sys\ arb} \dot{\lambda}; \\ \delta_{sys\ arb} \dot{\varphi} &= \delta^1_{sys\ arb} \dot{\varphi}; \\ \delta_{sys\ arb} \lambda &= \sum_{i=1}^{2} \delta^i_{sys\ arb} \lambda; \\ \delta_{sys\ arb} \varphi &= \delta^1_{sys\ arb} \varphi \end{aligned}\right\} \tag{2.161}$$

We shall examine the simplifications of the equations for angular velocity projection of the Earth's rotation.

For objects in motion in the vicinity of the arbitrary equator, the values ϕ_{arb} are relatively small, and therefore the equations with respect

to Ω_ξ, Ω_η, Ω_ζ may be simplified to the form

$$\left.\begin{aligned}\Omega_\xi &= \Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0}\sin\lambda_{arb};\\ \Omega_\eta &= \Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb} - \Omega_{\zeta 0}\varphi_{arb};\\ \Omega_\zeta &= (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\varphi_{arb} + \Omega_{\zeta 0}.\end{aligned}\right\} \qquad (2.162)$$

Here the following systematic errors will occur:

$$\left.\begin{aligned}\delta^{I}_{sys}\Omega_\xi &= 0;\\ \delta^{I}_{sys}\Omega_\eta &= (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})(1-\cos\varphi_{arb}) - \Omega_{\zeta 0}(\varphi_{arb} - \sin\varphi_{arb});\\ \delta^{I}_{sys}\Omega_\zeta &= (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})(\varphi_{arb} - \sin\varphi_{arb}) + \Omega_{\zeta 0}(1-\cos\varphi_{arb}).\end{aligned}\right\} \qquad (2.163)$$

Taking equation (2.46) into consideration, we obtain /77

$$\left.\begin{aligned}\delta^{I}_{sys}\Omega_\eta &= \Omega[(\cos\varphi_0\cos\Psi_0\sin\lambda_{arb} + \sin\varphi_0\cos\lambda_{arb})(1-\cos\varphi_{arb}) +\\ &\quad + \cos\varphi_0\sin\Psi_0(\varphi_0 - \sin\varphi_{arb})];\\ \delta^{I}_{sys}\Omega_\zeta &= \Omega[(\cos\varphi_0\cos\Psi_0\sin\lambda_{arb} + \sin\varphi_0\cos\lambda_{arb})(\varphi_{arb} - \sin\varphi_{arb}) -\\ &\quad - \cos\varphi_0\sin\Psi_0(1-\cos\varphi_{arb})].\end{aligned}\right\} \qquad (2.164)$$

It is obvious that the systematic errors do not exceed the values

$$\left.\begin{aligned}\delta^{I}_{sys}\Omega_\eta &\leqslant \Omega[(\sin\lambda_{arb} + \cos\lambda_{arb})(1-\cos\varphi_{arb}) + (\varphi_{arb} - \sin\varphi_{arb})];\\ \delta^{I}_{sys}\Omega_\zeta &\leqslant \Omega[(\sin\lambda_{arb} + \cos\lambda_{arb})(\varphi_{arb} - \sin\varphi_{arb}) - 1 + \cos\varphi_{arb}].\end{aligned}\right\} \qquad (2.165)$$

Where $\phi_{arb} \leqslant 1^\circ$ ($\xi \cong 111$ km), the values of these errors do not exceed 10^{-9} 1/sec, which is significantly less than the drift rate of contemporary gyroscopes.

For objects in flight at distances in which it is possible to assume that $\cos\lambda_{arb} \cong 1$, $\sin\lambda_{arb} \cong \lambda_{arb}$, equations (2.162) may be simplified to the form

$$\left.\begin{aligned}\Omega_\xi &= \Omega_{\xi 0} - \Omega_{\eta 0}\lambda_{arb};\\ \Omega_\eta &= \Omega_{\eta 0} + \Omega_{\xi 0}\lambda_{arb};\\ \Omega_\zeta &= \Omega_{\zeta 0}.\end{aligned}\right\} \qquad (2.166)$$

Here the following systematic errors will be introduced:

$$\left.\begin{aligned}\delta^{II}_{sys}\Omega_\eta &= \Omega_{\xi 0}(\lambda_{arb} - \sin\lambda_{arb}\cos\varphi_{arb}) + \Omega_{\eta 0}(1-\cos\lambda_{arb}\cos\varphi_{arb}) +\\ &\quad + \Omega_{\zeta 0}\sin\varphi_{arb};\\ \delta^{II}_{sys}\Omega_\xi &= \Omega_{\xi 0}(1-\cos\lambda_{arb}) - \Omega_{\eta 0}(\lambda_{arb} - \sin\lambda_{arb});\\ \delta^{II}_{sys}\Omega_\zeta &= -\Omega_{\xi 0}(\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\sin\varphi_{arb} + \Omega_{\zeta 0}(1-\cos\varphi_{arb}).\end{aligned}\right\} \qquad (2.167)$$

Assuming that $\Omega_{\xi 0} = \Omega_{\eta 0} = \Omega_{\zeta 0} = \Omega$, we obtain

$$\left.\begin{aligned}
\delta^{II}_{sys}\Omega_{\xi} &= \Omega(1-\cos\lambda_{arb}-\lambda_{arb}+\sin\lambda_{arb});\\
\delta^{II}_{sys}\Omega_{\eta} &= \Omega[\lambda_{arb}-(\sin\lambda_{arb}+\cos\lambda_{arb})\cos\varphi_{arb}+1+\sin\varphi_{arb}];\\
\delta^{II}_{sys}\Omega_{\zeta} &= \Omega[1-\cos\varphi_{arb}-(\sin\lambda_{arb}+\cos\lambda_{arb})\sin\varphi_{arb}].
\end{aligned}\right\} \qquad (2.168)$$

We may verify by means of equations (2.168) that for objects in motion near the arbitrary equator where L < 500 km, the INS systematic errors in the determination of Ω_{ξ}, Ω_{η}, Ω_{ζ}, do not exceed the inherent drift rate of contemporary gyroscopes. For INS objects in flight with distance L > 500 km, it is necessary to employ equations (2.162).

After simplifications of the equations for Ω_{ξ}, Ω_{η}, Ω_{ζ}, it is feasible /78
to estimate the values of the full systematic errors in accordance with the following expressions

$$\left.\begin{aligned}
\delta_{sys}\Omega_{\xi} &= \sum_{i}^{n}\delta^{i}_{sys}\Omega_{\xi};\\
\delta_{sys}\Omega_{\eta} &= \sum_{i}^{n}\delta^{i}_{sys}\Omega_{\eta};\\
\delta_{sys}\Omega_{\zeta} &= \sum_{i}^{n}\delta^{i}_{sys}\Omega_{\zeta}.
\end{aligned}\right\}$$

We shall examine the simplifications of the equations for the vector projections of the absolute angular velocity (2.48).

For INS objects in flight near the arbitrary equator

$$\left.\begin{aligned}
\omega_{a\eta} &= \Omega_{\eta};\\
\omega_{a\zeta} &= \Omega_{\zeta}-\dot{\lambda}_{sys}
\end{aligned}\right\} \qquad (2.169)$$

Here

$$\left.\begin{aligned}
\delta^{I}_{sys}\omega_{a\eta} &= -\dot{\lambda}_{arb}\sin\varphi_{arb}\\
\delta^{I}_{sys}\omega_{a\zeta} &= -\dot{\lambda}_{arb}(1-\cos\varphi_{arb}).
\end{aligned}\right\} \qquad (2.170)$$

A numerical analysis of equations (2.170) reveals that where $V \leqslant 1500$ m/sec, the systematic errors $\delta^{I}_{sys}\omega_{a\eta}$ and $\delta^{I}_{sys}\omega_{a\zeta}$ do not exceed the angular velocities of the inherent drift of contemporary gyroscopes. Where V > 1500 m/sec, it is feasible to calculate both $\sin\phi_{arb}$ as well as $\cos\phi_{arb}$ in equations for $\omega_{a\eta}$ and $\omega_{a\zeta}$.

Let us examine the simplifications of the equations for the angular velocities of platform inclination.

In INS's for objects in motion near the arbitrary equator at comparatively small distances, at which platform inclination angles ψ_a, ϑ_a, γ_a are small, the equations pertaining to $\dot{\phi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$ may be simplified to the form

$$\left.\begin{aligned}\dot{\psi}_a &= \omega_{a\eta} - \omega_{a\zeta}\gamma_a;\\ \dot{\vartheta}_a &= \omega_{a\eta}\gamma_a + \omega_{a\zeta};\\ \dot{\gamma}_a &= \omega_{a\xi} - (\omega_{a\eta} - \omega_{a\zeta}\gamma_a)\vartheta_a.\end{aligned}\right\} \qquad (2.171)$$

Here the following systematic errors will occur:

$$\left.\begin{aligned}\delta^I_{sys}\dot{\psi}_a &= \omega_{a\xi}\left(1 - \frac{\cos\gamma_a}{\cos\vartheta_a}\right) - \omega_{a\zeta}\left(\gamma_a - \frac{\sin\gamma_a}{\cos\vartheta_a}\right);\\ \delta^I_{sys}\dot{\vartheta}_a &= \omega_{a\eta}(\gamma_a - \sin\gamma_a) + \omega_{a\zeta}(1 - \cos\gamma_a);\\ \delta^I_{sys}\dot{\gamma}_a &= \omega_{a\zeta}(\gamma_a\vartheta_a - \sin\gamma_a\tan\vartheta_a) - \omega_{a\eta}(\vartheta_a - \cos\gamma_a\tan\vartheta_a).\end{aligned}\right\} \qquad (2.172)$$

In evaluating the error as a first approximation we shall assume that /79

$$\left.\begin{aligned}\omega_{a\xi} &= \Omega_\xi - \dot{\varphi}_{arb}\\ \omega_{a\eta} &= \Omega_\eta;\\ \omega_{a\zeta} &= \Omega_\zeta - \dot{\lambda}_{arb}\\ \Omega_\xi &\cong \Omega;\ \Omega_\eta \cong \Omega;\ \Omega_\zeta \cong \Omega.\end{aligned}\right\} \qquad (2.173)$$

We note that in fact equality of the last three approximations cannot be satisfied simultaneously, but they may be employed for approximate calculation.

Then

$$\left.\begin{aligned}\delta^I_{sys}\dot{\psi}_a &= (\Omega - \dot{\varphi}_{arb})\left(1 - \frac{\cos\gamma_a}{\cos\vartheta_a}\right) - (\Omega - \dot{\lambda}_{arb})\left(\gamma_a - \frac{\sin\gamma_a}{\cos\vartheta_a}\right);\\ \delta^I_{sys}\dot{\vartheta}_a &= \Omega_\eta(\gamma_a - \sin\gamma_a) + (\Omega - \dot{\lambda}_{arb})(1 - \cos\gamma_a);\\ \delta^I_{sys}\dot{\gamma}_a &= (\Omega - \dot{\lambda}_{arb})(\gamma_a\vartheta_a - \sin\gamma_a\tan\vartheta_a) - \Omega(\vartheta_a - \cos\gamma_a\tan\vartheta_a).\end{aligned}\right\} \qquad (2.174)$$

It may be shown through equations (2.174) that for objects with a velocity of $V \leqslant 500$ m/sec and with a distance of $L < 500$ km, systematic errors by the simplifications indicated do not exceed the angular velocity of the inherent drift of contemporary gyroscopes.

For objects in flight at velocities $V > 500$ m/sec and distances $L > 500$ km, it is feasible to calculate the values of the parameters ϕ_a, ϑ_a, γ_a in

accordance with equations (2.49).

We shall examine the simplified kinematic equations of an INS with a platform oriented in the arbitrary coordinate system.

Based on the foregoing, and with motion along the arbitrary equator, it is possible to recommend the following INS kinematic equations:

- For short-range objects

$$\left.\begin{aligned}
&\dot{V}_\xi = a_\xi - 2\Omega_\eta V_\zeta;\\
&\dot{V}_\zeta = a_\zeta + 2\Omega_\eta V_\xi;\\
&\dot{\lambda}_{arb} = \frac{V_\xi}{R_{ea}};\\
&\dot{\varphi}_{arb} = -\frac{V_\zeta}{R_{ea}};\\
&\dot{\psi}_a = \Omega_\eta - (\Omega_\zeta - \dot{\lambda}_{arb})\gamma_a;\\
&\dot{\vartheta}_a = \Omega_\eta \gamma_a + \Omega_\zeta - \dot{\lambda}_{arb}\\
&\dot{\gamma}_a = \Omega_\xi - \dot{\varphi}_{arb} - [\Omega_\eta - (\Omega_\zeta - \dot{\lambda}_{arb})\gamma_a]\,\vartheta_a;\\
&\Omega_\xi = \Omega_{\xi 0} - \Omega_{\eta 0}\lambda_{arb}\\
&\Omega_\eta = \Omega_{\xi 0}\lambda_{arb} + \Omega_{\eta 0};\\
&\Omega_\zeta = \Omega_{\zeta 0},
\end{aligned}\right\} \qquad (2.175)$$

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where $\Omega_{\xi 0}$, $\Omega_{\eta 0}$, $\Omega_{\zeta 0}$ are determined from equations (2.46) in accordance with the known geographic latitude (ϕ_0) and the course angle (Ψ_0) of the arbitrary equator at the point of reference.

It is apparent that an INS constructed in accordance with equations (2.175) may be a two-component system, since in the first two equations (2.175) the vertical velocity component V_η and altitude H are lacking;

- For medium-range objects,

$$\left.\begin{aligned}
&\dot{V}_\xi = a_\xi - \frac{V_\xi V_\eta}{R_{ea}} - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta);\\
&\dot{V}_\eta = a_\eta + \frac{V_\xi^2}{R_{ea}} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta;\\
&\dot{V}_\zeta = a_\zeta - \frac{V_\eta V_\zeta}{R_{ea}} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi);\\
&\dot{\lambda}_{arb} = \frac{1}{R_{ea} + H} V_\xi;\\
&\dot{H} = V_\eta;\\
&\dot{\varphi}_{arb} = -\frac{V_\xi}{R_{ea}};
\end{aligned}\right\} \qquad (2.176)$$

$$
\left.
\begin{aligned}
\dot{\psi}_a &= \frac{1}{\cos\vartheta_a}[\Omega_\eta \cos\gamma_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a];\\
\dot{\vartheta}_a &= \Omega_\eta \sin\gamma_a + (\Omega_\zeta - \dot{\lambda}_{arb})\cos\gamma_a;\\
\dot{\gamma}_a &= \Omega_\xi - \dot{\varphi}_{arb} - [\Omega_\eta \cos\gamma_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a]\tan\vartheta_a;\\
\Omega_\xi &= \Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0} - \sin\lambda_{arb};\\
\Omega_\eta &= \Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb} - \Omega_{\zeta 0}\varphi_{arb}\\
\Omega_\zeta &= (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\varphi_{arb} + \Omega_{\zeta 0},
\end{aligned}
\right\} \qquad (2.176)
$$

where $\Omega_{\xi 0}$, $\Omega_{\eta 0}$, $\Omega_{\zeta 0}$ are determined from equations (2.46).

It is apparent that an INS constructed in accordance with equations (2.176) must be a three-component system, since the equations for $\dot{V}_\xi$ and $\dot{V}_\zeta$ include the vertical velocity component $\dot{V}_\eta$ and cross links occur with the velocity vector components V_ξ, V_η, V_ζ.

It is necessary to state that the simplified kinematic equations cited /81
for the INS are formulated on the basis of a first approximation; it is therefore feasible to introduce additional refinements in accuracy in accordance with the technique described for objects having a kinematic parameters which are near the limits of the classification indicated.

§2.5. INS Systematic Error Equations

Systematic INS errors are caused by an imperfection in the system of kinematic equations.

Let us examine systematic INS errors caused by simplifications of the basic kinematic equations. In order to simplify the task we shall take as initial data the kinematic equations for a sphere as an approximation of the figure of the Earth.

Systematic INS errors may be calculated in accordance with data for the solution of the corresponding simplified system of kinematic equations, in accordance with known true values of the kinematic parameters required, or, in a first approximation, by means of solving the corresponding linearized equations which we call the equations of systematic errors.

It has already been pointed out that the calculation of INS systematic errors caused by simplifications is possible through the solving of the simplified and the full non-linear system of kinematic equations.

As a result, the instrumental (considering only systematic errors) and the true values, respectively, of kinematic parameters which enter into the system of equations will be obtained.

Systematic INS errors are determined as the difference between the instrumental and true values of the parameters which are of interest to us. It should be noted that the accuracy with which the systematic INS errors may be calculated is determined by the possibilities of employing a computer.

In addition to calculating systematic errors by means of solving the nonlinear system of INS kinematic equations, a method also exists which permits us to obtain the instantaneous error values in a first approximation. For this purpose it is necessary to solve the appropriate system of systematic error equations.

The equations may be derived by the linearization of the system of kinematic equations for the INS examined with consideration for partial systematic errors in the determination of each parameter of the left sides.

The coefficients of the linearized equations in a general case will be functions of the true values of the kinematic parameters which enter into the system and therefore they will change with time.

Systematic INS errors in the determination of all kinematic parameters are obtained directly as a result of solving the linearized system of kinematic equations.

The initial data for the solution are the true kinematic parameter values /82 which may be found by solving the complete system of kinematic equations with previously established values of relative accelerations, velocities and coordinates.

We shall examine the systematic error equations for an INS with a platform oriented in the geographic coordinate system.

Any type of simplification of the kinematic INS equations, as has already been pointed out, leads to the appearance of systematic errors in the determination of each parameter which enters into the kinematic equations. The values of these parameters - considering systematic errors - are designated with an asterisk, in order to distinguish these from the true values:

$$\left.\begin{array}{lll}
\dot{V}_H^*=\dot{V}_N+\Delta\dot{V}_{N\,\dot{\text{sys}}} & \dot{V}_H^*=\dot{V}_H+\Delta\dot{V}_{H\dot{\text{sys}}} & \dot{V}_E^*=\dot{V}_E+\Delta\dot{V}_{E\,\dot{\text{sys}}} \\
V_N^*=V_N+\Delta V_{N\dot{\text{sys}}} & V_H^*=V_H+\Delta V_{H\dot{\text{sys}}} & V_E^*=V_E+\Delta V_{E\,\dot{\text{sys}}} \\
\dot{\lambda}^*=\dot{\lambda}+\Delta\dot{\lambda}_{\dot{\text{sys}}} & \dot{H}^*=\dot{H}+\Delta\dot{H}_{\dot{\text{sys}}} & \dot{\varphi}^*=\dot{\varphi}+\Delta\dot{\varphi}_{\dot{\text{sys}}} \\
a_N^*=a_N+\Delta a_{N\,\dot{\text{sys}}} & a_H^*=a_H+\Delta a_{H\,\dot{\text{sys}}} & a_E^*=a_E+\Delta a_{E\,\dot{\text{sys}}} \\
\lambda^*=\lambda+\Delta\lambda_{\dot{\text{sys}}} & H^*=H+\Delta H_{\dot{\text{sys}}} & \varphi=\varphi+\Delta\varphi_{\dot{\text{sys}}} \\
\dot{\psi}_a^*=\dot{\psi}_a+\Delta\dot{\psi}_{a.\,\dot{\text{sys}}} & \dot{\vartheta}_a^*=\dot{\vartheta}_a+\Delta\dot{\vartheta}_{a.\,\dot{\text{sys}}} & \dot{\gamma}_a^*=\dot{\gamma}_a+\Delta\dot{\gamma}_{a.\,\dot{\text{sys}}} \\
\psi_a^*=\psi_a+\Delta\psi_a; & \vartheta_a^*=\vartheta_a+\Delta\vartheta_a; & \gamma_a^*=\gamma_a+\Delta\gamma_a,
\end{array}\right\} \qquad (2.177)$$

where $\dot{V}^*_N$; $\dot{V}^*_H$; $\dot{V}^*_E$ are instrument instantaneous values of the coefficients;
$\dot{V}_N$, $\dot{V}_H$, $\dot{V}_E$ are true instantaneous values of the parameters;
$\Delta\dot{V}_{N\,sys}$, $\Delta\dot{V}_{H\,sys}$, $\Delta\dot{V}_{E\,sys}$ are INS systematic errors in the determination of each parameter.

Instrument acceleration values a^*_N, a^*_H, a^*_E differ from the true values since the accelerometer measurement axes, when systematic errors are present, deviate from the directions of the trihedron axes (Figure 2.23):

$$\left.\begin{aligned}
a^*_N &= a_x = a_N \cos\Delta\psi_{a\,sys}\cos\Delta\vartheta_{a\,sys} + a_H \sin\Delta\vartheta_{a\,sys} \\
&\quad - a_E \sin\Delta\psi_{a\,sys}\cos\Delta\vartheta_{a\,sys} \\
a^*_H &= a_y = a_N(\sin\Delta\psi_{a\,sys}\sin\Delta\gamma_{a\,sys}\cos\Delta\psi_{a\,sys}\sin\Delta\vartheta_{a\,sys}\cos\Delta\gamma_{a\,sys}) + \\
&\quad + a_H\cos\Delta\vartheta_{a\,sys}\cos\Delta\gamma_{a\,sys} + a_E(\cos\Delta\psi_{a\,sys}\sin\Delta\gamma_{a\,sys} + \\
&\quad + \sin\Delta\psi_{a\,sys}\sin\Delta\vartheta_{a\,sys}\cos\Delta\gamma_{a\,sys}); \\
a^*_E &= a_z = a_N(\sin\Delta\psi_{a\,sys}\cos\Delta\gamma_{a\,sys} + \cos\Delta\psi_{a\,sys}\sin\Delta\vartheta_{a\,sys}\sin\Delta\gamma_{a\,sys}) - \\
&\quad - a_{sys}\cos\Delta\vartheta_{a\,sys}\sin\Delta\gamma_{a\,sys} + a_E(\cos\Delta\psi_{a\,sys}\cos\Delta\gamma_{a\,sys} - \\
&\quad - \sin\Delta\psi_{a\,sys}\sin\Delta\vartheta_{a\,sys}\sin\Delta\gamma_{a\,sys});
\end{aligned}\right\} \qquad (2.178)$$

where a_x, a_y, a_z are accelerations measured by the system accelerometers with deviation of their responsive axes from the axes of the arbitrary trihedron by the angle $\Delta\psi_{a\,sys}$, $\Delta\vartheta_{a\,sys}$, $\Delta\gamma_{a\,sys}$.

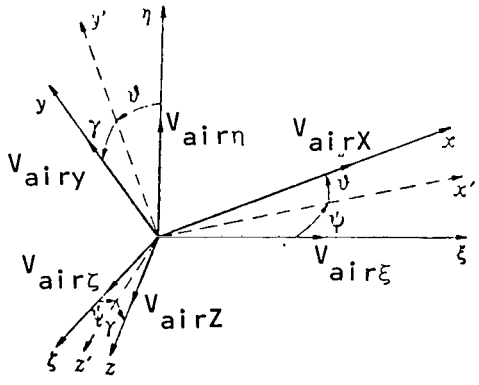

Figure 2.23

If we consider the angles $\Delta\psi_{a\,sys}$, $\Delta\vartheta_{a\,sys}$, $\Delta\gamma_{a\,sys}$ to be sufficiently small, then

$$\left.\begin{aligned}
a^*_N &\cong a_N + a_H(\vartheta^*_a - \vartheta_a) - a_E(\psi^*_a - \psi_a); \\
a^*_H &\cong a_N(\vartheta^*_a - \vartheta_a) + a_H + a_E(\gamma^*_a - \gamma_a); \\
a^*_E &\cong a_N(\psi^*_a - \psi_a) - a_H(\gamma^*_a - \gamma_a) + a_E,
\end{aligned}\right\} \qquad (2.179)$$

$$\left.\begin{aligned}
\Delta a_{N_{sys}} &= a_H\Delta\vartheta_{a\,sys} - a_E\Delta\psi_{a\,sys}; \\
\Delta a_{H_{sys}} &= -a_N\Delta\vartheta_{a\,sys} + a_E\Delta\gamma_{a\,sys}; \\
\Delta a_{E_{sys}} &= a_N\Delta\psi_{a\,sys} - a_E\Delta\gamma_{a\,sys}
\end{aligned}\right\} \qquad (2.180)$$

Kinematic INS equations for short-range objects formulated in accordance with the system of simplified equations (2.120), with consideration for the designations (2.177) and expressions (2.179) have the form

$$\left.\begin{aligned}
\dot{V}_N^* &= a_N - \frac{V_E^{*2}}{R_{ea}}\tan\varphi^* - 2\Omega V_E^* \sin\varphi^* + a_H(\dot{\vartheta}_a^* - \vartheta_a) - \\
&\quad - a_E(\dot{\psi}_a^* - \psi_a); \\
\dot{V}_E^* &= a_E + \frac{V_N^* V_E^*}{R_{ea}}\tan\varphi^* + 2\Omega V_N^* \sin\varphi^* + a_N(\dot{\psi}_a^* - \psi_a) - \\
&\quad - a_H(\gamma_a^* - \gamma_a); \\
\dot{\varphi}^* &= \frac{V_N^*}{R_{ea}}; \\
\dot{\varphi}^* &= \frac{V_N^*}{R_{ea}}; \\
\dot{\lambda}^* &= \frac{V_N^*}{R_{ea}\cos\varphi^*}; \\
\dot{\psi}_a^* &= (\dot{\lambda}^* + \Omega)\cos\varphi^* + \dot{\varphi}^*\gamma_a^*; \\
\dot{\vartheta}_a^* &= (\dot{\lambda}^* + \Omega)\sin\varphi^*\gamma_a^* - \dot{\varphi}^*; \\
\dot{\gamma}_a^* &= (\dot{\lambda}^* + \Omega) - [(\dot{\lambda}^* + \Omega)\sin\varphi^* - \dot{\varphi}^*\gamma_a^*]\vartheta_a^*.
\end{aligned}\right\} \qquad (2.181)$$

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Solution of the system (2.181) permits us to determine all the parameters with asterisks, which enter into the system. Initial data for the solution is provided by the true instantaneous values of the angles ψ_a, ϑ_a, γ_a and accelerations a_N, a_H, a_E. Under existing conditions the latter must be measured by the system accelerometers, but in the given case they are determined in advance together with the true values of the angles ψ_a, ϑ_a, γ_a and of the other parameters through the solution of the complete system of kinematic equations (2.38).

In turn, the value $\dot{V}_N(t)$, $\dot{V}_H(t)$, $\dot{V}_E(t)$, established by the kinematics of object motion, and the initial values of the true kinematic parameters serve as initial data for the solution of system (2.38).

Systematic INS errors may be determined as the differences

$$\left.\begin{aligned}
&\Delta\dot{V}_{N_{sys}} = \dot{V}_N^* - \dot{V}_N; \quad \Delta\dot{V}_{H_{sys}} = \dot{V}_H^* - \dot{V}_H; \quad \Delta\dot{V}_{E_{sys}} = \dot{V}_E^* - \dot{V}_E; \\
&\Delta V_{N_{sys}} = V_N^* - V_N; \quad \Delta V_{H_{sys}} = V_H^* - V_H; \quad \Delta V_{E_{sys}} = V_E^* - V_E; \\
&\Delta\varphi_{sys} = \varphi^* - \varphi; \quad \Delta H_{sys} = H^* - H; \quad \Delta\lambda_{sys} = \lambda^* - \lambda; \\
&\Delta\dot{\psi}_{a.sys} = \dot{\psi}_a^* - \dot{\psi}_a; \quad \Delta\dot{\vartheta}_{a.sys} = \dot{\vartheta}_a^* - \dot{\vartheta}_a; \quad \Delta\dot{\gamma}_{a.sys} = \dot{\gamma}_a^* - \dot{\gamma}_a; \\
&\Delta\psi_{a.sys} = \psi_a^* - \psi_a; \quad \Delta\vartheta_{a.sys} = \vartheta_a^* - \vartheta_a; \quad \Delta\gamma_{a\,sys} = \gamma_a^* - \gamma_a.
\end{aligned}\right\} \qquad (2.182)$$

We note that the calculation of systematic errors by means of the solution of the system of kinematic equations and the preparation of initial data is advantageously accomplished on an appropriate computer, since these equations are nonlinear.

For approximate evaluation of errors in an INS formulated in accordance with equations (2.120), it is necessary to derive a system of equations for systematic errors. In agreement with the technique outlined, this system is obtained by the linearization of equations (2.120) with consideration for partial systematic errors in the calculation of each parameter of the left sides. Allowing for equations (2.180), the systematic error equations in the operational form appear as /85

$$\left.\begin{aligned}
&p\Delta V_{N\,sys}+a_{1V_E}\Delta V_{E\,sys}+a_{1\varphi}\Delta\varphi_{sys}+a_{1\psi}\Delta\psi_{a.sys}+a_{1\vartheta}\Delta\vartheta_{a.sys}=\delta\dot{V}_{N\,sys};\\
&a_{2V_N}\Delta V_{N\,sys}+(p+a_{2V_E})\Delta V_{E\,sys}+a_{2\varphi}\Delta\varphi_{sys}+a_{2\psi}\Delta\psi_{a.sys}+\\
&\qquad+a_{2\gamma}\Delta\gamma_{a.sys}=\delta\dot{V}_{E\,sys};\\
&a_{3V_N}\Delta V_{N\,sys}+p\Delta\varphi_{sys}=\delta\dot{\varphi}_{sys};\\
&a_{4V_E}\Delta V_{E\,sys}+p\Delta\lambda_{sys}+a_{4\varphi}\Delta\varphi_{sys}=\delta\dot{\lambda}_{sys};\\
&a_{5\dot{\lambda}}p\Delta\lambda_{sys}+(a_{5\dot{\varphi}}p+a_{5\varphi})\Delta\varphi_{sys}+p\Delta\psi_{a.sys}+a_{5\gamma}\Delta\gamma_{a.sys}=\delta\dot{\psi}_{a\,sys};\\
&a_{6\dot{\lambda}}p\Delta\lambda_{sys}+(a_{6\dot{\varphi}}p+a_{6\varphi})\Delta\varphi_{sys}+p\Delta\vartheta_{a.sys}+a_{6\gamma}\Delta\gamma_{a.sys}=\delta\dot{\vartheta}_{a\,sys};\\
&a_{7\dot{\lambda}}p\Delta\lambda_{sys}+(a_{7\dot{\varphi}}p+a_{7\varphi})\Delta\varphi_{sys}+a_{7\vartheta}\Delta\vartheta_{a.sys}+(a_{7\gamma}+p)\Delta\gamma_{a.sys}=\delta\dot{\gamma}_{a\,sys},
\end{aligned}\right\}\qquad(2.183)$$

where the coefficients of the equations are

$$\left.\begin{aligned}
&a_{1V_E}=-2\frac{V_E}{R_{ea}}\tan\varphi+2\Omega\sin\varphi; && a_{2V_N}=-\frac{V_E}{R_{ea}}\tan\varphi-2\Omega\sin\varphi;\\
&a_{1\varphi}=\frac{V_E^2}{R_{ea}\cos^2\varphi}+2\Omega V_E\cos\varphi; && a_{2V_E}=\frac{-V_N\tan\varphi}{R_{ea}};\\
&a_{1\psi}=a_E; && a_{2\varphi}=\frac{-V_NV_E}{R_{ea}\cos^2\varphi}-2\Omega V_N\cos\varphi;\\
&a_{1\vartheta}=-a_H;\\
&a_{3V_N}=-\frac{1}{R_{ea}}; && a_{6\varphi}=-(\dot{\lambda}+\Omega)\gamma_a\cos\varphi;\\
&a_{4V_E}=-\frac{1}{R_{ea}\cos\varphi}; && a_{6\dot{\lambda}}=-\varphi\cdot\gamma_a;\\
&a_{4\varphi}=-\frac{V_E\sin\varphi}{R_{ea}\cos^2\varphi}; && a_{7\dot{\varphi}}=-\gamma_a\vartheta_a;\\
&a_{5\dot{\varphi}}=-\gamma_a; && a_{7\varphi}=(\dot{\lambda}+\Omega)\vartheta_a\cos\varphi;\\
&a_{5\varphi}=(\dot{\lambda}+\Omega)\gamma_a\sin\varphi;\\
&a_{5\dot{\lambda}}=-1; && a_{7\dot{\lambda}}=-1+\vartheta_a\varphi;\\
&a_{5\gamma}=-\dot{\varphi}; && a_{7\vartheta}=-\varphi\gamma_a+(\dot{\lambda}+\Omega)\varphi;\\
&a_{6\dot{\varphi}}=1; && a_{7\gamma}=-\dot{\varphi}\vartheta_a;\\
&a_{6\gamma}=-(\dot{\lambda}+\Omega)\varphi; && a_{2\psi}=-a_N;\quad a_{2\gamma}=a_N.
\end{aligned}\right\}\qquad(2.184)$$

The partial systematic errors on the right sides of the system equations (2.183), which are determined as the difference between the corresponding simplified (2.120) and full equations of the systems (2.38), are written in the form /86

$$\left.\begin{aligned}
\delta_{sys}\dot{V}_N &= -\frac{V_E^2 H}{R_{ea}^2}\tan\varphi + \frac{V_N V_H}{R_{ea}^2} H;\\
\delta_{sys}\dot{V}_E &= \frac{V_N V_E}{R_{ea}^2} H\tan\varphi + \frac{V_H V_E}{R_{ea}} + 2\Omega V_H \cos\varphi;\\
\delta_{sys}\dot{\varphi} &= V_N \frac{H}{R_{ea}(R_{ea}+H)};\\
\delta_{sys}\dot{\lambda} &= V_E \frac{H}{R_{ea}(R_{ea}+H)\cos\varphi};\\
\delta_{sys}\dot{\psi}_a &= (\dot{\lambda}+\Omega)\left(1-\frac{\cos\gamma_a}{\cos\vartheta_a}\right)+\dot{\varphi}\left(\gamma_a-\frac{\sin\gamma_a}{\cos\vartheta_a}\right);\\
\delta_{sys}\dot{\vartheta}_a &= (\dot{\lambda}+\Omega)(\gamma_a-\sin\gamma_a)-\dot{\varphi}(1-\cos\gamma_a);\\
\delta_{sys}\dot{\gamma}_a &= (\dot{\lambda}+\Omega)[(1-\cos\varphi)-\vartheta_a+\cos\gamma_a\tan\vartheta_a]+\\
&\quad +\dot{\varphi}(\gamma_a\vartheta_a-\sin\gamma_a\tan\vartheta_a).
\end{aligned}\right\} \qquad (2.185)$$

It is easy to see that the system of equations (2.183) for systematic error in the INS is linear with variable coefficients, the values of which are determined from equations (2.184). The true values of the kinematic parameters, which in turn are determined by the previously accomplished solution of the full system of kinematic equations (2.38), are the initial data for the computation of the coefficients of the equations, for the partial systematic errors, and for the solution of system (2.183).

Calculations accomplished with the aid of a computer reveal that for short-range objects the INS systematic errors may also be evaluated in constant values of the coefficients of equations (2.185), which are calculated in accordance with equations (2.181) with average kinematic parameter values.

In this calculation the systematic error values obtained are raised somewhat (by 10-15%) in comparison with the error values obtained through the solution of system (2.183) with variable coefficients.

Systematic error equations of INS's for medium-range objects, formulated in accordance with equations (2.121), may be derived in a similar fashion

$$\left.\begin{aligned}
p\Delta V_{N\,sys} + a_{1V_E}\Delta V_{E_{sys}} + a_{1\varphi}\Delta\varphi_{sys} + a_{1H}\Delta H_{sys} + a_{1\psi}\Delta\psi_{a.sys} +\\
+ a_{1\vartheta}\Delta\vartheta_{a.sys} = \delta_{sys}\dot{V}_N;\\
a_{2V_N}\Delta V_{N\,sys} + p\Delta V_{H\,sys} + a_{2V_E}\Delta V_{E\,sys} + a_{2\varphi}\Delta\varphi_{sys} + a_{2H}\Delta H +\\
+ a_{2\vartheta}\Delta\vartheta_{a.sys} + a_{2\gamma}\Delta\gamma_{a.sys} = \delta_{sys}\dot{V}_H;
\end{aligned}\right\} \qquad (2.186)$$

$$
\left.
\begin{aligned}
&a_{3V_N}\Delta V_{N_{sys}} + (p + a_{3V_E})\Delta V_{E_{sys}} + a_{3\varphi}\Delta\varphi_{sys} + a_{3H}\Delta H_{sys} + \\
&\qquad + a_{3\psi}\Delta\psi_{a_{sys}} + a_{3\gamma}\Delta\gamma_{a_{sys}} = \delta_{sys}\dot{V}_E;\\
&a_{4V_N}\Delta V_{N_{sys}} + p\Delta\varphi_{sys} + a_{4H}\Delta H_{sys} = \delta_{sys}\dot{\varphi};\\
&a_{5V_H}\Delta V_{H_{sys}} + p\Delta H = \delta_{sys}\dot{H};\\
&a_{6V_E}\Delta V_{E_{sys}} + a_{6\varphi}\Delta\varphi_{sys} + p\Delta\lambda_{sys} + a_{6H}\Delta H_{sys} = \delta_{sys}\dot{\lambda};\\
&(a_{7\dot{\varphi}}p + a_{7\varphi})\Delta\varphi_{sys} + a_{7\lambda}p\Delta\lambda_{sys} + p\Delta\psi_{a_{sys}} + a_{7\vartheta}\Delta\vartheta_{a_{sys}} + \\
&\qquad + a_{7\gamma}\Delta\gamma_{a_{sys}} = \delta_{sys}\dot{\psi}_a;\\
&(a_{8\dot{\varphi}}p + a_{8\varphi})\Delta\varphi_{sys} + a_{8\lambda}p\Delta\lambda_{sys} + p\Delta\vartheta_{a_{sys}} + a_{8\gamma}\Delta\sigma_{a_{sys}} = \delta_{sys}\dot{\vartheta}_a;\\
&(a_{9\dot{\varphi}}p + a_{9\varphi})\Delta\varphi_{sys} + a_{9\lambda}p\Delta\lambda_{sys} + a_{9\vartheta}\Delta\vartheta_{a_{sys}} + (p + a_{9\gamma})\Delta\gamma_{a_{sys}} = \delta_{sys}\dot{\gamma}_a,
\end{aligned}
\right\}
\qquad (2.186)
$$

/87

where the coefficients

$$
\left.
\begin{aligned}
a_{1V_E} &= -\frac{2V_E}{R_{ea}+H}\tan\varphi + \\
&\quad + 2\Omega\sin\varphi; & a_{2\vartheta} &= a_N;\\
a_{1\varphi} &= -\frac{V_E^2}{(R_{ea}+H)\cos^2\varphi} + & a_{2\gamma} &= -a_E;\\
&\quad + 2\Omega V_E\cos\varphi;\\
a_{1H} &= -\frac{V_E^2}{(R_{ea}+H)^2}\tan\varphi; & a_{2\varphi} &= 2\Omega V_E\sin\varphi;\\
a_{1\psi} &= a_E; & a_{2H} &= \frac{V_N^2}{(R_{ea}+H)^2} + \frac{V_E^2}{(R_{ea}+H)^2};\\
a_{1\vartheta} &= -a_H; & a_{3V_N} &= -\frac{V_E}{(R_{ea}+H)}\tan\varphi - \\
&&&\quad - 2\Omega\sin\varphi;\\
a_{2V_N} &= -2\frac{V_N}{R_{ea}+H}; & a_{3V_E} &= -\frac{V_H}{R_{ea}+H}\tan\varphi;\\
&& a_{3H} &= \frac{V_N V_E}{(R_{ea}+H)^2}\tan\varphi;\\
a_{2V_E} &= -2\frac{V_E}{R_{ea}+H} - & a_{3\varphi} &= -\frac{V_N V_E}{(R_{ea}+H)\cos^2\varphi} - \\
&\quad - 2\Omega\cos\varphi; & &\quad - 2\Omega V_N\cos\varphi;\\
a_{3H} &= +\frac{V_N V_E}{(R_{ea}+H)^2}\tan\varphi; & a_{7\varphi} &= \frac{(\dot{\lambda}+\Omega)}{\cos\vartheta_a}\sin\varphi\cos\gamma_a;
\end{aligned}
\right\}
\qquad (2.187)
$$

$$
\begin{aligned}
a_{4V_N} &= -\frac{1}{R_{ea}+H}; & a_{7\dot{\lambda}} &= -\cos\varphi\,\frac{\cos\gamma_a}{\cos\vartheta_a};\\
a_{3\psi} &= -a_N; & a_{7\vartheta_a} &= -\frac{[(\dot{\lambda}+\Omega)\cos\varphi\cos\gamma_a+\dot{\varphi}\sin\gamma_a]}{\cos^2\vartheta_a};\\
a_{3\gamma} &= a_H; & a_{7\gamma_a} &= -\left[\frac{\dot{\varphi}}{\cos\vartheta_a}\cos\gamma_a - \right.\\
& & & \left. -\frac{\sin\gamma_a}{\cos\vartheta_a}(\dot{\lambda}+\Omega)\cos\varphi\right];\\
a_{4H} &= \frac{V_H}{(R_{ea}+H)^2}; & a_{8\gamma} &= \cos\gamma_a;\\
a_{5V_H} &= -1; & a_{8\dot{\lambda}} &= -\sin\varphi\sin\gamma_a;\\
a_{6V_E} &= -\frac{1}{(R_{ea}+H)\cos\varphi}; & a_{8\varphi} &= -(\dot{\lambda}+\Omega)\sin\gamma_a\cos\varphi;\\
a_{6H} &= \frac{V_E}{(R_{ea}+H)^2\cos\varphi}; & a_{8\gamma} &= - \\
& & & -[(\dot{\lambda}+\Omega)\sin\varphi\cos\gamma_a+\dot{\varphi}\sin\gamma_a];\\
a_{6\varphi} &= -\frac{V_E\sin\varphi}{(R_{ea}+H)\cos^2\varphi}; & a_{9\dot{\varphi}} &= -\sin\gamma_a\tan\vartheta_a;\\
a_{7\dot{\varphi}} &= -\frac{\sin\gamma_a}{\cos\vartheta_a}; & a_{9\varphi} &= (\dot{\lambda}+\Omega)\sin\varphi+\\
a_{9\dot{\lambda}} &= -[\cos\varphi - & &+(\dot{\lambda}+\Omega)\cos\varphi\cos\gamma_a\tan\vartheta_a.\\
& -\sin\varphi\cos\gamma_a\tan\vartheta_a]; & & \\
a_{9\vartheta_a} &= & & \\
&= \frac{(\dot{\lambda}+\Omega)\sin\varphi\cos\gamma_a-\dot{\varphi}\sin\gamma_a}{\cos^2\vartheta_a}; & & \\
a_{9\gamma_a} &= -[(\dot{\lambda}+\Omega)\sin\varphi\sin\gamma_a+ & & \\
& +\dot{\varphi}\cos\gamma_a]\tan\vartheta_a; & &
\end{aligned}
\qquad (2.187)
$$

Partial systematic errors which enter into system (2.186), would with consideration for the change in the acceleration of gravity with respect to altitude, be determined as follows

$$
\begin{aligned}
\delta_{sys}\dot{V}_N &= \frac{V_N V_H}{R+H};\\
\delta_{sys}\dot{V}_H &= g\left[1-\frac{R_{ea}^2}{(R_{ea}+H)^2}\right];\\
\delta_{sys}\dot{V}_E &= \frac{V_H V_E}{R_{ea}+H}+2\Omega V_H\cos\varphi;
\end{aligned}
\qquad (2.188)
$$

$$\left.\begin{aligned}
\delta_{sys}\dot{\varphi}&=0;\\
\delta_{sys}\dot{H}&=0;\\
\delta_{sys}\dot{\lambda}&=0;\\
\delta_{sys}\dot{\psi}_a&=0;\\
\delta_{sys}\dot{\vartheta}_a&=0;\\
\delta_{sys}\dot{\gamma}_a&=0.
\end{aligned}\right\} \qquad (2.188)$$

Thus, INS systematic errors formulated in accordance with the system of kinematic equations (2.121) may be determined by solving the system (2.186).

Let us examine the systematic error equations for an INS with a platform oriented in the arbitrary coordinate system.

Parameter values which determine the kinematics of object motion in the arbitrary coordinate system, with consideration for systematic errors caused by simplification of the system of kinematic equations (2.50), may be presented in the form

$$\left.\begin{array}{lll}
\dot{V}^*_\xi=\dot{V}_\xi+\Delta\dot{V}_{\xi\,sys}; & \dot{V}^*_\eta=\dot{V}_\eta+\Delta\dot{V}_{\eta sys}; & \dot{V}^*_\zeta=\dot{V}_\zeta+\Delta\dot{V}_{\zeta sys}\\
\dot{V}^*_\xi=V_\xi+\Delta V_{\xi sys}; & V^*_\eta=V_\eta+\Delta V_{\eta sys}; & V^*_\zeta=\Delta_\zeta+\Delta V_{\zeta sys};\\
a^*_\xi=a_\xi+\Delta a_{\xi sys}; & a^*_\eta=a_\eta+\Delta\alpha_{\eta sys}{}^{1} & a^*_\zeta=a_\zeta+\Delta a_{\zeta sys};\\
\dot{\lambda}^*_{arb}=\dot{\lambda}_{arb}+\Delta\dot{\lambda}_{arb,\,m}; & \dot{H}^*=\dot{H}+\Delta\dot{H}_{sys}; & \dot{\varphi}^*_{arb}=\dot{\varphi}_{arb}+\Delta\dot{\varphi}_{arb,sys}\\
\lambda^*_{arb}=\lambda_{arb}+\Delta\lambda_{arb,\,m}; & H^*=H+\Delta H_{sys}; & \varphi^*_{arb}=\varphi_{arb}+\Delta\varphi_{arb,sys}\\
\dot{\psi}^*_a=\dot{\psi}_a+\Delta\dot{\psi}_{a\,sys}; & \dot{\vartheta}^*_a=\dot{\vartheta}_a+\Delta\dot{\vartheta}_{a\,sys}; & \dot{\gamma}^*_a=\dot{\gamma}_a+\Delta\dot{\gamma}_{a.\,sys}\\
\omega^*_{a\xi}=\omega_{a\xi}+\Delta\omega_{a\xi\,sys}; & \omega^*_{a\eta}=\omega_{a\eta}+\Delta\omega_{a\eta sys}; & \omega^*_{a\zeta}=\omega_{a\zeta}+\Delta\omega_{a\zeta\,sys}\\
\Omega^*_\xi=\Omega_\xi+\Delta\Omega_\xi; & \Omega^*_\eta=\Omega_\eta+\Delta\Omega_\eta; & \Omega^*_\zeta=\Omega_\zeta+\Delta\Omega_\zeta,
\end{array}\right\} \qquad (2.189)$$

where $\dot{V}^*_\xi$, $\dot{V}^*_\eta$, $\dot{V}^*_\zeta$ are instrument instantaneous values of the corresponding parameters, allowing for systematic errors;

$\dot{V}_\xi$, $\dot{V}_\eta$, $\dot{V}_\zeta$ are true instantaneous values for the corresponding parameters;

$\Delta\dot{V}_{\xi\,sys}$, $\Delta\dot{V}_{\eta\,sys}$, $\Delta\dot{V}_{\zeta\,sys}$ are systematic INS errors in the determination of the corresponding parameters.

It should be noted that the instrument acceleration values a^*_ξ, a^*_η, a^*_ζ represent indications by ideal accelerometers, the measurement axes of which are inclined from the axes of the arbitrary trihedron by the angles

[1] Translator's note: In the original Russian text, instrument acceleration α^*_η is defined as $\alpha^*_\eta = \alpha_\eta + \Delta V_{\eta\,sys}$.

$\Delta\Psi_{\text{a sys}}$, $\Delta\vartheta_{\text{a sys}}$, $\Delta\gamma_{\text{a sys}}$ and which are determined (Fig. 2.24) from the following relationships: /90

$$\left.\begin{aligned}
a_\xi^* = a_x = a_\xi \cos\Delta\psi_{\text{a.sys}} \cos\Delta\vartheta_{\text{a.sys}} + a_\eta \sin\Delta\vartheta_{\text{a.sys}} - \\
- a_\zeta \sin\Delta\psi_{\text{a.sys}} \cos\Delta\vartheta_{\text{a.sys}}; \\
a_\eta^* = a_y = a_\xi(\sin\Delta\psi_{\text{a.sys}} \sin\Delta\gamma_{\text{a.sys}} - \cos\Delta\psi_{\text{a.sys}} \sin\Delta\vartheta_{\text{a.sys}} \cos\Delta\gamma_{\text{a.sys}}) + \\
+ a_\eta \cos\Delta\vartheta_{\text{a.sys}} \cos\Delta\gamma_{\text{a.sys}} + a_\zeta(\cos\Delta\psi_{\text{a.sys}} \sin\Delta\gamma_{\text{a.sys}} + \\
+ \sin\Delta\psi_{\text{a.sys}} \sin\Delta\vartheta_{\text{a.sys}} \cos\Delta\gamma_{\text{a.sys}}); \\
a_\zeta^* = a_z = a_\xi(\sin\Delta\psi_{\text{a.sys}} \cos\Delta\gamma_{\text{a.sys}} + \cos\Delta\psi_{\text{a.sys}} \sin\Delta\vartheta_{\text{a.sys}} \sin\Delta\gamma_{\text{a.sys}}) - \\
- a_{a\eta} \cos\Delta\vartheta_{\text{a.sys}} \sin\Delta\gamma_{\text{a.sys}} + \\
+ a_\zeta(\cos\Delta\psi_{\text{a.sys}} \cos\Delta\gamma_{\text{a.sys}} - \sin\Delta\psi_{\text{a.sys}} \sin\Delta\vartheta_{\text{a.sys}} \sin\Delta\gamma_{\text{a.sys}}).
\end{aligned}\right\} \quad (2.190)$$

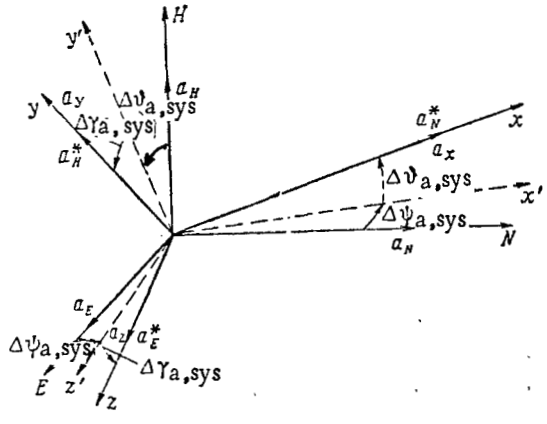

Figure 2.24

If we consider the angles $\Delta\psi_{\text{a sys}}$, $\Delta\vartheta_{\text{a sys}}$, $\Delta\gamma_{\text{a sys}}$ to be sufficiently small, then we obtain

$$\left.\begin{aligned}
a_\xi^* &\cong a_\xi + a_\eta(\vartheta_a^* - \vartheta_a) - a_\zeta(\psi_a^* - \psi_a); \\
a_\eta^* &\cong a_\eta - a_\xi(\vartheta_a^* - \vartheta_a) + a_\zeta(\gamma_a^* - \gamma_a); \\
a_\zeta^* &\cong a_\zeta + a_\xi(\psi_a^* - \psi_a) - a_\eta(\gamma_a^* - \gamma_a),
\end{aligned}\right\} \quad (2.191)$$

from which

$$\left.\begin{aligned}
\Delta a_\xi &= a_\eta \Delta\vartheta_{\text{a.sys}} - a_\zeta \Delta\psi_{\text{a.sys}}; \\
\Delta a_\eta &= a_\zeta \Delta\gamma_{\text{a.sys}} - a_\xi \Delta\vartheta_{\text{a.sys}}; \\
\Delta a_\zeta &= a_\xi \Delta\psi_{\text{a.sys}} - a_\eta \Delta\gamma_{\text{a.sys}}
\end{aligned}\right\} \quad (2.192)$$

The kinematic INS equations for short-range objects (2.175), which include consideration for systematic errors and also for equations (2.189) and (2.191), have the form: /91

$$\left.\begin{aligned}
\dot V_\xi^* &= a_\xi - 2\Omega_{\gamma 0}\dot V_\zeta^* + a_\eta(\vartheta_a^* - \vartheta_a) - a_\zeta(\psi_a^* - \psi_a); \\
\dot V_\zeta^* &= a_\zeta + 2\Omega_\eta^* V_\xi^* + a_\xi(\psi_a^* - \psi_a) - a_\eta(\gamma_a^* - \gamma_a); \\
\dot\lambda_{\text{arb}}^* &= \frac{1}{R_{\text{ea}}} V_\xi^*; \\
\dot\varphi_{\text{arb}}^* &= -\frac{1}{R_{\text{ea}}} V_\zeta^*; \\
\dot\psi_a^* &= \Omega_\eta^* - (\Omega_\zeta^* - \dot\lambda_{\text{arb}}^*)\gamma_a^*; \\
\dot\vartheta_a^* &= \Omega_\gamma \gamma_a^* + \Omega_\zeta^* - \lambda_{\text{arb}}^*
\end{aligned}\right\} \quad (2.193)$$

$$
\left.
\begin{aligned}
&\dot{\gamma}_a^* = \Omega_\xi^* - \varphi_{arb}^* - [\Omega_\eta^* - (\Omega_\zeta^* - \dot{\lambda}_{arb}^*)\gamma_a^*]\vartheta_a^*;\\
&\Omega_\xi^* = \Omega_{\xi 0} - \Omega_{\eta 0}\lambda_{arb}^*;\\
&\Omega_\eta^* = \Omega_{\zeta 0}\lambda_{arb}^* + \Omega_{\eta 0};\\
&\Omega_\zeta^* = \Omega_{\zeta 0},
\end{aligned}
\right\} \qquad (2.193)
$$

where values $\Omega_{\xi 0}$, $\Omega_{\eta 0}$, $\Omega_{\zeta 0}$ are determined from equations (2.46).

Solution of the system (2.193) permits us to determine all the parameters with asterisks which include systematic errors.

The true instantaneous values of the angle of ψ_a, ϑ_a, γ_a of the deviation of the $\xi\eta\zeta$ arbitrary trihedron from the $\xi_1\eta_1\zeta_1$ inertial trihedron and of accelerations a_ξ, a_η, a_ζ serve as initial data for the solution of this system.

These true parameter values, in conjunction with others which enter into system (2.193), may be determined by means of the preliminary solution of system (2.50) in accordance with known acceleration values $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$.

Systematic INS errors may be determined from equations (2.189) as the difference between instrument and true parameter values at the moment of time under consideration. They may be calculated approximately by solving the corresponding system of systematic error equations.

The latter may be derived by linearization of system (2.175), allowing for equations (2.192), and have the form:

$$
\left.
\begin{aligned}
&p\Delta V_{\xi\,sys} + a_{1V\zeta}\Delta V_{\zeta\,sys} + a_{1\psi}\Delta\psi_{arb,\,sys} + a_{1\vartheta}\Delta\vartheta_{a.\,sys} = \delta_{sys}\dot{V}_\xi;\\
&a_{2V\xi}\Delta V_{\xi\,sys} + p\Delta V_{\zeta\,sys} + a_{2\Omega_\eta}\Delta\Omega_{\eta\,sys} + a_{2\psi}\Delta\psi_{a.\,sys} + a_{2\gamma}\Delta\gamma_{a.\,sys} = \delta_{sys}\dot{V}_\zeta;
\end{aligned}
\right\} \qquad (2.194)
$$

/92

$$
\left.
\begin{aligned}
&p\Delta\lambda_{arb} + a_{3V\xi}\Delta V_{\xi\,sys} = \delta_{sys}\dot{\lambda}_{arb};\\
&p\Delta\varphi_{arb} + a_{4V\zeta}\Delta V_{\zeta\,sys} = \delta_{sys}\dot{\varphi}_{arb};\\
&a_{5\lambda}p\Delta\lambda_{arb,\,sys} + p\Delta\psi_{a.\,sys} + a_{5\gamma}\Delta\gamma_{a.\,sys} + a_{5\Omega_\eta}\Delta\Omega_{\eta\,sys} + a_{5\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot{\psi}_a;\\
&a_{6\lambda}p\Delta\lambda_{arb,\,sys} + p\Delta\vartheta_{a.\,sys} + a_{6\gamma}\Delta\gamma_{a.\,sys} + a_{6\Omega_\eta}\Delta\Omega_{\eta\,sys} + a_{6\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot{\vartheta}_a;\\
&a_{7\lambda}p\Delta\lambda_{arb,\,sys} + a_{7\dot{\varphi}}p\Delta\dot{\varphi}_{arb,\,sys} + a_{7\vartheta}\Delta\vartheta_{a.\,sys} + (p + a_{7\gamma})\Delta\gamma_{a.\,sys} +\\
&\qquad + a_{7\Omega_\xi}\Delta\Omega_{\xi\,sys} + a_{7\Omega_\eta}\Delta\Omega_{\eta\,sys} + a_{7\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot{\gamma}_a;\\
&a_{8\lambda_{arb}}\Delta\lambda_{arb,\,sys} + a_{8\Omega_\xi}\Delta\Omega_{\xi\,sys} = \delta_{sys}\Omega_\xi;\\
&a_{9\lambda_{arb}}\Delta\lambda_{arb,\,sys} + a_{9\Omega_\eta}\Delta\Omega_{\eta\,sys} = \delta_{sys}\Omega_\eta;\\
&a_{10\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\Omega_\zeta,
\end{aligned}
\right\} \qquad (2.195)
$$

where the coefficients of the equations are

$$
\left.\begin{aligned}
&a_{1V_\zeta}=2\Omega_{\eta 0}; && a_{6\gamma}=-\Omega_\eta;\\
&a_{1\varphi}=a_\zeta; && a_{6\Omega_\eta}=-\gamma_a;\\
&a_{1\vartheta}=-a_\eta; && a_{6\Omega_\zeta}=-1;\\
&a_{2V_\xi}=-2\Omega_\eta; && a_{7\lambda}=\gamma_a\vartheta_a;\\
&a_{2\Omega_\eta}=-2V_\xi; && a_{7\dot\varphi}=1;\\
&a_{2\psi}=-a_\xi; && a_{7\vartheta}=\Omega_\eta-(\Omega_\zeta-\dot\lambda_{arb})\gamma_a;\\
&a_{2\gamma}=-a_\eta; && a_{7\gamma_a}=-(\Omega_\zeta-\dot\lambda_{arb})\vartheta_a;\\
&a_{3V_\xi}=-\frac{1}{R_{ea}}; && a_{7\Omega_\xi}=-1;\\
&a_{4V_\zeta}=\frac{1}{R_{ea}}; && a_{7\Omega_\eta}=\vartheta_a;\\
&a_{5\dot\lambda}=-\gamma_a; && a_{7\Omega_\zeta}=-\gamma_a\vartheta_a;\\
&a_{5\gamma}=\Omega_\zeta-\dot\lambda_y; && a_{8\lambda}=\Omega_{\eta 0};\\
&a_{5\Omega_\eta}=-1; && a_{8\Omega_\xi}=1;\\
&a_{5\Omega_\zeta}=\gamma_a; && a_{9\lambda}=-\Omega_{\xi 0};\\
&a_{6\dot\lambda}=1; && a_{9\Omega_\eta}=1;\quad a_{10\Omega_\zeta}=1.
\end{aligned}\right\} \qquad (2.196)
$$

The partial systematic errors which enter into systems (2.194, 2.195) are determined as the difference between the right sides of the corresponding equations of system (2.175) and system (2.50). If we consider that $H \ll R_{ea}$, then

$$
\left.\begin{aligned}
&\delta_{sys}\dot V_\xi=\frac{V_\xi V_\zeta}{R_{ea}}\tan\varphi_y+\frac{V_\xi V_\eta}{R_{ea}}-2\Omega_\zeta V_\eta-2V_\zeta(\Omega_{\eta 0}-\Omega_\eta);\\
&\delta_{sys}\dot V_\zeta=\frac{V_\eta V_\zeta}{R_{ea}}-\frac{V_\xi^2}{R_{ea}}\tan\varphi_{arb}+2\Omega_\xi V_\eta;\\
&\delta_{sys}\dot\lambda_{arb}=\frac{V_\xi}{R_{ea}}\left(1-\frac{1}{\cos\varphi_{arb}}+\frac{H}{R_{ea}\cos\varphi_{arb}}\right)\cong\frac{V_\xi}{R_{ea}^2}H;\\
&\delta_{sys}\dot\varphi_{arb}=-\frac{V_\zeta}{R_{ea}}H;\\
&\delta_{sys}\dot\vartheta_a=\Omega_\eta\left(1-\frac{\cos\gamma_a}{\cos\vartheta_a}\right)-\Omega_\zeta\left(\gamma_a+\frac{\sin\gamma_a}{\cos\vartheta_a}\right)+\dot\lambda_{arb}\left(\gamma_a-\frac{\sin(\varphi_{arb}\gamma_a)}{\cos\vartheta_a}\right);\\
&\delta_{sys}\dot\vartheta_a=\Omega_\eta(\gamma_a-\sin\gamma_a)+\Omega_\zeta(1-\cos\gamma_a)-\dot\lambda_{arb}(1-\cos(\varphi_{arb}+\gamma_a)];\\
&\delta_{sys}\dot\gamma_a=-\Omega_\eta(\vartheta_a-\cos\gamma_a\tan\vartheta_a)+\Omega_\zeta(\gamma_a\vartheta_a-\cos\gamma_a\tan\vartheta_a)-\\
&\qquad-\dot\lambda_{arb}[\gamma_a\vartheta_a-\sin(\varphi_{arb}+\gamma_a)\tan\vartheta_a];\\
&\delta_{sys}\Omega_\xi=\Omega_{\xi 0}(1-\cos\lambda_{arb})-\Omega_{\eta 0}(\lambda_{arb}-\sin\lambda_{arb});\\
&\delta_{sys}\Omega_\eta=\Omega_{\xi 0}(\lambda_{arb}-\sin\lambda_{arb}\cos\varphi_{arb})+\Omega_{\eta 0}(1-\cos\lambda_{arb}\cos\varphi_{arb})+\\
&\qquad+\Omega_{\zeta 0}\sin\varphi_{arb};\\
&\delta_{sys}\Omega_\zeta=-(\Omega_{\xi 0}\sin\lambda_{arb}+\Omega_{\eta 0}\cos\lambda_{arb})\sin\varphi_{arb}+\Omega_{\zeta 0}(1-\cos\varphi_{arb}).
\end{aligned}\right\} \qquad (2.197)
$$

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The kinematic parameter values which enter into equations (2.196) and (2.197) are determined in advance by solving equations (2.50).

It is necessary to note that systematic INS errors may also be calculated with constant values of the coefficients of equations (2.194, 2.195), which are determined for a certain average point of the basic portion of the trajectory of object motion. Calculations reveal that error values here are increased (by 10-15%) in comparison with errors obtained through solution of the system with variable coefficients.

In turn, the kinematic INS equations (2.176) for medium-range objects, with consideration for systematic INS error and equations (2.70), have the form

$$
\begin{aligned}
\dot{V}^*_\xi &= a_\xi - \frac{V^*_\xi V^*_\eta}{R_{ea}} - 2(\Omega^*_\eta V^*_\zeta - \Omega^*_\zeta V^*_\eta) + a_\eta(\vartheta^*_a - \vartheta_a) - a_\zeta(\psi^*_a - \psi_a);\\
\dot{V}^*_\eta &= a_\eta + \frac{V^{*2}_\xi}{R_{ea}} - 2(\Omega^*_\zeta V^*_\xi - \Omega^*_\xi V^*_\zeta) - a_\xi(\vartheta^*_a - \vartheta_a) + \\
&\quad + a_\zeta(\vartheta^*_a - \gamma_a) + g_\eta;\\
\dot{V}^*_\zeta &= a_\zeta - \frac{V^*_\eta V^*_\zeta}{R_{ea}} - 2(\Omega^*_\xi V_\eta - \Omega^*_\eta V^*_\xi) + a_\xi(\psi^*_a - \psi_a) - a_\eta(\gamma^*_a - \gamma_a);\\
\dot{\lambda}^*_{arb} &= \frac{1}{R_{ea} + H} V^*_\xi;\\
\dot{H}^* &= V^*_\eta;\\
\dot{\varphi}^*_{arb} &= -\frac{1}{R_{ea}} V^*_\zeta;\\
\dot{\psi}^*_a &= \frac{1}{\cos\vartheta^*_a}[\Omega^*_\eta \cos\gamma^*_a - (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\sin\gamma^*_a];\\
\dot{\vartheta}^*_a &= \Omega^*_\eta \sin\gamma^*_a + (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\cos\gamma^*_a;\\
\dot{\gamma}^*_a &= \Omega^*_\xi - \dot{\varphi}^*_{arb} - [\Omega^*_\eta \cos\gamma^*_a - (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\sin\gamma^*_a]\tan\vartheta^*_a;\\
\Omega^*_\xi &= \Omega_{\xi 0}\cos\lambda^*_{arb} - \Omega_{\eta 0}\sin\lambda^*_{arb};\\
\Omega^*_\eta &= \Omega_{\xi 0}\sin\lambda^*_{arb} + \Omega_{\eta 0}\cos\lambda^*_{arb} - \Omega_{\zeta 0}\varphi^*_{arb};\\
\Omega^*_\zeta &= (\Omega_{\xi 0}\sin\lambda^*_{arb} + \Omega_{\eta 0}\cos\lambda^*_{arb})\varphi^*_{arb} + \Omega_{\zeta 0}.
\end{aligned}
\tag{2.198}
$$

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By solving the system (2.198), we determine the kinematic parameter values, taking into consideration systematic error. The latter may determined from equations (2.189).

The true instantaneous values of the angles ψ_a, ϑ_a, γ_a and accelerations a_ξ, a_η, a_ζ determined by the solution of equations (2.50) serve as initial data for the solution of system (2.198) as well as system (2.193).

Systematic INS errors may be calculated approximately by solving the system of equations of system errors derived on the basis of (2.176) and considering (2.189):

$$
\begin{aligned}
&(p+a_{1V_\xi})\Delta V_\xi + a_{1V_\eta}\Delta V_\eta + a_{1V_\zeta}\Delta V_{\zeta_{sys}} + a_{1\psi}\Delta\psi_{a.sys} + \\
&\quad + a_{1\vartheta}\Delta\vartheta_{a.sys} + a_{1\Omega_\eta}\Delta\Omega_{\eta\,sys} + a_{1\Omega_\zeta}\Delta\Omega_{\zeta sys} = \delta_{sys}\dot{V}_\xi; \\
&a_{2V_\xi}\Delta V_{\xi_{sys}} + p\Delta V_{\eta sys} + a_{2V_\zeta}\Delta V_{\zeta sys} + a_{2\vartheta}\Delta\vartheta_{a.sys} + a_{2\gamma}\Delta\gamma_{a.sys} + \\
&\quad + a_{2\Omega_\xi}\Delta\Omega_{\xi sys} - a_{2\Omega_\zeta}\Delta\Omega_{\zeta sys} = \delta_{sys}\dot{V}_\eta; \\
&a_{3V_\xi}\Delta V_{\xi sys} + a_{3V_\eta}\Delta V_{\eta sys} + (p+a_{3V_\zeta})\Delta V_{\zeta_{sys}} + a_{3\psi}\Delta\psi_{a.sys} + \\
&\quad + a_{3\gamma}\Delta\gamma_{a.sys} + a_{3\Omega_\xi}\Delta\Omega_{\xi sys} + a_{3\Omega_\eta}\Delta\Omega_{\eta sys} = \delta_{sys}\dot{V}_\zeta; \\
&a_{4V_\xi}\Delta V_{\xi_{sys}} + p\Delta\lambda_{arb,sys} + a_{4H}\Delta H_{sys} = \delta_{sys}\dot{\lambda}_{arb}; \\
&a_{5V_\eta}\Delta V_{\eta sys} + p\Delta H_{sys} = \delta_{sys}\dot{H}; \\
&a_{6V_\zeta}\Delta V_{\zeta sys} + p\Delta\varphi_{arb,sys} = \delta_{sys}\dot{\varphi}_{arb} \\
&a_{7\lambda}p\Delta\lambda_{arb,sys} + p\Delta\psi_{a.sys} + a_{7\vartheta}\Delta\vartheta_{a.sys} + a_{7\gamma}\Delta\gamma_{a.sys} + a_{7\Omega_\eta}\Delta\Omega_{\eta sys} + \\
&\quad + a_{7\Omega_\zeta}\Delta\Omega_{\zeta_{sys}} = \delta_{sys}\dot{\psi}_a; \\
&a_{8\lambda}p\Delta\lambda_{arb,sys} + p\Delta\vartheta_{a.sys} + a_{8\gamma}\Delta\gamma_{a.sys} + a_{8\Omega_\eta}\Delta\Omega_{\eta sys} + a_{8\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot{\vartheta}_a; \\
&a_{9\lambda}p\Delta\lambda_{arb,sys} + p\Delta\varphi_{arb,sys} + a_{9\vartheta}\Delta\vartheta_{a.sys} + (p+a_{9\gamma})\Delta\gamma_{a.sys} + \\
&\quad + a_{9\Omega_\xi}\Delta\Omega_{\xi sys} + a_{9\Omega_\eta}\Delta\Omega_{\eta sys} + a_{9\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot{\gamma}_a; \\
&a_{10\lambda}\Delta\lambda_{arb,sys} + a_{10\Omega_\xi}\Delta\Omega_{\xi sys} = \delta_{sys}\Omega_\xi; \\
&a_{11\lambda}\Delta\lambda_{arb} + a_{11\varphi}\Delta\varphi_{arb,sys} + a_{11\Omega_\eta}\Delta\Omega_{\eta sys} = \delta_{sys}\Omega_\eta; \\
&a_{12\lambda}\Delta\lambda_{arb,sys} + a_{12\varphi}\Delta\varphi_{arb,sys} + a_{12\Omega_\zeta}\Omega_{\zeta sys} = \delta_{sys}\Omega_\zeta,
\end{aligned}
\tag{2.199}
$$

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where the values of the coefficients are

$$
\begin{aligned}
a_{1V_\xi} &= \frac{V_\eta}{R_{ea}}; & a_{3\gamma} &= a_\eta; \\
a_{1V_\eta} &= -2\Omega_\eta + \frac{V_\xi}{R_{ea}}; & a_{3\Omega_\xi} &= 2V_\eta; \\
a_{1V_\zeta} &= 2\Omega_\eta; & a_{3\Omega_\eta} &= -2V_\xi; \\
a_{1\vartheta} &= -a_\eta; & a_{4V_\xi} &= -\frac{1}{R_{ea}+H}; \\
a_{1\psi} &= a_\zeta; & a_{4H} &= \frac{V_\xi}{(R_{ea}+H)^2}; \\
a_{1\Omega_\eta} &= 2V_\zeta; & a_{5V_\eta} &= -1; \\
a_{1\Omega_\zeta} &= -2V_\eta; & a_{6V_\zeta} &= \frac{1}{R_{ea}}; \\
a_{2V_\xi} &= 2\Omega_\zeta - 2\frac{V_\xi}{R_{ea}}; & a_{7\lambda} &= -\frac{\sin\gamma_a}{\cos\vartheta_a}; \\
a_{2V_\zeta} &= -2\Omega_\xi;
\end{aligned}
\tag{2.200}
$$

$$
\begin{aligned}
a_{2\vartheta} &= a_\xi; \\
a_{2\gamma} &= -a_\zeta; \\
a_{2\Omega_\xi} &= -2V_\zeta; \\
a_{2\Omega_\zeta} &= 2V_\xi; \\
a_{3V_\xi} &= -2\Omega_\eta; \\
a_{3V_\eta} &= \frac{V_\zeta}{R_{ea}} + 2\Omega_\xi; \\
a_{3V_\zeta} &= \frac{V_\eta}{R_{ea}}; \\
a_{3\psi} &= -a_\xi; \\
a_{8\gamma} &= (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a - \\
&\quad - \Omega_\eta \cos\gamma_a \\
a_{8\Omega_\eta} &= -\sin\gamma_a; \\
a_{8\Omega_\zeta} &= -\cos\gamma_a; \\
a_{9\vartheta} &= [\Omega_\eta \cos\gamma_a - \\
&\quad - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a]\frac{1}{\cos^2\vartheta_a}; \\
a_{9\gamma} &= -[\Omega_\eta \sin\gamma_a + \\
&\quad + (\Omega_\zeta - \dot{\lambda}_{arb})\cos\gamma_a]\tan\vartheta_a; \\
a_{9\Omega_\xi} &= -1; \\
a_{9\Omega_\eta} &= \cos\gamma_a \tan\vartheta_a; \\
a_{9\Omega_\zeta} &= -\sin\gamma_a \tan\vartheta_a; \\
a_{7\vartheta} &= -[\Omega_\eta \cos\gamma_a - (\Omega_\zeta - \\
&\quad - \dot{\lambda}_{arb})\sin\gamma_a]\frac{\sin\vartheta_a}{\cos^2\vartheta_a}; \\
a_{7\gamma} &= \frac{1}{\cos\vartheta_a}[\Omega_\eta \sin\gamma_a + \\
&\quad + (\Omega_\zeta - \dot{\lambda}_{arb})\cos\gamma_a); \\
a_{7\Omega_\eta} &= -\frac{\cos\gamma_a}{\cos\vartheta_a}; \\
a_{7\Omega_\zeta} &= \frac{\sin\vartheta_a}{\cos\vartheta_a}; \\
a_{8\lambda} &= \cos\gamma_a; \\
a_{9\lambda} &= \sin\gamma_a \tan\vartheta_a; \\
a_{10\lambda} &= \Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb}; \\
a_{10\Omega_\xi} &= 1; \\
a_{11\lambda} &= \Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0}\sin\lambda_{arb}; \\
a_{11\varphi} &= \Omega_{\zeta 0}; \\
a_{11\Omega_\eta} &= 1; \\
a_{12\lambda} &= -(\Omega_{\xi 0}\cos\lambda_{arb} - \\
&\quad - \Omega_{\zeta 0}\sin\lambda_{arb})\varphi_{arb}; \\
a_{12\varphi} &= -\Omega_{\xi 0}\sin\lambda_{arb} - \\
&\quad - \Omega_{\eta 0}\cos\lambda_{arb}; \\
a_{12\Omega_\zeta} &= 1.
\end{aligned}
\tag{2.200}
$$

The partial systematic errors, which enter into the system (2.199) and which are taken as the difference between the right sides of the corresponding equations of the system (2.176) and (2.50) are determined as follows: /96

$$
\begin{aligned}
\delta_{sys}\dot{V}_\xi &= \frac{V_\xi V_\zeta}{R_{ea}+H}\tan\varphi_{arb} - \frac{\dot{V}_\xi V_\eta}{R_{ea}(R_{ea}+H)}H; \\
\delta_{sys}\dot{V}_\eta &= \frac{V_\xi^2}{R_{ea}^2}H - \frac{V_\zeta^2}{R_{ea}+H} + g\left[1 - \frac{R^2}{(R_{ea}+H)^2}\right]; \\
\delta_{sys}\dot{V}_\zeta &= -\frac{V_\xi^2}{R_{ea}+H}\tan\varphi_{arb} - \frac{V_\eta V_\zeta}{R_{ea}^2}H; \\
\delta_{sys}\dot{\lambda}_{arb} &= \frac{V_\xi}{R_{ea}+H}\left(1 - \frac{1}{\cos\varphi_{arb}}\right); \\
\delta_{sys}\dot{\varphi}_{arb} &= \frac{V_\zeta}{R_{ea}^2}H;
\end{aligned}
\tag{2.201}
$$

$$
\begin{aligned}
&\delta_{sys}\dot{\psi}_a = \frac{1}{\cos\vartheta_a}\dot{\lambda}_{arb}[\sin\gamma_a - \sin(\varphi_{arb} + \gamma_a)];\\
&\delta_{sys}\dot{\vartheta}_a = -\dot{\lambda}_{arb}\sin\varphi_{arb}\sin\gamma_a - \dot{\lambda}_{arb}\cos\gamma_a(1-\cos\varphi_{arb});\\
&\delta_{sys}\dot{\gamma}_a = -\dot{\lambda}_{arb}[\sin\gamma_a - \sin(\varphi_{arb} + \gamma_a)]\tan\vartheta_a;\\
&\delta_{sys}\Omega_\xi = 0;\\
&\delta_{sys}\Omega_\eta = (\Omega_\xi\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})(1-\cos\varphi_{arb}) -\\
&\qquad -\Omega_{\zeta 0}(\varphi_{arb} - \sin\varphi_{arb});\\
&\delta_{sys}\Omega_\zeta = (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})(\varphi_{arb} - \sin\varphi_{arb}) +\\
&\qquad +\Omega_{\zeta 0}(1-\cos\varphi_{arb}).
\end{aligned}
\tag{2.201}
$$

The parameter values which enter into the coefficient expressions (2.200) and for partial systematic errors (2.201) are determined by the preliminary solution of the system (2.50) with known values $\dot{V}_\varepsilon(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$ and initial true parameter values. /97

It should be noted that system (2.199) fluctuates [20] as a result of systematic error ($\delta_{sys}\dot{V}_\eta$) and the determination of vertical acceleration, which leads to errors in the determination of velocity $\Delta\dot{V}_{\eta\ sys}$ and altitute ΔH_{sys}. In a first approximation these errors may be determined in accordance with the following equations:

$$
\begin{aligned}
&\Delta V_{\eta_{sys}} \cong \delta_{sys}\dot{V}_\eta t;\\
&\Delta H_{sys} \cong \frac{\delta_{sys}\dot{V}_\eta t^2}{2}.
\end{aligned}
$$

For example, where $\delta_{sys}\dot{V}_\eta = 10^{-4}$ m/sec^2 during time $t = 10^4$ sec $\Delta V_{\eta\ sys} =$ $= 1$ m/sec; $\Delta H_{sys} \cong 10$ km. Since parameters V_η and H are included in equations (2.176) for the calculation of $\dot{V}_\xi$ and $\dot{V}_\zeta$, then error $\delta_{sys}\dot{V}_\eta$ leads to an increase with time of errors in the determination of V_ξ and V_ζ, as a result of which systematic INS errors in the determination of λ_{arb} and ϕ_{arb} will also increase.

It follows from this that the three-component system, which has systematic errors in the determination of the vertical acceleration, is always unstable and during operation in the course of an extended period of time the errors may reach unpermissible values.

In order to remove this shortcoming of three-component INS's, it is necessary to provide for correction of parameters V_η and H in the system by means of devices based on other measurement principles, rather than the inertial navigation principle.

§2.6 INS Instrument Error Equations

Instrument INS errors of a given class will be determined by errors in accelerometers, gyroscopes, devices for platform inclination in inertial space (tracking systems or moment sensors) and by the computer.

The kinematic INS equations derived previously do not always take into consideration existing instrument errors.

Instrument errors for each INS parameter in the left sides of the equations are determined by errors in the computer devices which formulate the equations for the parameter, and by instrument errors in the determination of other parameters which are included in the right side of the equation.

We shall examine instrument INS errors, disregarding the inertia of system elements.

We shall designate the instrument values of the kinematic parameters includ- /98
ed in the INS equation system with the exponent "*".

Instrument errors are determined as the difference between instrument and true values, with consideration for systematic errors.

$$\left.\begin{aligned}
&\Delta\dot{V}_{N_{\text{in}}}=\dot{V}_N^*-\dot{V}_N-\Delta\dot{V}_{N_{\text{sys}}}; && \Delta\dot{V}_{H_{\text{in}}}=\dot{V}_H^*\dot{V}_H-\Delta\dot{V}_{H\text{sys}};\\
&\Delta\dot{V}_{E_{\text{in}}}=\dot{V}_E^*-\dot{V}_E-\Delta\dot{V}_{E\text{sys}}; && \Delta V_{N\text{in}}=V_N^*-V_N-\Delta V_{N\text{sys}};\\
&\Delta V_{H_{\text{in}}}=V_H^*-V_H-\Delta V_{H\text{sys}}; && \Delta V_{E\text{in}}=V_E^*-V_E-\Delta V_{E\text{sys}};\\
&\Delta\dot{\varphi}_{\text{in}}=\dot{\varphi}^*-\varphi-\Delta\dot{\varphi}_{\text{sys}}; && \Delta\dot{H}_{\text{in}}=\dot{H}^*-\dot{H}-\Delta\dot{H}_{\text{sys}};\\
&\Delta\dot{\lambda}=\dot{\lambda}^*-\dot{\lambda}-\Delta\dot{\lambda}_{\text{sys}}; && \Delta\varphi_{\text{in}}=\varphi^*-\varphi-\Delta\varphi_{\text{sys}};\\
&\Delta\dot{H}_{\text{in}}=H^*-H-\Delta\dot{H}_{\text{sys}}; && \Delta\lambda_{\text{in}}=\lambda^*-\lambda-\Delta\lambda_{\text{sys}};\\
&\Delta a_{N_{\text{in}}}=a_N^*-a_N; && \Delta a_{H\text{in}}=a_H^*-a_H;\\
&\Delta a_{E_{\text{in}}}=a_E^*-a_E; && \Delta\psi_{\text{a.in}}=\psi_{\text{a}}^*-\psi_{\text{a}}-\Delta\psi_{\text{a.sys}};\\
&\Delta\vartheta_{\text{a.in}}=\vartheta_{\text{a}}^*-\vartheta_{\text{a}}-\Delta\vartheta_{\text{a}}; && \Delta\gamma_{\text{a.in}}=\gamma_{\text{a}}^*-\gamma_{\text{a}}-\Delta\gamma_{\text{a}}.
\end{aligned}\right\} \qquad (2.202)$$

We note that instrument errors in individual system elements depend on many factors and are random time functions. However in this discussion we shall not use the mathematical apparatus of the theory of random functions. For an approximate evaluation of accuracy we may present errors in the form of certain nonrandom values.

We shall consider, in a first approximation, that these errors contain two components, one of which does not depend on the parameter measured, but the other is proportional to it. For example, accelerometer error

$$\Delta a=\Delta a_0+\delta_a a \qquad (2.203)$$

or gyroscope angular velocity gauge error

$$\Delta\omega = \Delta\omega_0 + \delta_\omega \omega, \tag{2.204}$$

We note that in an intensive analysis of instrument accuracy for navigation systems it is possible to use more complex models for errors in primary gauges. The following model of accelerometer errors along the x axis is reccommended as an example in [37]:

$$\Delta a_x = \Delta a_0 + k_1 a_x + k_2 a_x^2 + k_3 a_x^3 + k_4 a_y + \\ + k_5 a_y a_x + k_6 a_z + k_7 a_x a_z, \tag{2.205}$$

where Δa_0 is constant instrument zero drift;

a_y, a_z are non-gravitational acceleration components along the corresponding axes

k_1 is linear scale coefficient error;

k_2, k_3 are nonlinearity coefficients of the second and third order; /99

k_4, k_6 are coupling constants between zero error and accelerations normal to the responsive axis;

k_5, k_7 are coefficients established by the relationship of the scale factor and accelerations normal to the responsive axis.

The error models for the other primary gauges may be made more complex in similar fashion.

Of course the mathematical error models similar to (2.205) are more refined than (2.203) or (2.204) but at the same time they require a great deal of apriori information concerning the instruments themselves and to a significant degree complicate the analysis of the INS accuracy as a whole. Therefore during system synthesis, when the problem of the selection of specific primary gauges is being decided, it is frequently advantageous to be limited by approximate error models of the form (2.203) and (2.204).

In speaking of the mathematical error models of primary guages, one must keep in mind one very important, in our view, reservation concerning increasing INS instrument accuracy. This concept includes the possibility of solving the error equations or the dynamic equations of the primary gauges as a whole on board the object concurrently with the kinematic equations.

A designer of an accelerometer, gyroscope or a complex measurement subassembly often reaches a similar decision, after having developed a highly-sensitive instrument with stable but necessarily non-linear characteristics, or a device which requires a supplementary functional transformations of the output signals for the extraction of useful navigational information.

The problem of the advisability of the approach indicated, in one case or another, requires a specific solution.

INS instrument errors may be calculated in solving the system of kinematic equations with consideration for element errors. As a result of such a calculation it is possible to determine instrument values of the parameters included in the system in conjunction with which, and considering (2.202), the two errors are calculated.

True values of acceleration, errors in primary elements and basic values of the true kinematic parameters are the basic data for the solution of the system of kinematic equations.

The first approximation of INS instrument errors may be calculated by solving the instrument error equations. The latter are obtained as a result of the linearization of INS equations, which take into account instrument errors of all system elements.

It is feasible to employ the equations cited in section 2.1, in which the cross links between INS channels are not considered, for a coarse evaluation of instrumental INS errors in the determination of coordinates. These equations permit us to make a preliminary evaluation of the influence of the errors of each system element on the accuracy of system operation. Here all of the INS errors may be divided into the following three groups: /100

first - the measurement and calculation of accelerations;
second - angular changes in platform stabilization;
third - the determination of angular velocities (the influence of systematic platform drift).

It is possible to employ appropriate equations (2.9), (2.11), (2.12) for an evaluation of error values of each of these groups.

When errors are present in the individual elements of the INS, constructed in accordance with equations (2.120), instrument errors in the determination of all parameters included in the system will occur.

In this connection instrument acceleration values, allowing for equations (2.179) previously derived, are determined as follows

$$\left.\begin{aligned} a_N^* &= a_N + a_H(\vartheta_a^* - \vartheta_a) - a_E(\psi_{ra}^* - \psi_{ra}) + \Delta a_{N0} + \delta_{aN} a_N; \\ a_H^* &= a_H - a_N(\vartheta_a^* - \vartheta_a) + a_E(\gamma_a^* - \gamma_a) + \Delta a_{H0} + \delta_{aH} a_H; \\ a_E^* &= a_E + a_N(\psi_{ra}^* - \psi_{ra}) - a_H(\gamma_a^* - \gamma_a) + \Delta a_{E0} + \delta_{aE} a_E, \end{aligned}\right\} \qquad (2.206)$$

where Δa_{N0}, Δa_{H0}, Δa_{E0} are errors in the corresponding system accelerometers, which do not depend on the value of the parameter measured;
δ_{aN}, δ_{aH}, δ_{aE} are relative accelerometer errors.

It follows from equations (2.206) that instrument errors of the system in the determination of accelerations a_N, a_H, and a_E equal

$$\begin{aligned}
\Delta a_{N_{\text{in}}} &= a_H \Delta\vartheta_{\text{a. in}} - a_E \Delta\psi_{\text{a. in}} + \Delta a_{N0} + \delta_{aN} a_N;\\
\Delta a_{H_{\text{in}}} &= a_E \Delta\gamma_{\text{a. in}} + a_N \Delta\vartheta_{\text{a. in}} + \Delta a_{H0} + \delta_{aH} a_H;\\
\Delta a_{E_{\text{in}}} &= a_N \Delta\psi_{\text{a. in}} - a_H \Delta\gamma_{\text{a. in}} + \Delta a_{E0} + \delta_{aE} a_E.
\end{aligned} \tag{2.207}$$

It is obvious from the equations (2.207) that the errors sought are caused both by platform stabilization errors $\Delta\psi_{\text{a in}}$, $\Delta\vartheta_{\text{a in}}$, $\Delta\gamma_{\text{a in}}$, and by accelerometer errors

$$\Delta a_{N0},\ \Delta a_{H0},\ \Delta a_{E0},\ \delta_{a N},\ \delta_{a H},\ \delta_{aE}.$$

Kinematic INS equations for short-range objects, formulated in accordance with the system of simplified equations (2.120) and allowing for equations (2.206), have the form

$$\begin{aligned}
\dot V_N^* &= a_N - \frac{V_E^{*2}}{R_{\text{ea}}}\tan\varphi^* - 2\Omega V_E^* \sin\varphi^* + a_H(\vartheta_{\text{a}}^* - \vartheta_{\text{a}}) -\\
&\quad - a_E(\psi_{\text{a}}^* - \psi_{\text{a}}) + \Delta a_{N0} + \delta_{a N} a_N + \delta_{\text{co}\dot V_N}\dot V_N;\\
\dot V_E^* &= a_E + \frac{V_N^* V_E^*}{R_{\text{ea}}}\tan\varphi^* + 2\Omega V_N^* \sin\varphi^* + a_N(\psi_{\text{a}}^* - \psi_{\text{a}}) -\\
&\quad - a_H(\gamma_{\text{a}}^* - \gamma_{\text{a}}) + \Delta a_{E0} + \delta_{a E} a_E + \delta_{\text{co}\dot V_E}\dot V_E;\\
\dot\varphi^* &= \frac{1}{R_{\text{ea}}} V_N^* + \delta_{\text{co}\dot\varphi};\\
\dot\lambda^* &= \frac{1}{R_{\text{ea}}\cos\varphi^*} V_E^* + \delta_{\text{co}\dot\lambda};\\
\dot\psi_{\text{a}}^* &= (\dot\lambda^* + \Omega)\cos\varphi^* + \dot\varphi^*\gamma_{\text{a}}^* + \delta_{\text{co}\dot\psi}\dot\psi_{\text{a}} + \Delta\dot\psi_{\text{a}0};\\
\dot\vartheta_{\text{a}}^* &= (\dot\lambda^* + \Omega)\sin\varphi^*\gamma_{\text{a}}^* - \dot\varphi^* + \delta_{\text{co}\dot\vartheta}\dot\vartheta_{\text{a}} + \Delta\dot\vartheta_{\text{a}0};\\
\dot\gamma_{\text{a}}^* &= (\dot\lambda^* + \Omega) - [(\dot\lambda^* + \Omega^*)\sin\varphi^* - \dot\varphi^*\gamma_{\text{a}}^*]\vartheta_{\text{a}}^* + \delta_{\text{co}\dot\gamma}\dot\gamma_{\text{a}} + \Delta\dot\gamma_{\text{a}0};
\end{aligned} \tag{2.208}$$

where $\delta_{\text{co}\dot V_N}$, $\delta_{\text{co}\dot V_E}$, $\delta_{\text{co}\dot\phi}$ represent computer error in the formulation of the appropriate equations; /101

$\Delta\dot\psi_{\text{a0}}$, $\Delta\dot\vartheta_{\text{a0}}$, $\Delta\dot\gamma_{\text{a0}}$ are angular velocities of platform drift in inertial space.

In addition, in solving the system (2.208) we must consider that

$$
\begin{aligned}
V_N^* &= V_{N0} + \int_0^t \dot{V}_N^* dt + \Delta V_{N0} + \delta_{co_{V_N}} V_N;\\
V_E^* &= V_{E0} + \int_0^t \dot{V}_E^* dt + \Delta V_{E0} + \delta_{co_{V_E}} V_E;\\
\varphi^* &= \varphi_0 + \int_0^t \dot{\varphi}^* dt + \Delta\varphi_0 + \delta_{co_\varphi}\varphi;\\
\lambda^* &= \lambda_0 + \int_0^t \dot{\lambda}^* dt + \Delta\lambda_0 + \delta_{co_\lambda}\lambda;\\
\psi_a^* &= \psi_0 + \int_0^t \dot{\psi}_a^* dt + \Delta\psi_{a0} + \delta_{co_\psi}\psi_a;\\
\vartheta_a^* &= \vartheta_0 + \int_0^t \dot{\vartheta}_a^* dt + \Delta\vartheta_{a0} + \delta_{co_\vartheta}\vartheta_a;\\
\gamma_a^* &= \gamma_a + \int_0^t \dot{\gamma}_a^* dt + \Delta\gamma_{a0} + \delta_{co_\gamma}\gamma_a,
\end{aligned}
\qquad (2.209)
$$

where V_{N0}, V_{E0}, ϕ_0, λ_0, ψ_{a0}, ϑ_{a0}, γ_{a0} are initial parameter values; /102
ΔV_{N0}, ΔV_{E0}, $\Delta\phi_0$, etc. are errors in the determination and in the introduction into the INS of the corresponding parameters at the starting point (INS display errors);
δ_{coV_N}, $\delta_{co_{V_E}}$, $\delta_{co\phi}$, $\delta_{co\lambda}$, $\delta_{co\psi}$, $\delta_{co\vartheta}$, $\delta_{co\gamma}$ are relative errors in the integration of the corresponding parameters.

We determine the instrument values of the parameter which include systematic and instrument error through the solution of system (2.208).

The instantaneous true values of the angles $\psi_a(t)$, $\vartheta_a(t)$, $\gamma_a(t)$ and accelerations $a_N(t)$, $a_E(t)$, $a_H(t)$ serve as initial data for the solution of this system just as for the system (2.181). These values are calculated by solving the system (2.38) in accordance with known data of the trajectory kinematics [for example, accelerations $\dot{V}_N(t)$, $\dot{V}_H(t)$, $\dot{V}_E(t)$].

INS instrument errors are determined in accordance with equations (2.202).

In turn, kinematic INS equations for medium range objects, constructed in accordance with the system of simplified equations (2.121), and taking element errors into consideration, will have the following form,

$$
\begin{aligned}
\dot{V}_N^* &= a_N - \frac{V_E^{*2}}{R_{ea}+H^*}\tan\varphi^* - 2\Omega V_E^*\sin\varphi^* + a_H(\vartheta_a^* - \vartheta_a) - \\
&\quad - a_E(\psi_a^* - \psi_a) + \Delta a_{N0} + \delta_{aN}a_N + \delta_{\dot{V}_N}\dot{V}_N; \\
\dot{V}_H^* &= a_H + \frac{V_N^{*2}}{R_{ea}+H^*} + \frac{V_E^{*2}}{R_{ea}+H^*} + 2\Omega V_E^*\cos\varphi^* + g_H - \\
&\quad - a_N(\vartheta_a^* - \vartheta_a) + a_E(\gamma_a^* - \gamma_a) + \Delta a_{H0} + \delta_{aH}a_H + \delta_{\dot{V}_H}\dot{V}_H; \\
\dot{V}_E^* &= a_E + \frac{V_N^* V_E^*}{R_{ea}+H^*}\tan\varphi^* + 2\Omega V_N^*\sin\varphi^* + a_H(\psi_a^* - \psi_a) - \\
&\quad - a_H(\gamma_a^* - \gamma_a) + \Delta a_{E0} + \delta_{aE}a_E + \delta_{\dot{V}_E}\dot{V}_E; \\
\dot{\varphi}^* &= \frac{V_N^*}{R_{ea}+H^*} + \delta_{\dot{\varphi}}\dot{\varphi}; \\
\dot{H}^* &= V_H^*; \\
\dot{\lambda}^* &= \frac{V_E^*}{(R_{ea}+H^*)\cos\varphi^*} + \delta_{\dot{\lambda}}\dot{\lambda}; \\
\dot{\psi}_a^* &= \frac{1}{\cos\vartheta_a^*}[(\dot{\lambda}^* + \Omega)\cos\varphi^*\cos\gamma_a^* + \dot{\varphi}^*\sin\gamma_a^*] + \delta_{\dot{\psi}}\dot{\psi}_a + \Delta\psi_{a0}; \\
\dot{\vartheta}_a^* &= (\dot{\lambda}^* + \Omega)\sin\varphi^*\sin\gamma_a^* - \dot{\varphi}^*\cos\gamma_a^* + \delta_{\dot{\vartheta}}\dot{\vartheta}_a + \Delta\vartheta_{a0}; \\
\dot{\gamma}_a^* &= (\dot{\lambda}^* + \Omega)\cos\varphi^* - [(\dot{\lambda}^* + \Omega)\sin\varphi^*\cos\gamma_a^* - \dot{\varphi}^*\sin\gamma_a^*]\tan\vartheta_a^* + \\
&\quad + \delta_{\dot{\gamma}}\dot{\gamma}_a + \Delta\dot{\gamma}_{a0},
\end{aligned}
\tag{2.210}
$$

where /103

$$
V_H^* = V_{H0} + \int_0^t \dot{V}_H^*\,dt + \Delta V_{H0} + \delta_{V_H}V_H;
$$

$$
H^* = H_0 + \int_0^t V_H^*\,dt + \Delta H_0 + \delta_H H.
$$

The method of calculation set forth for instrument errors permits us to investigate the influence of each element and of any combination of these elements on the operational accuracy of the system as a whole.

Let us examine the instrument error equations for an INS with a platform oriented in the geographic coordinate system.

INS instrument error equations for short-range objects are derived by the linearization of equations (2.120) with consideration for instrument errors, and in the operational form will appear as follows:

$$
\begin{aligned}
&p\Delta V_{N_{in}}+a_{1V_E}\Delta V_E+a_{1\varphi}\Delta\varphi_{in}+a_{1\psi}\Delta\psi_{a.in}+a_{1\vartheta}\Delta\vartheta_{a.in}=\delta_{in}\dot V_N;\\
&a_{2V_N}\Delta V_{N_{in}}+(p+a_{2V_E})\Delta V_{E\,in}+a_{2\varphi}\Delta\varphi_{in}+a_{2\psi}\Delta\psi_{a.in}+\\
&\quad+a_{2\gamma_a}\Delta\gamma_{a.in}=\delta_{in}\dot V_E;\\
&a_{3V_N}\Delta V_{N_{in}}+p\Delta\varphi_{in}=\delta_{in}\dot\varphi;\\
&a_{4V_E}\Delta V_{E\,in}+a_{4\varphi}\Delta\varphi_{in}+p\Delta\lambda_{in}=\delta_{in}\dot\lambda;\\
&a_{5\dot\varphi}p\Delta\varphi_{in}+a_{5\dot\lambda}p\Delta\lambda_{in}+p\Delta\psi_{a.in}+a_{6\gamma}\Delta\gamma_{a.in}=\delta_{in}\dot\psi_a;\\
&a_{6\dot\lambda}p\Delta\lambda_{in}+(a_{6\dot\varphi}p+a_{6\varphi})\Delta\varphi_{in}+p\Delta\vartheta_{a.in}+a_{6\gamma}\Delta\gamma_{a.in}=\delta_{in}\dot\vartheta_a;\\
&(a_{7\dot\varphi}p+a_{7\varphi})\Delta\varphi_{in}+a_{7\dot\lambda}p\Delta\lambda_{in}+a_{7\vartheta_a}\Delta\vartheta_{a.in}+(p+a_{7\gamma})\Delta\gamma_{a.in}=\delta_{in}\dot\gamma_a,
\end{aligned}
\tag{2.211}
$$

where INS partial instrument errors included in the first parts of the system of equations (2.211) equal, in a first approximation:

$$
\begin{aligned}
&\delta_{in}\dot V_N=\Delta a_{N0}+\delta_{aN}a_N+\delta_{co\dot V_N}\dot V_N;\\
&\delta_{in}\dot V_E=\Delta a_{E0}+\delta_{aE}a_E+\delta_{co\dot V_E}\dot V_E;\\
&\delta_{in}\dot\varphi=\delta_{co\dot\varphi}\dot\varphi;\\
&\delta_{in}\dot\lambda=\delta_{co\dot\lambda}\dot\lambda;\\
&\delta_{in}\dot\psi_a=\delta_{co\dot\psi}\dot\psi_a+\Delta\dot\psi_{a0};\\
&\delta_{in}\dot\vartheta_a=\delta_{co\dot\vartheta}\dot\vartheta_a+\Delta\dot\vartheta_{a0};\\
&\delta_{in}\dot\gamma_a=\delta_{co\dot\gamma}\dot\gamma_a+\Delta\dot\gamma_{a0}.
\end{aligned}
\tag{2.212}
$$

The coefficients of the equations (2.211) are calculated similarly to the coefficients of the systematic error equations in accordance with the equations (2.184). In addition, in solving the system (2.211), it is necessary to consider that /104

$$
\begin{aligned}
&\Delta V_{N\,in}=\int_0^t\Delta\dot V_{N\,in}dt+\Delta V_{N0}+\delta_{co\,V_N}V_N;\\
&\Delta V_{E_{in}}=\int_0^t\Delta\dot V_{E\,in}dt+\Delta V_{E0}+\delta_{co\,V_E}V_E;\\
&\Delta\varphi_{in}=\int_0^t\Delta\dot\varphi_{in}dt+\Delta\varphi_{in_0}+\delta_{co\,\varphi}\varphi;\\
&\Delta\lambda_{in}=\int_0^t\Delta\dot\lambda_{in}dt+\Delta\varphi_{in_0}+\delta_{co\,\lambda}\lambda;
\end{aligned}
\tag{2.213}
$$

$$\Delta\psi_a = \int_0^t \Delta\dot{\psi}_{a.in}\,dt + \Delta\psi_{a0} + \delta_{co\psi}\psi_a;$$
$$\Delta\vartheta_a = \int_0^t \Delta\dot{\vartheta}_{a.in}\,dt + \Delta\vartheta_{a0} + \delta_{co\vartheta}\vartheta_a; \qquad (2.213)$$
$$\Delta\gamma_a = \int_0^t \Delta\dot{\gamma}_{a.in}\,dt + \Delta\gamma_{a0} + \delta_{co\gamma}\gamma_a,$$

where ΔV_{N0}, ΔV_{E0} are errors in the determination and in the introduction of initial values of the kinematic parameters desired;
$\delta_{coV_N}V_N$, $\delta_{coV_E}V_E$, etc. are kinematic parameter integration errors.

The INS type dependencies may be determined by the solution of system (2.211).

It is apparent that the system of instrument error equations (2.211) are linear with variable coefficients, the values of which depend on the true instantaneous values of the kinematic motion parameters. /105

This system is crosslinked with instrument errors in the determination of the horizontal velocity components $\Delta V_{N\ in}$, $\Delta V_{E\ in}$ of coordinates $\Delta\phi_{in}$, $\Delta\lambda_{in}$ and with the stabilization errors of accelerometer measurement axes $\Delta\psi_{a\ in}$ $\Delta\vartheta_{a\ in}$, $\Delta\gamma_{a\ in}$; therefore instrument errors cannot be calculated exactly without considering these crosslinks.

The basic data for solution of system (2.211) encompasses errors in individualsystem elements, including display errors and errors in the introduction of initial data, true instantaneous values of the kinematic parameters desired, which are included in the equations for coefficients; these values may be determined in advance by solving the system (2.38).

It is not difficult to verify that INS instrument error equations for medium-range objects (2.121) have the form

$$\begin{aligned}
&p\Delta V_{N\,in} + a_{1V_E}\Delta V_{E\,in} + a_{1\varphi}\Delta\varphi_{in} + a_{1H}\Delta H_{in} + a_{1\psi}\Delta\psi_a + \\
&\quad + a_{1\vartheta_a}\Delta\vartheta = \delta_{in}\dot{V}_H; \\
&a_{2V_N}\Delta V_{N\,in} + p\Delta V_{H\,in} + a_{2V_E}\Delta V_{E\,in} + a_{2\varphi}\Delta\varphi_{in} + a_{2H}\Delta H_{in} + \\
&\quad + a_{2\vartheta}\Delta\vartheta + a_{2\gamma}\Delta\gamma = \delta_{in}\dot{V}_H; \qquad (2.214) \\
&a_{3V_N}\Delta V_{N\,in} + (p + a_{3V_E})\Delta V_{E\,in} + a_{3\varphi}\Delta\varphi + a_{3H}\Delta H_{in} + \\
&\quad + a_{3\psi}\Delta\psi + a_{3\gamma}\Delta\gamma = \delta_{in}\dot{V}_E;
\end{aligned}$$

$$
\begin{aligned}
&a_{4V_N}\Delta V_{N_{in}} + p\Delta\varphi_{in} + a_{4H}\Delta H_{in} = \delta_{in}\dot{\varphi};\\
&a_{5V_H}\Delta V_{H_{in}} + p\Delta H_{in} = \delta_{in}\dot{H};\\
&a_{6V_E}\Delta V_{E_{in}} + p\Delta\lambda_{in} + a_{6\varphi}\Delta\varphi_{in} + a_{6H}\Delta H_{in} = \delta_{in}\dot{\lambda};\\
&a_{7\lambda}p\Delta\lambda_{in} + (a_{7\dot{\varphi}}p + a_{7\varphi})\Delta\varphi_{in} + p\Delta\psi_{a.in} + a_{7\vartheta}\Delta\vartheta_{a.in} + a_{7\gamma}\Delta\gamma_{a.in} = \delta_{in}\dot{\psi}_a;\\
&a_{8\lambda}p\Delta\lambda_{in} + (a_{8\dot{\varphi}}p + a_{8\varphi})\Delta\varphi_{in} + p\Delta\vartheta_{a.in} + a_{8\gamma}\Delta\gamma_{a.in} = \delta_{in}\dot{\vartheta}_a;\\
&(a_{9\dot{\varphi}}p + a_{9\varphi})\Delta\varphi_{in} + a_{9\lambda}p\Delta\lambda_{in} + a_{9\vartheta}\Delta\vartheta_{a.in} + (p + a_{9\gamma})\Delta\gamma_{a.in} = \delta_{in}\dot{\gamma}_a,
\end{aligned}
\qquad (2.214)
$$

where the coefficients of the equations are determined by the relationships (2.87), and partial instrument errors by (2.212).

Equations (2.213) must be considered in the solution of the system (2.214).

We shall examine the kinematic equations of an INS with a platform oriented in the arbitrary coordinate system, considering instrument errors.

Instrument parameter values obtained from an INS with a platform oriented in the arbitrary coordinate system, with consideration for systematic and instrument errors, respectively, equal: /106

$$
\begin{aligned}
&\dot{V}^*_\xi = \dot{V}_\xi + \Delta\dot{V}_{\xi in} + \Delta\dot{V}_{\xi sys}; && \dot{V}^*_\eta = \dot{V}_\eta + \Delta\dot{V}_{\eta in} + \Delta\dot{V}_{\eta sys};\\
&\dot{V}^*_\zeta = \dot{V}_\zeta + \Delta\dot{V}_{\zeta in} + \Delta\dot{V}_{\zeta sys}; && V^*_\xi = V_\xi + \Delta V_{\xi in} + \Delta V_{\xi sys};\\
&V^*_\eta = V_\eta + \Delta V_{\eta in} + \Delta V_{\eta sys}; && V^*_\zeta = V_\zeta + \Delta V_{\zeta in} + \Delta V_{\zeta sys};\\
&a^*_\xi = a_\xi + \Delta a_{\xi in} + \Delta a_{\xi sys}; && a^*_\eta = a_\eta + \Delta a_{\eta in} + \Delta a_{\eta sys};\\
&a^*_\zeta = a_\zeta + \Delta a_{\zeta in} + \Delta a_{\zeta sys}; && \dot{\lambda}^*_{arb} = \dot{\lambda}_{arb} + \Delta\dot{\lambda}_{arb,in} + \Delta\dot{\lambda}_{arb,sys};\\
&\dot{H}^* = \dot{H} + \Delta\dot{H}_{in} + \Delta\dot{H}_{sys}; && \dot{\varphi}^*_{arb} = \dot{\varphi}_{arb} + \Delta\dot{\varphi}_{arb,in} + \Delta\dot{\varphi}_{arb,sys};\\
&\lambda^*_{arb} = \lambda_{arb} + \Delta\lambda_{arb,in} + \Delta\lambda_{arb,sys}; && H^* = H + \Delta H_{in} + \Delta H_{sys};\\
&\varphi^*_{arb} = \varphi_{arb} + \Delta\varphi_{arb,in} + \Delta\varphi_{arb,sys}; && \dot{\psi}^*_a = \dot{\psi}_a + \Delta\dot{\psi}_{a,sys} + \Delta\dot{\psi}_{a,sys}\\
&\vartheta^*_a = \vartheta_a + \Delta\vartheta_{a.sys} + \Delta\vartheta_{a.sys} && \dot{\gamma}^*_a = \dot{\gamma}_a + \Delta\gamma_{a.sys} + \Delta\gamma_{a.sys}\\
&\omega^*_{a\xi} = \omega_{a\xi} + \Delta\omega_{a\xi in} + \Delta\omega_{a\xi sys}; && \omega^*_{a\eta} = \omega_{a\eta} + \Delta\omega_{a\eta in} + \Delta\omega_{a\eta sys};\\
&\omega^*_{a\zeta} = \omega_{a\zeta} + \Delta\omega_{a\zeta in} + \Delta\omega_{a\zeta sys}; && \Omega^*_\xi = \Omega_\xi + \Delta\Omega_{\xi in} + \Delta\Omega_{\xi sys};\\
&\Omega^*_\eta = \Omega_\eta + \Delta\Omega_{\eta in} + \Delta\Omega_{\eta sys}; && \Omega^*_\zeta = \Omega_\zeta + \Delta\Omega_{\zeta in} + \Delta\Omega_{\zeta sys}.
\end{aligned}
\qquad (2.215)
$$

With consideration for equations (2.191), instrument acceleration values a^*_ξ, a^*_η, a^*_ζ respectively equal

$$\left.\begin{aligned} a_\xi^* &= a_\xi + a_\eta(\vartheta_a^* - \vartheta_a) - a_\zeta(\psi_a^* - \psi_a) + \Delta a_{\xi 0} + \delta_{a\xi} a_\xi; \\ a_\eta^* &= a_\eta - a_\xi(\vartheta_a^* - \vartheta_a) + a_\zeta(\gamma_a^* - \gamma_a) + \Delta a_{\eta 0} + \delta_{a\eta} a_\eta; \\ a_\zeta^* &= a_\zeta + a_\xi(\psi_a^* - \psi_a) - a_\eta(\gamma_a^* - \gamma_a) + \Delta a_{\zeta 0} + \delta_{a\zeta} a_\zeta, \end{aligned}\right\} \qquad (2.216)$$

where $\Delta a_{\xi 0}$, $\Delta a_{\eta 0}$, $\Delta a_{\zeta 0}$ are constant error components of the corresponding system accelerometers; $\delta_{a\xi}$, $\delta_{a\eta}$, $\delta_{a\zeta}$, are relative accelerometer errors.

We determine from equations (2.216) instrument errors

$$\left.\begin{aligned} \Delta a_{\xi_{in}} &= a_\eta \Delta\vartheta_{a.in} - a_\zeta \Delta\psi_{a.in} + \Delta a_{\xi 0} + \delta_{a\xi} a_\xi; \\ \Delta a_{\eta_{in}} &= a_\zeta \Delta\gamma_{a.in} + a_\xi \Delta\vartheta_{a.in} - \Delta a_{\eta 0} + \delta_{a\eta} a_\eta; \\ \Delta a_{\zeta_{in}} &= a_\xi \Delta\psi_{a.in} - a_\eta \Delta\gamma_{a.in} + \Delta a_{\zeta 0} + \delta_{a\zeta} a_\zeta. \end{aligned}\right\} \qquad (2.217)$$

INS kinematic equations (2.175) for short-range objects, allowing for systematic and instrument errors, in considering equations (2.125) and (2.216) have the form:

$$\left.\begin{aligned} \dot V_\xi^* &= a_\xi - 2\Omega_{\eta 0} V_\zeta^* + a_\eta(\vartheta_a^* - \vartheta_a) - a_\zeta(\psi_a^* - \psi_a) + \delta_{in}\dot V_\xi; \\ \dot V_\zeta^* &= a_\zeta + 2\Omega_\eta^* V_\xi^* + a_\xi(\psi_a^* - \psi_a) - a_\eta(\gamma_a^* - \gamma_a) + \delta_{in}\dot V_\zeta; \\ \dot\lambda_{arb}^* &= \frac{1}{R_{ea}} V_\xi^* + \delta_{in}\dot\lambda_{arb}; \\ \dot\varphi_{arb}^* &= -\frac{1}{R_{ea}} V_\zeta^* + \delta_{in}\dot\varphi_{arb}; \\ \dot\psi_a^* &= \Omega_\eta^* - (\Omega_\zeta^* - \dot\lambda_{arb}^*)\gamma_a^* + \delta_{in}\dot\psi_a + \Delta\dot\psi_{a0}; \\ \dot\vartheta_a^* &= \Omega_\eta^*\gamma_a + \Omega_\zeta^* - \dot\lambda_{arb}^* + \delta_{in}\dot\vartheta_a + \Delta\dot\vartheta_{a0}; \\ \dot\gamma_a^* &= \Omega_\xi^* - \dot\varphi_{arb}^* - [\Omega_\eta^* - (\Omega_\zeta^* - \dot\lambda_{arb}^*)\gamma_a^*]\vartheta_a^* + \delta_{in}\dot\gamma_a + \Delta\dot\gamma_{a0}; \\ \Omega_\xi^* &= \Omega_{\zeta 0}^* - \Omega_{\eta 0}^*\lambda_{arb}^* + \delta_{in}\Omega_\xi; \\ \Omega_\eta^* &= \Omega_{\xi 0}^*\lambda_{arb}^* + \Omega_{\eta 0}^* + \delta_{in}\Omega_\eta; \\ \Omega_\zeta^* &= \Omega_{\zeta 0}^* + \delta_{in}\Omega_\zeta. \end{aligned}\right\} \qquad (2.218)$$

Here $\Delta\dot\psi_{a0}$, $\Delta\dot\vartheta_{a0}$, $\Delta\dot\gamma_{a0}$ are angular velocities of accelerometer platform drift in inertial space caused by inherent drift of the gyroscope elements; /107

$$\Omega^*_{\xi 0}=\Omega\cos\varphi^*_0\cos\Psi^*_0;$$
$$\Omega^*_{\eta 0}=\Omega\sin\varphi^*_0;$$
$$\Omega^*_{\zeta 0}=-\Omega\cos\varphi_0\sin\Psi^*_0;$$
$$\varphi^*_0=\varphi_0+\Delta\varphi_0;$$
$$\psi^*_0=\Psi_0+\Delta\Psi_0,$$

where $\Delta\phi_0$ is error in the determination and introduction into the system of the value of the geographic latitude at the initial point; $\Delta\Psi_0$ is error in the determination and introduction into the system of the initial course angle value.

In addition,

$$\begin{aligned}
V^*_\xi&=V_{\xi 0}+\int_0^t \dot V^*_\xi dt+\Delta V_{\xi 0}+\delta_{co V_\xi}V_\xi;\\
V^*_\zeta&=V_{\zeta 0}+\int_0^t \dot V^*_\zeta dt+\Delta V_{\zeta 0}+\delta_{co V_\zeta}V_\zeta;\\
\varphi^*_{arb}&=\varphi_{arb\,0}+\int_0^t \dot\varphi^*_{arb}dt+\Delta\varphi_{arb\,0}+\delta_{co\varphi}\varphi_{arb};\\
\lambda^*_{arb}&=\lambda_{arb\,0}+\int_0^t \dot\lambda^*_{arb}dt+\Delta\lambda_{arb\,0}+\delta_{co\lambda}\lambda_{arb};\\
\psi^*_a&=\psi_{a0}+\int_0^t \dot\psi^*_a dt+\Delta\psi_{a0}+\delta_{co\psi}\psi_a+\Delta\psi_a;\\
\vartheta^*_a&=\vartheta_{a0}+\int_0^t \dot\vartheta^*_a dt+\Delta\vartheta_{a0}+\delta_{co\vartheta}\vartheta_a+\Delta\vartheta_a;\\
\gamma^*_a&=\gamma_{a0}+\int_0^t \dot\gamma^*_a dt+\Delta\gamma_{a0}+\delta_{co\gamma}\gamma_a+\Delta\gamma_a,
\end{aligned}\qquad (2.219)$$

where $\Delta\psi_a$, $\Delta\vartheta_a$, $\Delta\gamma_a$ are errors in platform inclination devices; /108
$\Delta V_{\xi 0}$, $\Delta V_{\zeta 0}$, $\Delta\phi_{arb\ 0}$, $\Delta\lambda_{arb\ 0}$, $\Delta\psi_{a0}$, $\Delta\vartheta_{a0}$, $\Delta\gamma_{a0}$ are INS display errors at the starting point and in the introduction into the system of initial data;
$\delta_{co V\xi}$, $\delta_{co V\zeta}$, $\delta_{co\phi}$, $\delta_{co\lambda}$, $\delta_{co\psi}$, $\delta_{co\vartheta}$, $\delta_{co\gamma}$ are relative integration errors of the corresponding parameters.

Partial INS instrument errors on the right side of equations (2.218) equal:

$$
\begin{aligned}
&\delta_{\text{in}}\dot{V}_\xi = \Delta a_{\xi 0} + \delta_{a\xi} a_\xi + \delta_{\text{co}\dot{V}_\xi}\dot{V}_\xi;\\
&\delta_{\text{in}}\dot{V}_\zeta = \Delta a_{\zeta 0} + \delta_{a\zeta} a_\zeta + \delta_{\text{co}\dot{V}_\zeta}\dot{V}_\zeta;\\
&\delta_{\text{in}}\dot{\lambda}_{\text{arb}} = \delta_{\text{co}\dot{\lambda}}\dot{\lambda}_{\text{arb}};\\
&\delta_{\text{in}}\dot{\varphi}_{\text{arb}} = \delta_{\text{co}\dot{\varphi}}\dot{\varphi}_{\text{arb}};\\
&\delta_{\text{in}}\dot{\psi}_{\text{in}} = \delta_{\text{co}\dot{\psi}}\dot{\psi}_a + \Delta\dot{\psi}_{a0};\\
&\delta_{\text{in}}\dot{\vartheta}_a = \delta_{\text{co}\dot{\vartheta}}\dot{\vartheta}_a + \Delta\dot{\vartheta}_{a0};\\
&\delta_{\text{in}}\dot{\gamma}_a = \delta_{\text{co}\dot{\gamma}}\dot{\gamma}_a + \Delta\dot{\gamma}_{a0};\\
&\delta_{\text{in}}\Omega_\xi = \delta_{\text{co}\Omega_\xi}\Omega_\xi;\\
&\delta_{\text{in}}\Omega_\eta = \delta_{\text{co}\Omega_\eta}\Omega_\eta;\\
&\delta_{\text{in}}\Omega_\zeta = \delta_{\text{co}\Omega_\zeta}\Omega_\zeta,
\end{aligned}
\qquad (2.220)
$$

where $\delta_{\text{co}\dot{V}_\xi}$, $\delta_{\text{co}\dot{V}_\zeta}$, $\delta_{\text{co}\dot{\lambda}}$, $\delta_{\text{co}\dot{\varphi}}$, $\delta_{\text{co}\dot{\psi}}$, $\delta_{\text{co}\dot{\vartheta}}$, $\delta_{\text{co}\dot{\gamma}}$, $\delta_{\text{co}\Omega_\xi}$, $\delta_{\text{co}\Omega_\eta}$, $\delta_{\text{co}\Omega_\zeta}$ are relative errors in the corresponding computer devices.

Instrument parameter values which include both systematic as well as instrument element errors are determined by solving the system (2.218) and considering equations (2.219). The basic data for the solution of system (2.218) encompasses the true instantaneous values ψ_a, ϑ_a, γ_a of the deviation angles of the $\xi\eta\zeta$ arbitrary trihedron and accelerations a_ξ, a_η, a_ζ, which are subject to measurement by system accelerometers. These parameters are determined in advance by the solution of system (2.50). /109

The time dependencies of INS instrument errors may be determined from equations (2.215) in accordance with systematic error values which must be calculated in advance.

Kinematic INS equations (2.176) for medium-range objects, with consideration for systematic and instrument errors, as well as for equations (2.215) and (2.216) will have the form

$$
\begin{aligned}
\dot{V}^*_\xi &= a_\xi - \frac{V^*_\xi V_\eta}{R_{\text{ea}}} - 2(\Omega^*_\eta V^*_\zeta - \Omega^*_\zeta V^*_\eta) + a_\eta(\vartheta^*_a - \vartheta_a) -\\
&\quad - a_\zeta(\psi^*_a - \psi_a) + \delta_{\text{in}}\dot{V}_\xi;\\
\dot{V}^*_\eta &= a_\eta + \frac{V^{*2}_\xi}{R_{\text{ea}}} - 2(\Omega^*_\zeta V^*_\xi - \Omega^*_\xi V^*_\zeta) - a_\xi(\vartheta^*_a - \vartheta_a) +\\
&\quad + a_\zeta(\gamma^*_a - \gamma_a) + \delta_{\text{in}}\dot{V}_\eta + g_\eta;\\
\dot{V}^*_\zeta &= a_\zeta - \frac{V^*_\eta V^*_\zeta}{R_{\text{ea}}} - 2(\Omega^*_\xi V_\eta - \Omega^*_\eta V^*_\xi) + a_\xi(\psi^*_a - \psi_a) -\\
&\quad - a_\eta(\gamma^*_a - \gamma_a) + \delta_{\text{in}}\dot{V}_\zeta;
\end{aligned}
\qquad (2.22_1)
$$

$$
\begin{aligned}
&\dot{\lambda}^*_{arb} = \frac{1}{R_{ea}+H^*}V^*_\xi + \delta_{in}\dot{\lambda}_{arb};\\
&\dot{H}^* = V^*_\eta;\\
&\dot{\varphi}^*_{arb} = -\frac{1}{R_{ea}}V^*_\zeta + \delta_{in}\dot{\varphi}_{arb};\\
&\dot{\psi}^*_a = \frac{1}{\cos\vartheta^*_a}[\Omega^*_\eta\cos\gamma^*_a - (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\sin\gamma^*_a] + \Delta\dot{\psi}_{a0} + \delta_{in}\dot{\psi}_a;\\
&\dot{\vartheta}^*_a = \Omega^*_\eta\sin\gamma^*_a + (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\cos\gamma^*_a + \Delta\dot{\vartheta}_{a0} + \delta_{in}\dot{\vartheta}_a;\\
&\dot{\gamma}^*_a = \Omega^*_\xi - \dot{\varphi}^*_{arb} - [\Omega^*_\eta\cos\gamma^*_a - (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\sin\gamma^*_a]\tan\vartheta^*_a +\\
&\qquad + \delta_{in}\dot{\gamma}_a + \Delta\dot{\gamma}_{a0};\\
&\Omega^*_\xi = \Omega^*_{\xi 0}\cos\lambda^*_{arb} - \Omega^*_{\eta 0}\sin\lambda^*_{arb} + \delta_{in}\Omega_\xi;\\
&\Omega^*_\eta = \Omega^*_{\xi 0}\sin\lambda^*_{arb} + \Omega^*_{\eta 0}\cos\lambda^*_{arb} - \Omega^*_{\zeta 0}\varphi^*_{arb} + \delta_{in}\Omega_\eta;\\
&\Omega^*_\zeta = (\Omega^*_{\xi 0}\sin\lambda^*_{arb} + \Omega^*_{\eta 0}\cos\lambda^*_{arb})\varphi^*_{arb} + \Omega^*_{\zeta 0} + \delta_{in}\Omega_\zeta.
\end{aligned}
\tag{2.221}
$$

Through the solution of system (2.221) with the aid of equation (2.219) we may determine the instrument parameter values obtained from an INS during object motions along a trajectory established with respect to the surface of the earth. The instrument error values for each moment of time are easily determined from equations (2.215). The same parameters as those for the solution of the system (2.218) serve as initial data for the solution of system (2.221). /110

It is necessary to know that the presence of instrument INS errors in the determination of vertical acceleration $\dot{V}\eta$ leads not only to errors in the determination of the INS vertical velocity and altitude, but also to errors in the determination of all other parameters which are included in equations (2.221), since the latter have crosslinks with many parameters.

Instrument errors in an INS with a platform oriented in the arbitrary coordinate system may be determined in a first approximation, just as for the geographic coordinate system, by a solution of the corresponding system of error equations.

These equations (2.175) for short-range objects have the following form:

$$
\begin{aligned}
&p\Delta V_{\xi in} + a_{1V_\zeta}\Delta V_{\zeta in} + a_{1\psi}\Delta\psi_{a.in} + a_{1\vartheta}\Delta\vartheta_{a.in} = \delta_{in}\dot{V}_\xi;\\
&a_{2V_\xi}\Delta V_{\xi in} + p\Delta V_{\zeta in} + a_{22_\eta}\Delta\Omega_{\eta in} + a_{2\psi}\Delta\psi_{a.in} + a_{2\gamma}\Delta\gamma_{a.in} = \delta_{in}\dot{V}_\zeta;\\
&p\lambda_{arb} + a_{3V_\zeta}\Delta V_{\xi in} = \delta_{in}\dot{\lambda}_{arb};\\
&p\varphi_{arb} + a_{4V_\zeta}\Delta V_{\zeta in} = \delta_{in}\dot{\varphi}_{arb};\\
&a_{5\lambda}p\lambda_{arb.in} + p\Delta\psi_{a.in} + a_{5\gamma}\Delta\gamma_{a.in} + a_{5\Omega_\eta}\Delta\Omega_{\eta in} + a_{5\Omega_\zeta}\Delta\Omega_{\zeta in} = \delta_{in}\dot{\psi}_a;
\end{aligned}
\tag{2.222}
$$

$$
\begin{aligned}
&a_{6\lambda}p\Delta\lambda_{a,in}+p\Delta\vartheta_{a,in}+a_{6\gamma}\Delta\gamma_{a,in}+a_{6\varrho_\eta}\Delta\Omega_{\eta in}+a_{6\varrho_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\dot{\vartheta}_a;\\
&a_{7\lambda}p\Delta\lambda_{arb,in}+a_{7\varphi}p\Delta\varphi_{arb,in}+a_{7\vartheta}\Delta\vartheta_{a,in}+(p+a_{7\gamma})\Delta\gamma_{a,in}+\\
&\qquad+a_{7\varrho_\xi}\Delta\Omega_{\xi in}+a_{7\varrho_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\dot{\gamma}_a;\\
&a_{8\lambda}\Delta\lambda_{arb\,arb,in}+a_{8\varrho_\xi}\Delta\Omega_{\xi in}=\delta_{in}\Omega_\xi;\\
&a_{9\lambda}\Delta\lambda_{arb\,arb,in}+a_{9\varrho_\eta}\Delta\Omega_{\eta in}=\delta_{in}\Omega_\eta;\\
&a_{10\varrho_\zeta}\Delta\Omega_\zeta=\delta_{in}\Omega_\zeta.
\end{aligned}
\tag{2.222}
$$

Coefficients of equations (2.222) are determined in accordance with (2.196), and partial instrument errors which are included in the right sides of the equations are determined from (2.220).

In addition, in solving equations (2.222), it is necessary to consider equations (2.219), which permit us to examine the influence of INS and display errors at the starting point.

Basic data for the solution of system (2.222) are furnished by individual element errors, INS display errors (including errors in the introduction of initial data), and also by the true values of the kinematic parameters determined by means of solving system (2.50). /111

INS instrument error equations (2.176) for medium-range objects will be as follows:

$$
\begin{aligned}
&(p+a_{1V_\xi})\Delta V_{\xi in}+a_{1V_\eta}\Delta V_{\eta in}+a_{1V_\zeta}\Delta V_{\zeta in}+a_{1\psi}\Delta\psi_{a,in}+\\
&\qquad+a_{1\vartheta}\Delta\vartheta_{a,in}+a_{1\varrho_\eta}\Delta\Omega_{\eta in}+a_{1\varrho_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\dot{V}_\xi;\\
&a_{2V_\xi}\Delta V_{\xi in}+p\Delta V_{\eta in}+a_{2V_\zeta}\Delta V_{\zeta in}+a_{2\vartheta}\Delta\vartheta_{a,in}+a_{2\gamma}\Delta\gamma_{a,in}+\\
&\qquad+a_{2\varrho_\xi}\Delta\Omega_{\xi in}+a_{2\varrho_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\dot{V}_\eta;\\
&a_{3V_\xi}\Delta V_{\xi in}+a_{3V_\eta}\Delta V_{\eta in}+(p+a_{3V_\zeta})\Delta V_{\zeta in}+a_{3\psi}\Delta\psi_{a,in}+\\
&\qquad+a_{3\gamma}\Delta\gamma_{a,in}+a_{3\varrho_\xi}\Delta\Omega_{\xi in}+a_{3\varrho_\eta}\Delta\Omega_{\eta in}=\delta_{in}\dot{V}_\zeta;\\
&a_{4V_\xi}\Delta V_{\xi in}+p\Delta\lambda_{arb,in}+a_{4H}\Delta H_{in}=\delta_{in}\dot{\lambda}_{arb};\\
&a_{5V_\eta}\Delta V_{\eta in}+p\Delta H_{in}=\delta_{in}\dot{H};\\
&a_{6V_\zeta}\Delta V_{\zeta in}+p\Delta\varphi_{arb,in}=\delta_{in}\dot{\varphi}_{arb};\\
&a_{7\lambda}p\Delta\lambda_{arb,in}+p\Delta\psi_{a,in}+a_{7\vartheta}\Delta\vartheta_{a,in}+a_{7\gamma}\Delta\gamma_{a,in}+a_{7\varrho_\eta}\Delta\Omega_{\eta in}+\\
&\qquad+a_{7\varrho_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\dot{\psi}_a;\\
&a_{8\lambda}p\Delta\lambda_{arb,in}+p\Delta\vartheta_{a,in}+a_{8\gamma}\Delta\gamma_{a,in}+a_{8\varrho_\eta}\Delta\Omega_{\eta in}+
\end{aligned}
\tag{2.223}
$$

$$
\begin{aligned}
&+a_{82\zeta}\Delta\Omega_{\zeta\,in}=\delta_{in}\dot{\vartheta}_a;\\
a_{9\lambda}p\Delta\lambda_{arb,in}&+p\Delta\varphi_{arb,in}+a_{99}\Delta\vartheta_{a,in}+(p+a_{9\gamma})\Delta\gamma_{a,in}+\\
&+a_{99_\xi}\Delta\Omega_{\xi in}+a_{99_\eta}\Delta\Omega_{\eta in}+a_{99_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\dot{\psi}_a;\\
a_{10\lambda}\Delta\lambda_{arb,in}&+a_{10\Omega_\xi}\Delta\Omega_{\zeta in}=\delta_{in}\Omega_\xi;\\
a_{11\lambda}\Delta\lambda_{arb,in}&+a_{11\varphi}\Delta\varphi_{arb,in}+a_{11\Omega_\eta}\Delta\Omega_{\eta in}=\delta_{in}\Omega_\eta;\\
a_{12\lambda}\Delta\lambda_{arb,in}&+a_{12\varphi}\Delta\varphi_{arb,in}+a_{12\Omega_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\Omega_\zeta.
\end{aligned}
\qquad (2.223)
$$

Equation coefficient values are determined by the relationships (2.200), and partial instrument errors $\delta_{in}\dot{V}_\xi, \delta_{in}\dot{V}_\eta, \delta_{in}\dot{V}_\zeta$, by (2.220). Initial data preparation technique in the calculation of instrument errors in the system under consideration are analogous to the same operations for INS short-range objects.

§2.7 Systematic and Instrument INS Errors for Short-Range Objects

Object flight distance is L = 500 km. The average speed on the basic segment of the trajectory is V ≅ 400 m/sec, H ≤ 1000 m; and flight time is t ≅ 1000 sec. The assumed program trajectory of object motion in the vertical plane is shown in Figure 2.25. /112

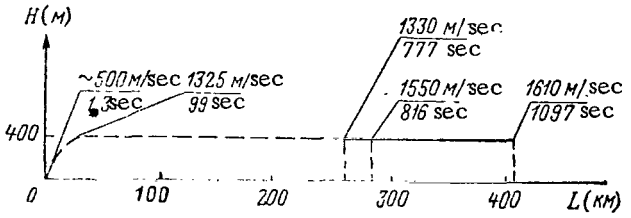

Figure 2.25. The Trajectory of Object Motion in the Vertical Plane.

The inertial navigation system intended for installation on the given object is constructed in accordance with kinematic equations (2.175). This is a two-component system with a gyroscope platform having three degrees of freedom with power gyroscope stabilization. The platform carries two accelerometers A_x, A_z, the measurement axes of which lie in one plane and are mutually perpendicular. The accelerometer axes are stabilized in space by the platform along the ξ and ζ axes of the ξηζ coordinate trihedron, which is also oriented perpendicular to the arbitrary equator. During the flight process the platform is inclined by the application of appropriate correcting moments on the gyroscope elements in accordance with signals $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$, obtained from the output of the continuously-acting computer.

A point at latitude $\phi_0 = 60°$ was chosen as the initial point of the route. It was assumed that the object was started along the programmed orthodrome which forms angle $\phi_0 = 45°$ with the meridian of the starting point.

In order to simplify the problem the trajectory of object motion in the vertical plane was divided by flight time into separate segments.

Uniformity of acceleration components was assumed in the middle of each segment, from which parameter values $V_\xi(t)$, $V_\eta(t)$, $V_\zeta(t)$, $\lambda_{arb}(t)$, $\phi_{arb}(t)$, $H(t)$ were determined.

Through the solution of equation (2.50), with the aid of known parameter values $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$, $V_\xi(t)$, $V_\eta(t)$, $V_\zeta(t)$, $\lambda_{arb}(t)$, $\phi_{arb}(t)$, $H(t)$, we determined the corresponding absolute acceleration vectors $a_\xi(t)$, $a_\eta(t)$, $a_\zeta(t)$, and the angles of rotation $\psi_a(t)$, $\vartheta_a(t)$, $\gamma_a(t)$ of the arbitrary trihedron which occur during object motion along the trajectory established. The equation system coefficients and partial INS systematic errors were determined after this in accordance with equations (2.196) and (2.197).

The results of the solution of the system (2.195) are presented in the form of graphs of the time dependencies of INS systematic errors in the measurement of horizontal velocity components $\Delta V_{\xi\ sys}$, $\Delta V_{\zeta\ sys}$ of the center of mass and in the measurement of object arbitrary coordinates $\Delta\lambda_{arb\ sys}$, $\Delta\phi_{arb\ sys}$. /113

It is apparent from the graphs that with the passage of flight time INS systematic errors in the determination of λ_{arb} and ϕ_{arb} increase. At the end of the trajectory these values are

$$\Delta V_{\zeta sys} \cong 0.08 \text{ m/sec}; \quad \Delta\lambda_{arb\ sys} \cong 3''; \quad \Delta\phi_{arb\ sys} = 2.4''.$$

Based on these calculations it is possible to maintain that the equation system (2.193) completely satisfies INS operational accuracy requirements.

The results of the solution of system (2.195) with constant values for the coefficients and for the partial systematic errors (their values were taken as arithmetic averages over the segment) are shown in Figure 2.26 as a dashed line.

It is apparent from the graphs that INS systematic errors calculated by means of solving equations with constant coefficients have values close to the values of systematic errors obtained as a result of solving equations with variable coefficients. /114

Based on what has been stated, INS systematic errors for short-range objects should be evaluated by means of solving the system of linear equations with variable coefficients.

Let us employ the following technique to determine instrument errors.

The system of kinematic equations (2.218) of the INS under consideration and involving a given short-range object, with the presence of errors, was derived previously. In solving this system, equations (2.219) were considered.

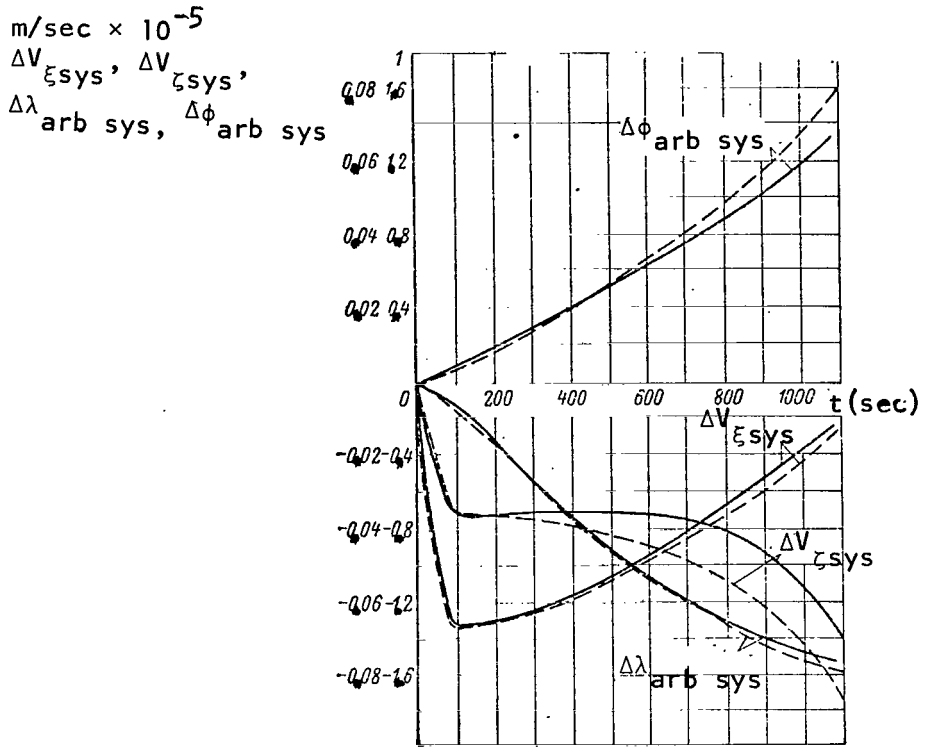

Figure 2.26. INS Systematic Error Graphs.

INS partial instrument error values, included in system (2.218) are determined by equations (2.220) in a first approximation.

In turn, system element errors, included in equations (2.219) and (2.220) are shown in Table 2.1.

Instrument errors in the INS under consideration are determined from instrument values of motion parameters V^*_ξ, V^*_ζ, λ^*_{arb}, ϕ^*_{arb}, obtained as a result of the solution of the system of non-linear differential equations (2.218), with consideration for equations (2.219).

The initial data for the solution of this system is furnished by the true values of the kinematic parameters $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$, $V_\xi(t)$, $V_\eta(t)$, $V_\zeta(t)$, $\lambda_{arb}(t)$, $H(t)$, $\phi_{arb}(t)$ and by INS partial instrument errors.

INS instrument errors in the determination of object ground speed components $\Delta V_{\xi in}$ and $\Delta V_{\eta in}$ and of arbitrary coordinates $\Delta\lambda_{arb\ in}$, $\Delta\phi_{arb\ in}$ with various values for partial instrument error were calculated for each moment in time in accordance with (2.215). /116

TABLE 2.1. ELEMENT ERRORS

Error	Designation	Numerical Error Value
The constant component of accelerometer error	$\Delta a_{\xi 0}$, $\Delta a_{\zeta 0}$	$0.5 \cdot 10^{-4} g$
Relative accelerometer error	$\delta_{a\xi}$, $\delta_{a\zeta}$	0.1%
Error in the calculation of equations with respect to $\dot{V}_{\xi}$ and $\dot{V}_{\zeta}$	$\delta_{co\dot{V}_{\xi}}$, $\delta_{co\dot{V}_{\zeta}}$	-
Error in the determination and introduction of initial object velocities	$\Delta V_{\xi 0}$, $\Delta V_{\zeta 0}$	0.125 m/sec
Error in the device which integrates accelerations $\dot{V}_{\xi}$ and $\dot{V}_{\zeta}$	$\delta_{coV_{\xi}}$, $\gamma_{coV_{\zeta}}$	0.01%
Computer error $\dot{\lambda}_{arb}$ and $\dot{\phi}_{arb}$	$\delta_{co\dot{\lambda}_{arb}}$, $\delta_{co\dot{\phi}_{arb}}$	-
Error in the introduction of the starting point into the arbitrary coordinate system	$\Delta\lambda_{arb0}$, $\Delta\phi_{arb0}$	$4.2 \cdot 10^{-5}$ rad
Error in the device which integrates $\dot{\lambda}_{arb}$ and $\dot{\phi}_{arb}$	$\delta_{co\lambda_{arb}}$, $\delta_{co\phi_{arb}}$	0.01%
Angular velocity of systematic platform drift caused by imperfection in gyroscope elements	$\Delta\dot{\psi}_{a0}$, $\Delta\dot{\vartheta}_{a0}$, $\Delta\dot{\gamma}_{a0}$	$0.5 \cdot 10^{-6}$ 1/sec
Computer error ψ_a, ϑ_a, γ_a	$\delta_{co\psi}$, $\delta_{co\vartheta}$, $\delta_{co\gamma}$	-
Azimuth error in initial platform display	$\Delta\psi_{a0}$	$5.8 \cdot 10^{-4}$ (2)
Error in inital platform display with respect to the plane of the horizon	$\Delta\vartheta_{a0}$, $\Delta\gamma_{a0}$	$2.9 \cdot 10^{-4}$ (1)
Angular parameter error in the platform inclination device (static error in the angular inclination of the platform)	$\Delta\psi_a$, $\Delta\vartheta_a$, $\Delta\gamma_a$	$1.45 \cdot 10^{-4}$ (0.5)

The influence of error in INS in separate elements on the accuracy of system operation is evaluated by solving system (2.218) for various variants in partial instrument errors.

The results of calculation of INS instrument errors in the determination of ground speed components $\Delta V_{\xi\ in}$, $\Delta V_{\zeta\ in}$ and of objects coordinates $\Delta\lambda_{arb}$, $\Delta\phi_{arb}$ at the end of the trajectory, caused by errors in each system element, are presented in Table 2.2 for the different solution variants (there are eleven variants in all).

Figures 2.27-2.30 show the time dependencies of INS instrument errors from which the influence of the errors of separate system elements on true kinematic parameter errors are apparent. The number of curves shown on the graphs correspond to the numbers of solution variants shown in Table 2.2.

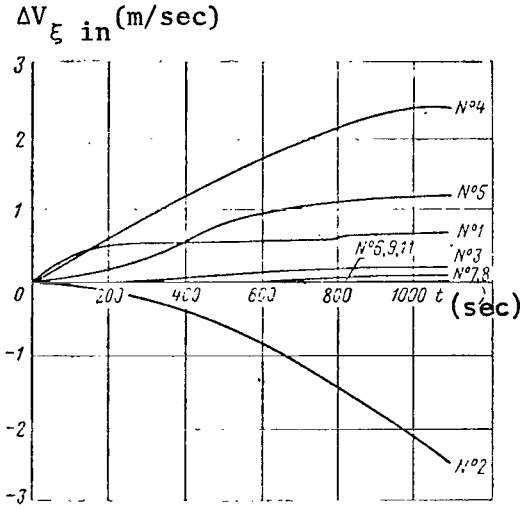

Figure 2.27. Graphs of INS Partial Instrument Errors in the Measurement of V_{ξ}.

In an analysis of the results of the calculation, it is possible to make the following conclusions:

1. The maximum influence on operational accuracy of the INS under consideration is caused by errors in the accelerometer which measures the longitudinal accelerations, and by initial display errors in platform stabilization in the longitudinal and lateral planes.

INS errors caused by each of the elements indicated do not exceed the following values toward the end of the object controlled flight trajectory:

$$\Delta V_{\xi in} < 2.4 \text{ m/sec}; \qquad \Delta V_{\zeta in} < 2.3 \text{ m/sec};$$
$$\Delta\lambda_{\xi arb\ in} < 1'; \qquad \Delta\phi_{\zeta arb\ in} < 0.6'.$$

2. Errors in the integration of lateral accelerations and velocities, as well as errors in initial display and as in platform stabilization exert less influence on the operation of the INS under consideration.

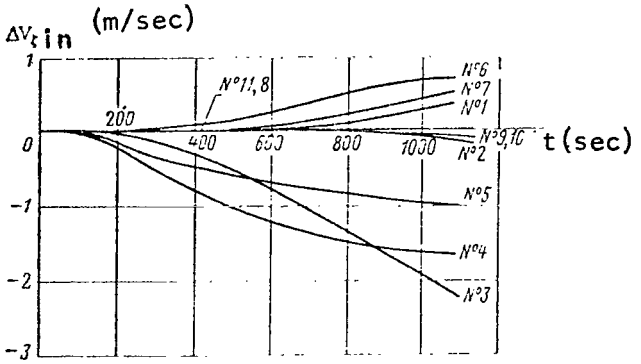

Figure 2.28. Graphs of INS Partial Instrument Errors in the Measurement of V_ζ.

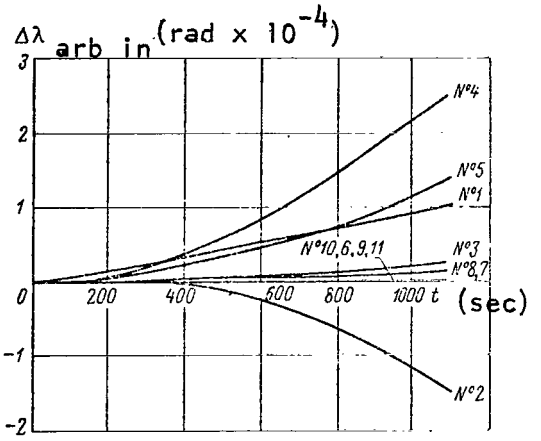

Figure 2.29. Graphs of INS Partial Instrument Errors in the Measurement of λ_{arb}.

INS error as a result of these elements does not exceed

$$\Delta V_{\xi in} < 0.05 \text{ m/sec}; \qquad \Delta V_{\zeta in} < 0.11 \text{ m/sec};$$
$$\Delta \lambda_{arb} < 0.2''; \qquad \Delta \phi_{arb\ in} < 0.2''.$$

Therefore it is possible to reduce somewhat the accuracy requirements of these elements without influencing significantly the operational accuracy of the INS as a whole.

TABLE 2.2. INSTRUMENT ERRORS OF INS'S OF THE 1 CLASS.

Name of Element and Error	Partial Instrument Error	Equation Variant	INS Error at the end of the Trajectory			
			ΔV_ξ in m/sec	ΔV_ζ in m/sec	$\Delta\varphi$ arb in	$\Delta\lambda$ arb in
Accelerometer A_x	$\Delta a_{ax} = 0{,}5\cdot10^{-4}g$ $\delta_{ax} = 0.1\,\%$	1	0,63	0,29	$-0{,}12\cdot10^{-4}$ ($\approx3''$)	$0{,}979\cdot10^{-4}$ (0,3′)
Systematic platform drift in the longitudinal plane	$\Delta\dot{\vartheta}_{a0}=0{,}5\cdot10^{-6}$	2	−2,50	−0,14	$9{,}3\cdot10^{-5}$ ($\approx2''$)	$-1{,}5\cdot10^{-4}$ (0,5′)
Systematic platform drift in the lateral plane	$\Delta\dot{\gamma}_{a0}=0{,}5\cdot10^{-6}$	3	0,14	−2,26	$1{,}4\cdot10^{-4}$ ($\approx5'$)	$0{,}22\cdot10^{-4}$ ($\approx4''$)
Error in initial platform display	$\Delta\psi_{a0}=2{,}9\cdot10^{-4}$ $\Delta\dot{\vartheta}_{a0}=-2{,}9\cdot10^{-4}$ $\Delta\gamma_{a0}=-2{,}9\cdot10^{-4}$	4	2,39	−1,75	$1{,}75\cdot10^{-4}$ ($\approx5'$)	$2{,}5\cdot10^{-4}$ ($\approx1'$)

Tr. Note: Commas indicate decimal points.

Static platform inclination error in angular parameters	$\delta_{\psi_a}=\delta_{\theta_a}=\delta_{\gamma_a}=1{,}45\cdot10^{-4}$	5	1,22	—0,99	$1{,}02\cdot10^{-4}$ (0,3′)	$1{,}3\cdot10^{-4}$ (0,5′)
Accelerometer A_z	$\Delta a_z=0{,}5\cdot10^{-4}g$ $\delta_{az}=0{,}1\%$	6	0,01	0,67	$-5{,}07\cdot10^{-5}$ (≈10″)	$-0{,}23\cdot10^{-6}$ (0,05″)
Integrator $\dot{V}_\xi$	$\delta_{co V_\xi}=0.01\%$	7	0,1	0,46	$-1{,}2\cdot10^{-5}$ (≈2,5)	$8{,}70\cdot10^{-6}$ (≈2,0″)
Integrator $\dot{\lambda}_{arb}$	$\delta_{co\lambda_{arb}}=0.01\%$	8	0,1	—0,02	$2{,}21\cdot10^{-6}$ (≈0,5′)	$6{,}59\cdot10^{-6}$ (≈1″)
Integrator $\dot{V}_\zeta$	$\delta_{co V_\zeta}=0.01\%$	9	0,04	0,11	$0{,}89\cdot10^{-6}$ (0,2″)	$0{,}9\cdot10^{-7}$ (≈0,02″)
Systematic platform drift in azimuth	$\Delta\psi_{a0}=0{,}5\cdot10^{-6}$ (0,18 °/hr)	10	0,02	0,12	$7{,}78\cdot10^{-6}$ (≈1,5″)	$1{,}58\cdot10^{-6}$ (0,2″)
Integrator $\dot{\phi}_{arb}$	$\delta_{co\phi_{arb}}=0.01\%$	11	0,00	0,00	$0{,}12\cdot10^{-8}$ (0,002″)	0,0000000

Tr. Note: Commas indicate decimal points.

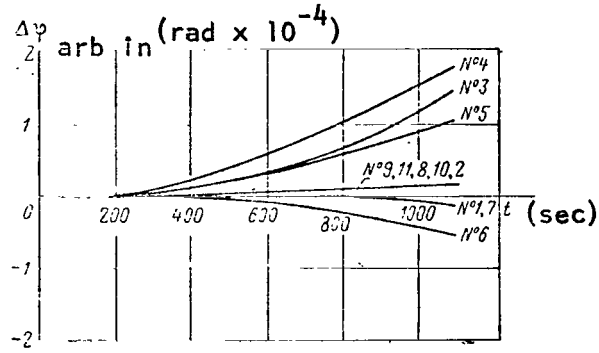

Figure 2.30. Graphs of Partial Systematic Errors in the Measurement of ϕ_{arb}.

In view of the fact that maximum influence on INS operational accuracy is exerted by errors in accelerometers and platform stabilization, it is feasible to accomplish the initial coarse calculation while considering these errors alone. In an evaluation of INS error caused by other elements, it is feasible to investigate the influence of errors in the latter on platform stabilization or on the determination of accelerations in accordance with which it is possible to judge INS operational accuracy.

CHAPTER III

THE KINEMATICS OF AN INERTIAL NAVIGATION SYSTEM WITH A PLATFORM STABILIZED IN INERTIAL SPACE.

§3.1. The Principle Possibility of Constructing an INS.

Errors in an INS with a platform oriented in the earth coordinate system are to a great degree determined by platform stabilization errors. /121

The satisfaction of high requirements established for INS operational accuracy is to a significant degree limited by difficulties in the development and manufacture of precision correction platforms.

Therefore a specific practical interest is involved in the development of such INS's which drop the platform inclination requirement for the stabilization of platform in the earth coordinate system (the geographic, arbitrary, and other systems).

Investigations reveal that it is possible to construct an INS with a platform which carries accelerometers and which does not rotate in inertial space. Here devices for platform inclination or platform correction in the earth coordinate system are not required, but the platform must be stabilized with a sufficient degree of accuracy in inertial space.

In such a system errors will be avoided which are caused by the platform inclination devices which introduce errors both directly and through the gyroscope elements the operation of which they influence.

We shall examine the kinematic equations of a single-component INS.

Let us assume that an object with a platform stabilized in inertial space and bearing two accelerometers A_x and A_y is in motion in a plane relative to the fixed earth sphere (Figure 3.1).

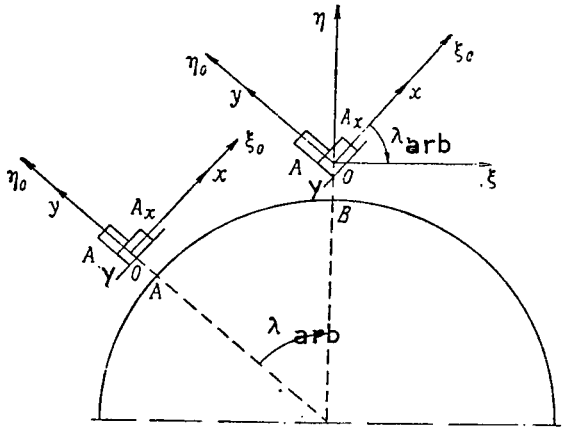

Figure 3.1.

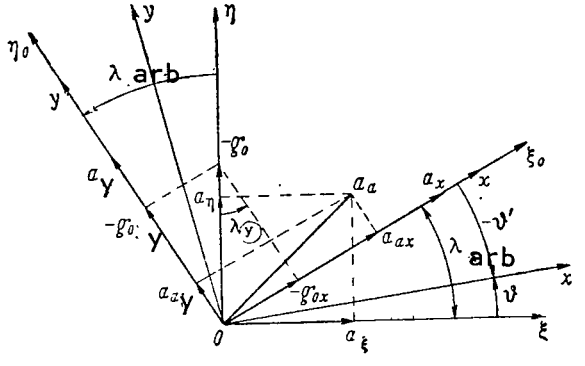

Figure 3.2.

The x, y accelerometer measurement axes are mutually perpendicular and are established before the initiation of motion along the $\xi_0\eta_0$ axes of the arbitrary dihedron; in the process of motion their angular position in inertial space remains fixed.

The angular position of the platform and of the accelerometer measurement axes relative to the plane of the horizon at any time during motion (point B) is determined by the arbitrary coordinate λ_{arb} of object location with respect to starting point A. /123

The vector projections of absolute acceleration equal (Figure 3.2)

$$\begin{aligned} a_{ax} &= a_x - g_{0x}; \\ a_{ay} &= a_y - g_{0y}, \end{aligned} \tag{3.1}$$

where g_{0x}, g_{0y} are earth gravity projections;
a_x, a_y are non-gravitational acceleration components measured by the corresponding accelerometers A_x and A_y.

It is apparent from Figure 3.2 that

$$\begin{aligned} g_{0x} &= g_0 \sin \lambda_{arb} \\ g_{0y} &= g_0 \cos \lambda_{arb} \end{aligned} \tag{3.2}$$

then

$$\begin{aligned} a_{ax} &= a_x - g_0 \sin \lambda_{arb} \\ a_{ay} &= a_y - g_0 \cos \lambda_{arb} \end{aligned} \tag{3.3}$$

Since we are interested in the motion parameters with respect to the surface of the earth, we shall find the projections of these accelerations on the $\xi\eta$ axes of the dyhedron:

$$\begin{aligned} a_\xi &= a_{ax} \cos \lambda_{arb} - a_{ay} \sin \lambda_{arb}; \\ a_\eta &= a_{ax} \sin \lambda_{arb} + a_{ay} \cos \lambda_{arb} \end{aligned} \tag{3.4}$$

Having substituted values a_{ax} and a_{ay} from equations (3.3) in the equation (3.4), we obtain

$$\begin{aligned} a_\xi &= a_x \cos \lambda_{arb} - a_y \sin \lambda_{arb} \\ a_\eta &= a_x \sin \lambda_{arb} + a_y \cos \lambda_{arb} - g_0. \end{aligned} \tag{3.5}$$

It is apparent from equations (3.5) that in a determination of horizontal a_ξ and vertical a_η vector acceleration components, there is no requirement to stabilize the accelerometers with respect to the plane of the horizon and vertically, for a_ξ and a_η may be calculated from equations (3.5), if a_x and a_y and λ_{arb} are known.

In order to determine the arbitrary coordinate λ_{arb}, it is sufficient to utilize the first equation of the system (3.5), which, when the following dependency is taken into consideration

$$a_\xi = \ddot{\lambda}_{arb} R_{ea} \tag{3.6}$$

has the form

$$\ddot{\lambda}_{arb} = \frac{1}{R_{ea}} (a_x \cos\lambda_{arb} - a_y \sin\lambda_{arb}). \tag{3.7}$$

It is easy to see that equation (3.7) is nonlinear. Basic data for the solution of this equation is furnished by accelerometer indications a_x and a_y and by the initial values $\dot{\lambda}_{arb0}$ and λ_{arb0} of the parameters desired, which for simplicity we shall consider to be equal to zero.

The block diagram of a single-component INS with a fixed platform (Figure 3.3) shows the connection between the accelerometers A_x and A_y, installed on the platform, with the computer (VU). /124

It is apparent from the block diagram that the single-component INS with a fixed platform is closed with respect to parameter λ_{arb} in the same way as in an INS of the I class. This closure is accomplished in the system computer which models mathematically the conditions corresponding to the Schuler pendulum, in place of a physical model, which is present in the INS of the I class.

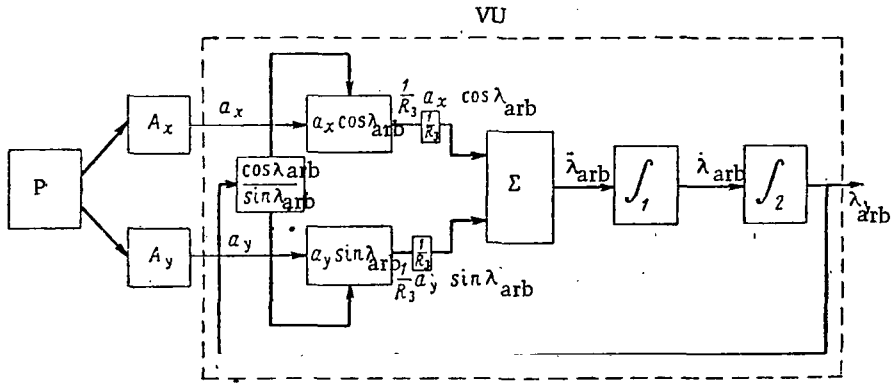

Figure 3.3. Block Diagram of a Single-Component INS with a Platform Stabilized in Inertial Space.

It should be noted that by means of the INS examined, it is possible to determine also the instantaneous values of the angles of deviation of the longitudinal axis of object x, in which the system is installed, from the plane of the horizon (Figure 3.2) by solving the equation

$$\vartheta = -(\lambda_{arb} - \vartheta'), \tag{3.8}$$

where ϑ is the angle of inclination of the object longitudinal axis from the plane of the horizon; ϑ' is the angle of inclination of the object longitudinal axis with respect to the fixed platform.

Thus the requirement for having a vertical gyroscope on board the object is avoided.

It is obvious that INS instrument errors will be caused by errors in the accelerometers, in the computer devices, and by platform stabilization errors in inertial space.

If the A_x and A_y accelerometers measure accelerations a_x and a_y with errors Δa_x, Δa_y, then this will lead to the appearance of errors in the measurement of the λ_{arb} coordinate.

The instrument parameter values a_x^*, a_y^*, λ_{arb}^* are determined as follows: /125

$$\left.\begin{aligned} a_x^* &= a_x + \Delta a_x; \\ a_y^* &= a_y + \Delta a_y; \\ \lambda_{arb}^* &= \lambda_{arb} + \Delta\lambda_{arb} \end{aligned}\right\} \tag{3.9}$$

Here the equation (3.7) is rewritten in the following form:

$$\ddot{\lambda}^*_{arb} = \frac{1}{R_{ea}}(a^*_x \cos\lambda^*_{arb} - a^*_y \sin\lambda^*_{arb}). \tag{3.10}$$

Having placed the values a^*_x, a^*_y, λ^*_{arb} from equations (3.9) in the last equation, after transformations, considering $\Delta\lambda_{arb} = -\Delta\vartheta_a$ and equations (3.5), we obtain

$$\Delta\ddot{\lambda}_{arb} + \frac{a_\eta + g_0}{R_{ea}}\Delta\lambda_{arb} \cong \frac{1}{R_{ea}}(\Delta a_x \cos\lambda_{arb} - \Delta a_y \sin\lambda_{arb}). \tag{3.11}$$

It is apparent that the left side of the equation (3.11) represents (where $a_\eta = 0$) the Schuler pendulum equation, and the right side represents a perturbation factor which disturbs pendulum equilibrium.

Just as stated previously, accelerometer errors may be presented in the following manner:

$$\left.\begin{aligned} \Delta a_x &= \Delta a_{x0} + \delta_a a_x; \\ \Delta a_y &= \Delta a_{y0} + \delta_a a_y. \end{aligned}\right\} \tag{3.12}$$

Allowing for (3.12), equation (3.11) will have the form

$$\left.\begin{aligned} \Delta\ddot{\lambda}_{arb} + \frac{a_\eta + g_0}{R_{ea}}\Delta\lambda_{arb} = \frac{1}{R_{ea}}[(\Delta a_{x0}\cos\lambda_{arb} - \Delta a_{y0}\sin\lambda_{arb}) + \\ + \delta_a(a_x\cos\lambda_{arb} - a_y\sin\lambda_{arb})]. \end{aligned}\right\} \tag{3.13}$$

We shall designate

$$\left.\begin{aligned} \Delta a_{x0}\cos\lambda_{arb} - \Delta a_{y0}\sin\lambda_{arb} &= \Delta a_{\xi 0}; \\ \delta_a(a_x\cos\lambda_{arb} - a_y\sin\lambda_{arb}) &= \delta_a a_\xi, \end{aligned}\right\} \tag{3.14}$$

and then the error equations will be

$$\Delta\ddot{\lambda}_{arb} + \frac{a_\eta + g_0}{R_{ea}}\Delta\lambda_{arb} = \frac{1}{R_{ea}}(\Delta a_{\xi 0} + \delta_a a_\xi), \tag{3.15}$$

or

$$\Delta\ddot{\lambda}_{arb} + \frac{a_\eta + g_0}{R_{ea}}\Delta\lambda_{arb} = \frac{\Delta a_\xi}{R_{ea}}, \tag{3.16}$$

where

$$\Delta a_\xi = \Delta a_{\xi 0} + \delta_a a_\xi.$$

Where $\Delta a_\xi = \Delta a = \text{const}$, $a_\eta = 0$, the solution to equation (3.16) will be as follows: /126

$$\Delta\lambda_{arb} = \frac{\Delta a}{g_0}\left(1 - \cos\sqrt{\frac{g_0}{R_{ea}}}\cdot t\right). \tag{3.17}$$

This expression is similar to expression (2.9), and a graph of its functions is shown in Figure 3.4.

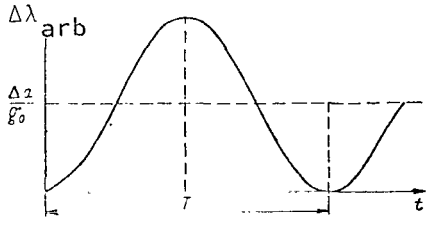

Figure 3.4. A Graph of $\Delta\lambda_{arb}(t)$ Error Changes for a Single Component INS.

It is apparent from equation (3.17) that INS error in the determination of the λ_{arb} coordinate, caused by constant accelerometer errors, has a constant component

$$\Delta\lambda_{arb0} = \frac{\Delta a}{g_0} \tag{3.18}$$

and a variable component

$$\Delta\lambda_{arb0} = \frac{\Delta a}{g_0}\cos\sqrt{\frac{g_0}{R_{ea}}}\cdot t; \tag{3.19}$$

and the latter changes with a period of

$$T=2\pi\sqrt{\frac{R_{ea}}{g_0}} \tag{3.20}$$

and with an amplitude of

$$\Delta\lambda_m=\frac{\Delta a}{g_0}. \tag{3.21}$$

It is obvious that the maximum INS error value in the determination of λ_{arb} will not exceed

$$\Delta\lambda_{arb\ m}\leqslant 2\frac{\Delta a}{g_0}$$

Thus error in an INS with a platform fixed in inertial space, caused by constant accelerometer error, in a first approximation will change in accordance with the same law which similarly governs error in an INS with a "corrected" platform (Section 2.1).

If, for example, object motion on the basic segment of the trajectory is /127

$$a_\xi=0{,}5\,g,\ \Delta a_{\xi 0}=5\cdot 10^{-5}\,g,\ \delta_a=0{,}05\%,$$

then INS error in the determination of the arbitrary coordinate will not exceed

$$\Delta\lambda_{arb}<\Delta\lambda_{arb\,m}\cong 2'.$$

On the initial segment of the trajectory where $a_\xi \cong 10g$, error amplitude is $\Delta\lambda_{arb\ m} = 15'$, but, considering that in a majority of cases the time of object motion on the acceleration segment is comparatively short (≈100 sec), we conclude from (3.17) that INS error in the determination of the arbitrary coordinate at the end of the acceleration segment will not exceed 8".

It is apparent from the considerations mentioned that errors in acceleration measurement exert a significant influence on the measurement accuracy of the arbitrary coordinate, especially under conditions of extended acceleration effects (of the main segment). Due to the short times involved in the effects of high accelerations on the acceleration segment, arbitrary coordinate

measurement error does not involve high values.

If, in the process of INS operation, errors occur in platform stabilization in inertial space (Figure 3.5), then instrument acceleration values will be

$$a_\xi^* = a_x \cos(\lambda_{arb}^* + \Delta\vartheta) - a_y \sin(\lambda_{arb}^* + \Delta\vartheta), \tag{3.22}$$

where $\Delta\vartheta$ is platform inclination from the initial position.

From equation (3.22) and considering (3.6), we have

$$\ddot{\lambda}_{arb}^* = \frac{1}{R_{ea}}[a_x \cos(\lambda_{arb}^* + \Delta\vartheta) - a_y \sin(\lambda_{arb}^* + \Delta\vartheta)]. \tag{3.23}$$

After linearization, considering equations (3.4) and (3.5), we have

$$\Delta\ddot{\lambda}_{arb} + \frac{a_\eta + g_0}{R_{ea}} \Delta\lambda_{arb} = -\frac{a_\eta + g_0}{R_{ea}} \Delta\vartheta. \tag{3.24}$$

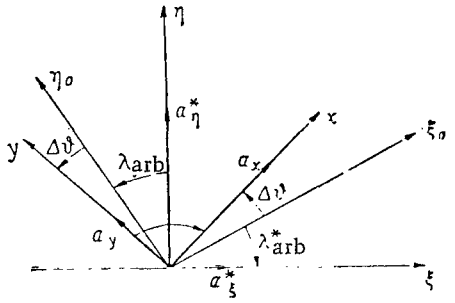

Figure 3.5.

If $\Delta\vartheta = \Delta\vartheta_0 = \text{const}$ and $a_\eta = 0$, then the solution of equation (3.24) will be as follows:

$$\Delta\lambda_{arb} = \Delta\vartheta_0 \left(\cos\sqrt{\frac{g_0}{R_{ea}}}\, t - 1\right). \tag{3.25}$$

It is apparent from equation (3.25) that /128 where constant error $\Delta\vartheta_0$ in platform stabilization is present, error in the determination of the arbitrary coordinate will have a constant component

$$\Delta\lambda_{arb}^0 = -\Delta\vartheta_0$$

and a variable component

$$\Delta\lambda_{arb} = \Delta\vartheta_0 \cos \sqrt{\frac{g_0}{R_{ea}}} \cdot t, \tag{3.26}$$

which changes with a Schuler pendulum period and amplitude

$$\Delta\lambda_m = \Delta\vartheta_0.$$

The maximum value of the INS error in the determination of the arbitrary coordinate is

$$\Delta\lambda_{arb\,m} \cong 2\Delta\vartheta_0. \tag{3.27}$$

The graph of the $\Delta\lambda_{arb}(t)$ function for this case is shown in Figure 3.6.

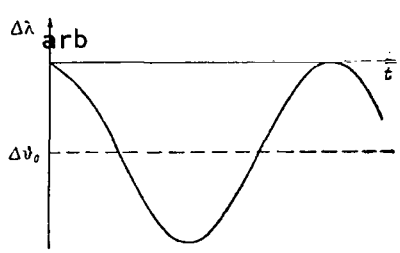

Figure 3.6. Graph of $\Delta\lambda_{arb}(t)$ error change for a single-component INS.

With the presence of systematic inherent platform drift in inertial space with constant angular velocity $\Delta\dot{\vartheta}_0$, we have

$$\Delta\ddot{\lambda}_{arb} + \frac{a_\eta + g_0}{R_{ea}} \Delta\lambda_{arb} = -\frac{a_\eta + g_0}{R_{ea}} \Delta\dot{\vartheta}_0 t. \tag{3.28}$$

in this case INS error in the determination of the arbitrary coordinate is defined as

$$\Delta\lambda_{arb} = \Delta\dot{\vartheta}_0 \left(\sqrt{\frac{R_{ea}}{g_0}} \sin \sqrt{\frac{g_0}{R_{ea}}} \cdot t - t \right). \tag{3.29}$$

It is apparent from (3.29) that with the presence of platform drift with constant angular velocity, the average value of INS error in the determination of the $\Delta\lambda_{arb0}$ arbitrary coordinate will increase along the modulus in proportion to the time of motion

$$\Delta\lambda_{arb\,av} \cong \Delta\dot{\vartheta}_0 t,$$

and the component

$$\Delta\lambda_{\text{arb}\sim} = \Delta\dot{\vartheta}_0 \sqrt{\frac{R_{ea}}{g_0}} \sin \sqrt{\frac{g_0}{R_{ea}}} \cdot t \tag{3.30}$$

will change with Schuler pendulum period and amplitude /129

$$\Delta\lambda_{\text{arb}^m} = \Delta\dot{\vartheta}_0 \sqrt{\frac{R_{ea}}{g_0}}. \tag{3.31}$$

For example, if $\Delta\dot{\vartheta}_0$ = 0.1 deg/hr, then for time t = 20 min $\Delta\lambda_{\text{arb av}} \cong 2'$, and $\Delta\lambda_{\text{arb}} \cong 1''$.

It is not difficult to note that the main portion of the error fits the average component, proportional to the time of motion.

It is apparent from the entire discussion above that errors in an INS with a platform stabilized in inertial space are caused by errors in basic system elements, and in a first approximation these errors change in accordance with the same laws which are applicable to the INS with a platform stabilized in the earth coordinate system. In contrast to the latter, in an INS with a "fixed" platform, the kinematics of the Schuler pendulum is not physically modelled (with a corrected platform and accelerometers), but is mathematically modelled in the system computer assembly. A coordinate dihedron only is materialized, "fixed" in inertial space, from which all calculations are derived.

§3.2. The Kinematic Equations of a Three-Component INS.

Let us assume that an object equipped with the INS under consideration is in motion along a trajectory established with respect to the surface of the earth from point A to point B (Figure 3.7) at an altitude which in a general case is a time function H(t).

The position of the object on the surface of the earth will be determined in the arbitrary coordinate system, since this system may be used for object navigation in any areas of the earth.

We shall introduce into the examination the $\xi_1\eta_1\zeta_1$ inertial, the $\xi\eta\zeta$ arbitrary, and the xyz accelerometer trihedrons. The orientation of the $\xi_1\eta_1\zeta_1$ and the $\xi\eta\zeta$ trihedrons is similar to that indicated in section 2.2.

In turn, the xyz accelerometer trihedron is oriented in the process of motion in parallel to the $\xi_1\eta_1\zeta_1$ axes of the inertial trihedron.

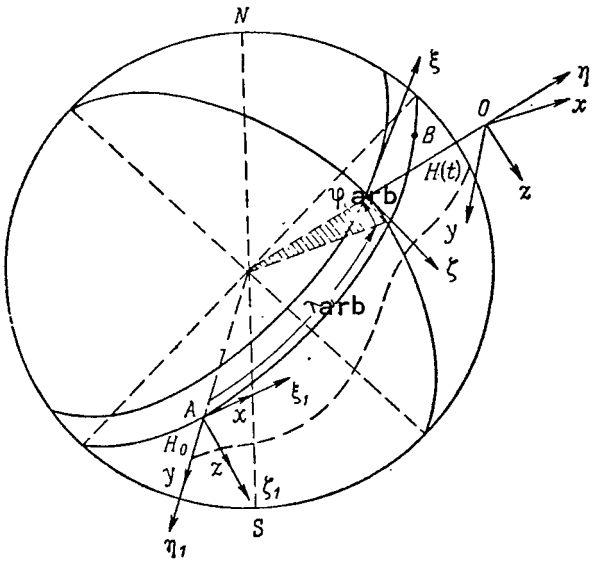

Figure 3.7.

The derivation of the kinematic equations for an INS with a "fixed" platform in inertial space is similar to the derivation of equations for an INS with a platform oriented in the earth coordinate system, and is reduced in practice to the derivation of equations which characterize the kinematics of the motion of the accelerometer trihedron along a trajectory established with respect to the surface of the earth. The latter circumstance indicates that the accelerometer trihedron, in motion, will participate in the rotation of the earth; therefore in deriving these equations, this migratory motion must be taken into consideration.

In addition, in determining the motion of this trihedron with respect to the surface of the earth, it is possible to utilize the kinematic motion equations perviously derived (2.50) for the arbitrary coordinate trihedron by supplementing these with equations defining the position of the accelerometer trihedron with respect to the arbitrary trihedron. /130

In fact, in a "fixed" platform INS the angular position of the accelerometer measurement axes are fixed in inertial space, and therefore the accelerations a_ξ, a_η, a_ζ, included in the system (2.50), will not be measured directly by the system accelerometers; they must be calculated.

But in order to compute a_ξ, a_η, a_ζ from accelerometer data a_x, a_y, a_z it is necessary to know the angles ψ_a, ϑ_a, γ_a which define the position of the arbitrary trihedron with respect to the inertial trihedron axes, along which the accelerometer measurement axes are stabilized.

The angles ψ_a, ϑ_a, γ_a cannot be measured directly on the object, but they may be calculated by solving the corresponding kinematic equations which must be derived.

The instantaneous values a_ξ, a_η, a_ζ, of the acceleration projections which are included in the first three equations of the system (2.50) for $\dot{V}_\xi$, $\dot{V}_\eta$, $\dot{V}_\zeta$ may be determined (Figure 3.8) if the instantaneous values of the projections of this vector on the $\xi_1\eta_1\zeta_1$ (a_x, a_y, a_z) axes of the inertial trihedron are known: /131

$$
\begin{aligned}
a_\xi &= a_x \cos\psi_a \cos\vartheta_a + a_y \sin\vartheta_a - a_z \sin\psi_a \cos\vartheta_a;\\
a_\eta &= a_x(\sin\psi_a \sin\gamma_a - \cos\psi_a \sin\vartheta_a \cos\gamma_a) + a_y \cos\vartheta_a \cos\gamma_a -\\
&\quad + a_z(\cos\psi_a \sin\gamma_a + \sin\psi_a \sin\vartheta_a \cos\gamma_a);\\
a_\zeta &= a_x(\sin\psi_a \cos\gamma_a + \cos\psi_a \sin\vartheta_a \sin\gamma_a) -\\
&\quad - a_y \cos\vartheta_a \sin\gamma_a + a_z(\cos\psi_a \cos\gamma_a - \sin\psi_a \sin\vartheta_a \sin\gamma_a),
\end{aligned}
\tag{3.32}
$$

where ψ_a, ϑ_a, γ_a are angles which characterize the rotation of the $\xi\eta\zeta$ arbitrary trihedron with respect to the $\xi_1\eta_1\zeta_1$ inertial trihedron;
a_x, a_y, a_z are accelerations measured by the system accelerometers while they are stabilized along the inertial trihedron axes.

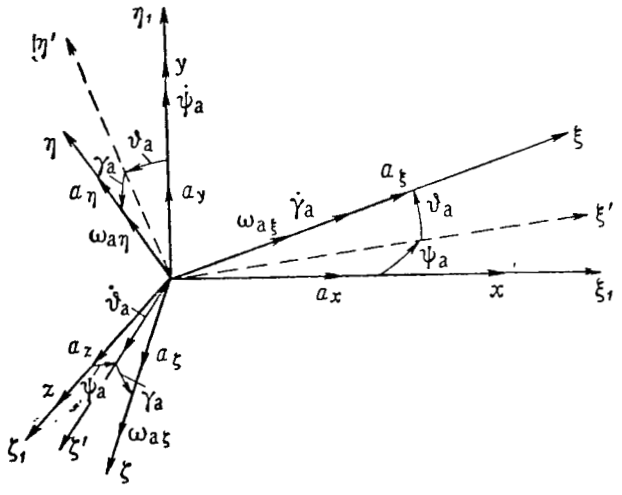

Figure 3.8

Rotation of the $\xi\eta\zeta$ arbitrary trihedron with respect to inertial space is caused by the rotation of the earth and object motion with respect to the surface of the earth. The vector projections of the angular velocity of this rotation ($\omega_{a\xi}$, $\omega_{a\eta}$, $\omega_{a\zeta}$) on the arbitrary trihedron axes are determined by equations previously derived (2.48).

The angular velocities $\dot\psi_a$, $\dot\vartheta_a$, $\dot\gamma_a$ may be determined (see Figure 3.8) from the known equations /132

$$
\left.
\begin{aligned}
\dot\psi_a &= \frac{1}{\cos\vartheta_a}(\omega_{a\eta}\cos\gamma_a - \omega_{a\zeta}\sin\gamma_a);\\
\dot\vartheta_a &= \omega_{a\eta}\sin\gamma_a + \omega_{a\zeta}\cos\gamma_a;\\
\dot\gamma_a &= \omega_{a\xi} - (\omega_{a\eta}\cos\gamma_a - \omega_{a\zeta}\sin\gamma_a)\tan\vartheta_a,
\end{aligned}
\right\}
\tag{3.33}
$$

which are similar to equations (2.49), but in this connection the angles ψ_a, ϑ_a, γ_a are, by definition, different.

The system of kinematic equations for an INS with a "fixed" platform in inertial space may be derived from the motion equation (2.50) of the arbitrary trihedron with respect to the surface of the earth, allowing for equations (3.32) and (3.33):

$$\dot{V}_\xi = a_\xi - \frac{V_\xi V_\eta \cos\varphi_{arb}}{R_{arb\,p} + H\cos\varphi_{arb}} - \frac{V_\xi V_\zeta \sin\varphi_{arb}}{R_{arb\,p} + H\cos\varphi_{arb}} - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta) + g_\xi(\varphi_{arb}, \lambda_{arb}, H);$$

$$\dot{V}_\eta = a_\eta + \frac{V_\xi^2 \cos\varphi_{arb}}{R_{arb\,p} + H\cos\varphi_{arb}} + \frac{V_\zeta^2}{R_{arb,\,m} + H} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta(\varphi_{arb}, \lambda_{arb}, H);$$

$$\dot{V}_\zeta = a_\zeta + \frac{V_\xi^2 \sin\varphi_{arb}}{R_{arb\,p} + H\cos\varphi_{arb}} - \frac{V_\eta V_\zeta}{R_{arb,\,m} + H} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\zeta) + g_\zeta(\varphi_{arb}, \lambda_{arb}, H);$$

$$a_\xi = a_x\cos\psi_a\cos\vartheta_a + a_y\sin\vartheta_a - a_z\sin\psi_a\cos\vartheta_a;$$

$$a_\eta = a_x(\sin\psi_a\sin\gamma_a - \cos\psi_a\sin\vartheta_a\cos\gamma_a) + a_y\cos\vartheta_a\cos\gamma_a + a_z(\cos\psi_a\sin\gamma_a + \sin\psi_a\sin\vartheta_a\cos\gamma_a);$$

$$\dot{a}_\zeta = a_x(\sin\psi_a\cos\gamma_a + \cos\psi_a\sin\vartheta_a\sin\gamma_a) - a_y\cos\vartheta_a\sin\gamma_a + a_z(\cos\psi_a\cos\gamma_a - \sin\psi_a\sin\vartheta_a\sin\gamma_a);$$

$$\dot{\lambda}_{arb} = \frac{\dot{V}_\xi}{R_{arb\,p} + H\cos\varphi_{arb}}; \qquad (3.34)$$

$$\dot{H} = V_\eta;$$

$$\dot{\varphi}_{arb} = -\frac{V_\zeta}{R_{arb,\,m} + H};$$

$$\dot{\psi}_a = \frac{1}{\cos\vartheta_a}[\Omega_\eta\cos\gamma_a - \Omega_\zeta\sin\gamma_a + \dot{\lambda}_{arb}\sin(\varphi_{arb} + \gamma_a)];$$

$$\dot{\vartheta}_a = (\Omega_\eta + \dot{\lambda}_{arb}\sin\varphi_{arb})\sin\gamma_a + (\Omega_\zeta - \dot{\lambda}_{arb}\cos\varphi_{arb})\cos\gamma_a;$$

$$\dot{\gamma}_a = \Omega_\xi - \dot{\varphi}_{arb}[\Omega_\eta\cos\gamma_a - \Omega_\zeta\sin\gamma_a + \dot{\lambda}_{arb}\sin(\varphi_{arb} + \gamma_a)]\tan\vartheta_a;$$

$$\Omega_\xi = \Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0}\sin\lambda_{arb};$$

$$\Omega_\eta = (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\cos\varphi_{arb} - \Omega_{\zeta 0}\sin\varphi_{arb};$$

$$\Omega_\zeta = (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\sin\varphi_{arb} + \Omega_{\zeta 0}\cos\varphi_{arb};$$

$$\Omega_{\xi 0} = \Omega\cos\varphi_0\cos\Psi_0;$$

$$\Omega_{\eta 0} = \Omega\sin\varphi_0;$$

$$\Omega_{\zeta 0} = -\Omega\cos\varphi_0\sin\Psi_0.$$

It is apparent that system (3.34), just as system (2.50), is nonlinear with cross links along all parameters which enter into the system, including also accelerations measured by the accelerometers. Therefore an INS of the class under consideration must be a three-component system. /133

By solving the system (3.34) we find the instantaneous values of the following kinematic parameters: velocity vector projections V_ξ, V_η, V_ζ, arbitrary coordinates λ_{arb}, ϕ_{arb} and flight altitude H.

Basic data for solutions of the system (3.34) include accelerometer indications a_x, a_y, a_z, initial motion parameter values desired $V_{\xi 0}$, $V_{\eta 0}$, $V_{\zeta 0}$, λ_{arb0}, H_0, ϕ_{arb0}, the geographic latitude value ϕ_0 and the course angle Ψ_0 of the arbitrary equator at the starting point.

If we compare the system of kinematic equations (3.34) for an INS of the given class with the system of equations (2.50) for an INS with a corrected platform, then it is apparent that the former differs from the latter only by the equations required for the calculation of a_ξ, a_η, a_ζ, which in an INS with a "fixed" platform are not measured directly.

An INS of the given class permits us to determine the angular position of the object x_1, y_1, z_1 axes with respect to the surface of the earth (Figure 3.9), i.e. yaw angle ψ, pitch angle ϑ and bank angle γ.

For this purpose it is necessary to solve in addition the following system of equations

$$\left.\begin{aligned}
\cos\vartheta\cos\psi &= \cos\vartheta'\cos\vartheta_a\cos(\psi'-\psi_a)+\sin\vartheta'\sin\vartheta_a;\\
\cos\vartheta\cos\gamma &= \cos\gamma'\cos\gamma_a\cos(\vartheta'-\vartheta_a)+\sin\gamma'\sin\gamma_a;\\
\cos\gamma\cos\psi &= \cos\psi'\cos\psi_a\cos(\gamma'-\gamma_a)+\sin\psi'\sin\psi_a,
\end{aligned}\right\} \qquad (3.35)$$

where ψ', ϑ', γ' are object deviation angles with respect to the platform.

With small values of these angles

$$\left.\begin{aligned}
\psi &\cong \psi'-\psi_a;\\
\vartheta &\cong \vartheta'-\vartheta_a;\\
\gamma &\cong \gamma'-\gamma_a.
\end{aligned}\right\} \qquad (3.36)$$

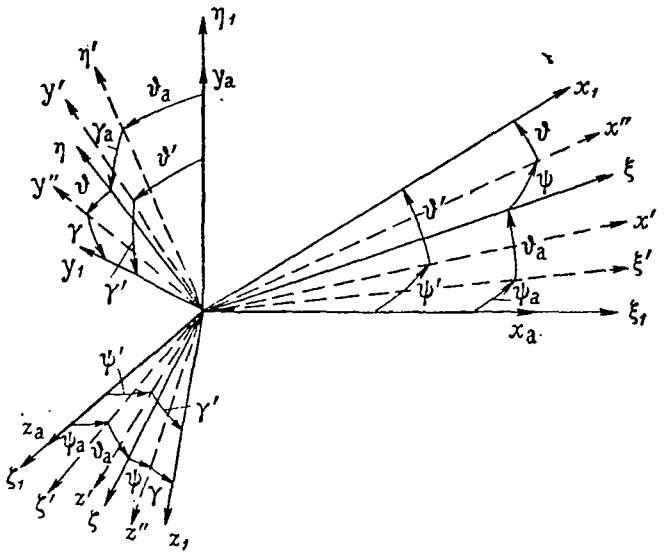

Figure 3.9.

In introducing into the INS computer the parameters which characterize object motion with respect to the medium (for example, air speed, angle of attack and slip), it is possible also to determine the parameters which characterize motion of the medium with respect to the surface of the earth. In this connection it is necessary to solve the system of kinematic equations previously derived (2.56).

It is apparent from the expanded INS block diagram (Figure 3.10), constructed on the basis of the system of kinematic equations derived (3.34), that a system of this class, just as the system of the class previously described, it closed with respect to parameters λ_{arb}, ϕ_{arb}, $\dot{\lambda}_{arb}$, $\dot{\phi}_{arb}$.

In contrast to an INS with a platform oriented in the earth coordinate system, which models the kinematics of the Schuler pendulum physically, the INS under consideration with a "fixed" platform models the kinematics of this pendulum mathematically. In this system only the inertial coordinate trihedron is materialized on the object, and all angular parameters which are included in the INS system of equations are calculated with respect to this trihedron.

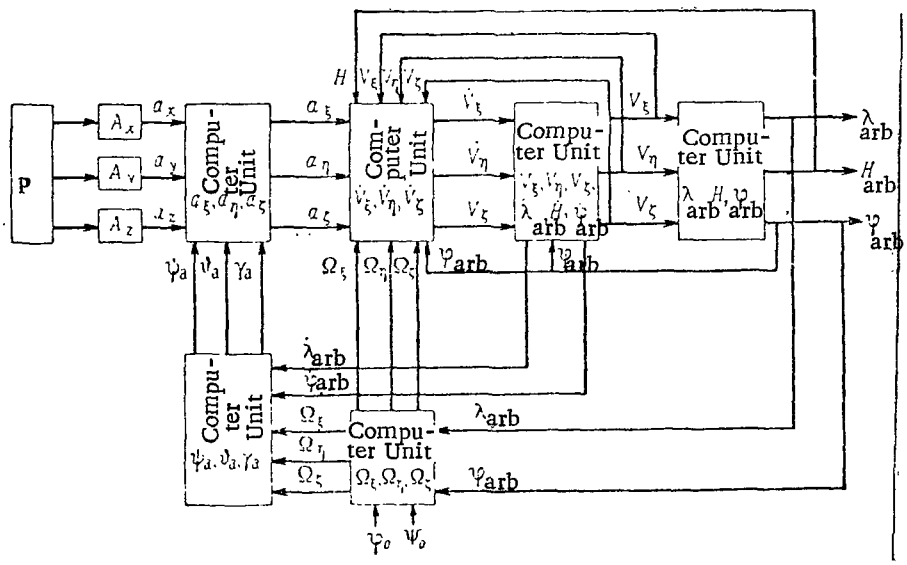

Figure 3.10. Expanded INS Block Diagram.

The fundamental electrokinematic diagram of an INS with a platform stabilized in inertial space is shown in Figure 3.11. Power stabilization is provided for in the platform which carries the accelerometer unit. The block diagram of an INS is shown in Figure 3.12.

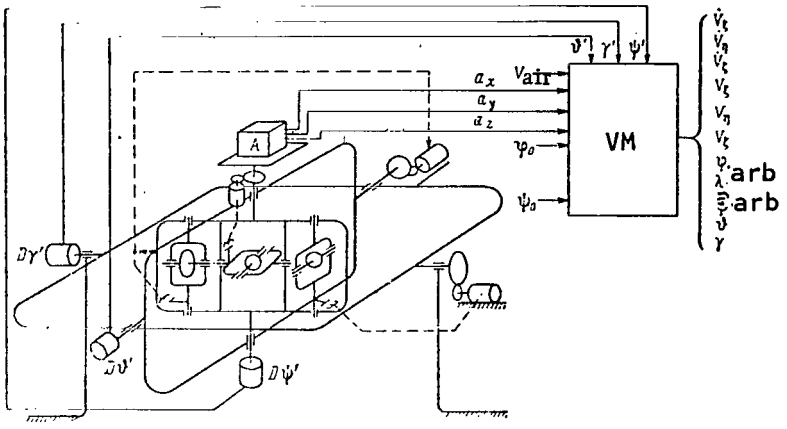

Figure 3.11. Electrokinematic INS Diagram.

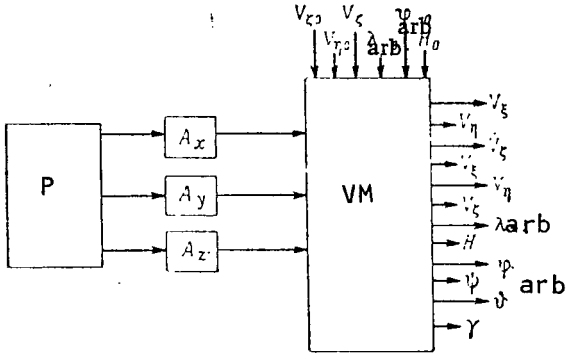

Figure 3.12. Block Diagram of a Three-Component INS.

§3.3. The Simplification of INS Kinematic Equations.

The system of kinematic equations (3.34) for an INS with a platform which is "fixed" in inertial space may be simplified to a greater or lesser degree when applied to specific objects.

Here just as in the case for the simplification of equations (2.50), we must consider not only the navigational accuracy requirements of an object for which the system is intended, but also the errors of separate elements which may be used in the system (particularly their dead zones).

We may employ the technique shown in Chapter II and used for the simplification of kinematic equations for INS's of the I class in order to simplify equations of the system (3.34).

Considering the fact that system (3.34) differs from system (2.50) by additional equations with respect to accelerations a_ξ, a_η, a_ζ, we may use the results cited in Section 2.5, after analyzing the possibility of simplifying the equation with respect to a_ξ, a_η, a_ζ.

For INS on objects in motion in which the values of the angles ψ_a, ϑ_a, γ_a included in equation (3.32) are sufficiently small, the equations may be simplified to the form /137

$$\left.\begin{aligned} a_\xi &\cong a_x + a_y\vartheta_a - a_z\psi_a;\\ a_\eta &\cong -a_x\vartheta_a + a_y + a_z\gamma_a;\\ a_\zeta &\cong a_x\psi_a - a_y\gamma_a + a_z. \end{aligned}\right\} \qquad (3.37)$$

Here the following systematic errors will occur in the determination of accelerations:

$$\left.\begin{aligned} \delta^I a_\xi &= a_x(1-\cos\psi_a\cos\vartheta_a) + a_y(\vartheta_a - \sin\vartheta_a) - \\ &\quad - a_z(\psi_a - \sin\psi_a\cos\vartheta_a);\\ \delta^I a_\eta &= -a_x(\vartheta_a + \sin\psi_a\sin\gamma_a - \cos\psi_a\sin\vartheta_a\cos\gamma_a) + \\ &\quad + a_y(1-\cos\vartheta_a\cos\gamma_a) + a_z(\gamma_a - \cos\psi_a\sin\gamma_a - \\ &\quad - \sin\psi_a\sin\vartheta_a\cos\gamma_a);\\ \delta^I a_\zeta &= a_x(\psi_a - \sin\psi_a\cos\gamma_a - \cos\psi_a\sin\vartheta_a\sin\gamma_a) - \\ &\quad - a_y(\gamma_a - \cos\vartheta_a\sin\gamma_a) + a_z(1-\cos\psi_a\cos\gamma_a + \\ &\quad + \sin\psi_a\sin\vartheta_a\sin\gamma_a). \end{aligned}\right\} \qquad (3.38)$$

For an approximate evaluation of these errors, considering equations (2.171) and (2.173), we take

$$\left.\begin{aligned}\dot{\psi}_a &\cong \Omega_\eta;\\ \dot{\vartheta}_a &\cong \Omega_\zeta - \dot{\lambda}_{arb}\\ \dot{\gamma}_a &\cong \Omega_\xi - \dot{\varphi}_{arb},\end{aligned}\right\} \tag{3.39}$$

or, assuming ϕ_{arb} to be small in value, after integration we obtain

$$\left.\begin{aligned}\psi_a &\leqslant \Omega t;\\ \vartheta_a &\leqslant \Omega t - \lambda_{arb}\\ \gamma_a &\leqslant \Omega t.\end{aligned}\right\} \tag{3.40}$$

If we consider that for many short-range objects on the basic horizontal segment of the trajectory

$$\begin{aligned}a_x &\leqslant 0{,}1g;\\ a_y &\cong g;\\ a_z &\leqslant 0{,}1g,\end{aligned}$$

then considering (3.40) we obtain as an approximation /138

$$\left.\begin{aligned}\delta^1 a_\xi &\cong g\,[\Omega t - \lambda_{arb} - \sin(\Omega t - \lambda_{arb})];\\ \delta^1 a_\eta &\cong g\,[1 - \cos(\Omega t - \lambda_{arb})\cos\Omega t];\\ \delta^1 a_\zeta &\cong g\,[\Omega t - \cos(\Omega t - \lambda_{arb})\sin\Omega t].\end{aligned}\right\} \tag{3.41}$$

For objects in motion near the arbitrary equator at distance L < 300 km during time t < 1,000 sec, the systematic errors indicated will fall within the limits of the dead zones of comtemporary accelerometers.

For objects in flight near the arbitrary equator at distance 300 km < L < < 500 km in the course of t < 1,000 sec, the values of angles ϑ_a, γ_a will be such that it is feasible to assume

$$\left.\begin{aligned} a_x &\cong g \sin \vartheta_a; \\ a_y &\cong g \cos \vartheta_a; \\ a_z &\cong g \sin \gamma_a. \end{aligned}\right\} \qquad (3.42)$$

We note that in equation (3.42) the given values are not projections of the acceleration of gravity, but are applicable limited values for a rough calculation of the non-gravitational component of object acceleration expressed in fractions of g.

In addition, $\vartheta_a > \gamma_a$ and $\vartheta_a > \psi_a$, then

$$\left.\begin{aligned} &\cos \vartheta_a \neq 1; \quad \cos \psi_a \cong 1; \quad \cos \gamma_a \cong 1; \\ &\sin \vartheta_a \neq \vartheta_a; \quad \sin \psi_a = \psi_a; \quad \sin \gamma_a = \gamma_a. \end{aligned}\right\} \qquad (3.43)$$

With consideration for equations (3.43), the equations with respect to a_ξ, a_η, a_ζ may be presented in the following form:

$$\left.\begin{aligned} a_\xi &= a_x \cos \vartheta_a + a_y \sin \vartheta_a - a_z \psi_a \cos \vartheta_a; \\ a_\eta &= a_x (\psi_a \gamma_a - \sin \vartheta_a) + a_y \cos \vartheta_a + a_z (\gamma_a + \psi_a \sin \vartheta_a); \\ a_\zeta &= a_x (\psi_a + \gamma_a \sin \vartheta_a) - a_y \gamma_a \cos \vartheta_a + a_z. \end{aligned}\right\} \qquad (3.44)$$

The simplifications cited cause systematic errors in the INS in the calculation of a_ξ, a_η, a_ζ, the values of which are determined from the following equations:

$$\left.\begin{aligned} \delta^{II} a_\xi &= a_x \cos \vartheta_a (1 - \cos \psi_a) - a_z \cos \vartheta_a (\psi_a - \sin \psi_a); \\ \delta^{II} a_\eta &= a_x [(\psi_a \gamma_a - \sin \psi_a \sin \gamma_a) - \sin \vartheta_a (1 - \cos \psi_a - \cos \gamma_a)] + \\ &\quad + a_y \cos \vartheta_a (1 - \cos \gamma_a) + a_z [(\gamma_a - \cos \psi_a \sin \gamma_a) + \\ &\quad + \sin \vartheta_a (\psi_a - \sin \psi_a \cos \gamma_a)]; \\ \delta^{II} a_\zeta &= a_x [\psi_a - \sin \psi_a \cos \gamma_a + (\gamma_a - \sin \gamma_a \cos \psi_a) \sin \vartheta_a - \\ &\quad - a_{ay} \cos \vartheta_a (\gamma_a - \sin \gamma_a) + a_z [1 - \cos \psi_a \cos \gamma_a - \\ &\quad - \sin \psi_a \sin \vartheta_a \sin \gamma_a]. \end{aligned}\right\} \qquad (3.45)$$

By substituting the values a_x, a_y, a_z from (3.42) in the last equation and considering (3.40), and also the small value of ϕ_{arb}, we obtain /139

$$\left.\begin{aligned}
\delta^{II}a_\xi &= \frac{g}{2}[\sin 2(\Omega t-\lambda_{arb})(1-\cos\Omega t)-2\sin\Omega t\cos(\Omega t-\lambda_{arb})\times \\
&\qquad \times(\Omega t-\sin\Omega t)]; \\
\delta^{II}a_\eta &\cong g\sin(\Omega t-\lambda_{arb})\{\Omega^2t^2-\sin^2\Omega t[1+\sin(\Omega t-\lambda_{arb})]\}+ \\
&+g\cos^2(\Omega t-\lambda_{arb})(1-\cos\Omega t)+g\sin\Omega t[(\Omega t-\sin\Omega t\cos\Omega t)\times \\
&\qquad \times[1+\sin(\Omega t-\lambda_{arb})]\}; \\
\delta^{II}a_\zeta &\cong g\left\{\sin(\Omega t-\lambda_{arb})\left(\Omega t-\frac{1}{2}\sin^2\Omega t\right)[1+\sin(\Omega t-\lambda_{arb})-\right. \\
&\left.-\cos^2(\Omega t-\lambda_{arb})(\Omega t-\sin\Omega t)+\sin^3\Omega t[1+\sin(\Omega t-\lambda_{arb})]\right\}.
\end{aligned}\right\} \qquad (3.46)$$

An evaluation of errors according to equations (3.46) reveals that for objects in flight near the arbitrary equator at distance L < 500 km with velocities V < 500 m/sec, systematic error values fall within the limits of the dead zones of contemporary accelerometers.

Therefore it is feasible to accomplish INS calculations for these objects in accordance with equations (3.44).

In turn, it may be shown that in equations for objects in motion near the arbitrary equator where 500 km < L < 1,500 km and 500 m/sec < V < 1,500 m/sec, the instantaneous values of accelerations a_ξ, a_η, a_ζ may be advantageously determined by using the equations

$$\left.\begin{aligned}
a_\xi &= a_x\cos\psi_a\cos\vartheta_a+a_y\sin\vartheta_a-a_z\psi_a\cos\vartheta_a; \\
a_\eta &= a_x(\psi_a\gamma_a-\sin\vartheta_a)+a_y\cos\vartheta_a\cos\gamma_a+a_z(\gamma_a+\psi_a\sin\vartheta_a); \\
a_\zeta &= a_x(\psi_a+\sin\vartheta_a\gamma_a)-a_y\gamma_a\cos\vartheta_a+a_z\cos\psi_a\cos\gamma_a.
\end{aligned}\right\} \qquad (3.47)$$

The following systematic errors will occur here in the determination of accelerations:

$$\delta^{III}a_\xi = -g\sin\Omega t\cos(\Omega t-\lambda_{arb})(\Omega t-\sin\Omega t);$$

$$\delta^{III}a_\eta = g\left\{\sin(\Omega t-\lambda_{arb})[\Omega^2 t^2-\sin^2\Omega t[1+\sin(\Omega t-\lambda_{arb})]]+\right.$$
$$\left.+\sin\Omega t\left[\left(\Omega t-\frac{1}{2}\sin 2\Omega t\right)[1+\sin(\Omega t-\lambda_{arb})]\right]\right\};$$
$$\delta^{III}a_\zeta = g\left\{\sin(\Omega t-\lambda_{arb})\left[\left(\Omega t-\frac{1}{2}\sin\Omega t\right)[1+\sin(\Omega t-\lambda_{arb})]\right]-\right.$$
$$\left.-[\cos^2(\Omega t-\lambda_{arb})(\Omega t-\sin\Omega t)+\sin^3\Omega t\sin(\Omega t-\lambda_{arb})]\right\}. \qquad (3.48)$$

Based on the foregoing it is possible to recommend the following system of kinematic equations with consideration for the results of section 2.5 for INS short-range objects in motion near the arbitrary equator: /140

$$\begin{aligned}
&\dot V_\xi = a_\xi - 2\Omega_{\eta_0}V_\zeta;\\
&\dot V_\zeta = a_\zeta + 2\Omega_\eta V_\xi;\\
&a_\xi = a_x\cos\vartheta_a + a_y\sin\vartheta_a - a_z\psi_a\cos\vartheta_a;\\
&a_\zeta = a_x(\psi_a+\sin\vartheta_a\gamma_a) - a_y\cos\vartheta_a\gamma_a + a_z;\\
&\dot\lambda_{arb} = \frac{V_\xi}{R_{ea}};\\
&\dot\varphi_y = -\frac{V_\zeta}{R_{ea}};\\
&\dot\psi_a = \Omega_\eta - (\Omega_\zeta - \dot\lambda_{arb})\gamma_a;\\
&\dot\vartheta_a = \Omega_{\eta_1}\gamma_a + \Omega_\zeta - \dot\lambda_{arb}\\
&\dot\gamma_a = \Omega_\xi - \dot\varphi_y - [\Omega_\eta - (\Omega_\zeta - \dot\lambda_{arb})\gamma_a]\vartheta_a;\\
&\Omega_\xi = \Omega_{\xi 0} - \Omega_{\eta_1 0}\lambda_{arb}\\
&\Omega_\eta = \Omega_{\xi 0}\lambda_{arb} + \Omega_{\eta 0};\\
&\Omega_\zeta = \Omega_{\zeta 0}.
\end{aligned} \qquad (3.49)$$

It is apparent that an INS constructed in accordance with the system (3.49) may be a two-component system, since in the first two equations the vertical velocity component V_η is lacking; it should be noted however that in order to determine instantaneous values of the parameters a_ξ, and a_ζ it is necessary to have three accelerometers in the system.

The equations assume the following form for INS medium-range objects, in flight near the arbitrary equator:

$$\dot{V}_\xi = a_\xi - \frac{V_\xi V_\eta}{R_{ea}} - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta);$$

$$\dot{V}_\eta = a_\eta + \frac{V_\xi^2}{R_{ea}} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta;$$

$$\dot{V}_\zeta = a_\zeta - \frac{V_\eta V_\zeta}{R_{ea}} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi);$$

$$a_\xi = a_x \cos\psi_a \cos\vartheta_a + a_y \sin\vartheta_a - a_z\psi_a \cos\vartheta_a;$$

$$a_\eta = a_x(\psi_a\gamma_a - \sin\vartheta_a) + a_y \cos\vartheta_a \cos\gamma_a + a_z(\gamma_a + \psi_a \sin\vartheta_a);$$

$$a_\zeta = a_x(\psi_a + \gamma_a \sin\vartheta_a) - a_y \cdot \gamma_a \cos\vartheta_a + a_z \cos\psi_a \cos\gamma_a;$$

$$\dot{\lambda}_{arb} = \frac{1}{R_{ea} + H} V_\xi; \qquad (3.50)$$

$$\dot{H} = V_\eta;$$

$$\dot{\varphi}_y = -\frac{1}{R_{ea}} V_\zeta;$$

$$\dot{\psi}_a = \frac{1}{\cos\vartheta_a}[\Omega_\eta \cos\gamma_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a];$$

$$\dot{\vartheta}_a = \Omega_\eta \sin\gamma_a + (\Omega_\zeta - \dot{\lambda}_{arb})\cos\gamma_a;$$

$$\dot{\gamma}_a = \Omega_\xi - \dot{\varphi}_{arb} - [\Omega_\eta \cos\gamma_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a]\tan\vartheta_a;$$

$$\Omega_\xi = \Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0}\sin\lambda_{arb}$$

$$\Omega_\eta = \Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb} - \Omega_{\zeta 0}\varphi_{arb};$$

$$\Omega_\zeta = (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\varphi_{arb} + \Omega_{\zeta 0}.$$

It is obvious that an INS constructed in accordance with equations (3.50) must be a three-component system, since the equations for $\dot{V}_\xi$ and $\dot{V}_\zeta$ include the vertical velocity component V_η and cross links occur along the velocity vector components V_ξ, V_η, V_ζ, as well as along the acceleration vector components a_x, a_y, a_z, measured by the system accelerometers. /141

In constructing an INS it is also necessary to consider that errors in the determination of vertical acceleration $\dot{V}_\eta$ and vertical velocity V_η lead to an increase in errors not only in the determination of altitude, but also in the determination of the ϕ_{arb} and λ_{arb} coordinates.

A quantitative analysis reveals that where flight time t>3000 sec in the system, the parameters V_η and H must be corrected according to other measuring instruments or systems.

§3.4. INS Systematic Error Equations

Let us examine systematic errors caused by kinematic equation simplifications for an INS with a platform which is "fixed" in inertial space.

In constrast to errors involved in INS's of the I class, systematic errors in the INS's under consideration will be influenced by systematic equation simplifications with respect to accelerations a_ξ, a_η, a_ζ.

The same methods that were employed in section 2.5 may be used for the calculation of systematic errors.

The derivation of the INS kinematic equations with consideration for systematic errors and the derivation of the systematic error equations, as well as the technique for the determination of these errors, are similar to the explanation in Section 2.5.

The INS kinematic equations for short-range objects, allowing for systematic errors, may be derived from the system (3.49) and after transformations will have the following form: /142

$$
\begin{aligned}
&\dot{V}^*_\xi = a^*_\xi - 2\Omega^*_{\eta 0} V^*_\zeta;\\
&\dot{V}^*_\zeta = a^*_\zeta + 2\Omega^*_{\eta} V^*_\xi;\\
&a^*_\xi = a_x \cos\vartheta^*_a + a_y \sin\vartheta^*_a - a_z\psi^*_a \cos\vartheta^*_a;\\
&a^*_\zeta = a_x(\psi^*_a + \gamma^*_a \sin\vartheta^*_a) - a_y\gamma^*_a \cos\vartheta^*_a + a_z;\\
&\dot{\lambda}^*_{arb} = \frac{1}{R_{ea}} V^*_\xi;\\
&\dot{\varphi}^*_{arb} = -\frac{1}{R_{ea}} V^*_\zeta;\\
&\dot{\psi}^*_a = \Omega^*_\eta - (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\gamma^*_a;\\
&\dot{\vartheta}^*_a = \Omega^*_\eta \gamma^*_a + \Omega^*_\zeta - \dot{\lambda}^*_{arb}\\
&\dot{\gamma}^*_a = \Omega^*_\zeta - \dot{\varphi}^*_{arb} - [\Omega^*_\eta - (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\gamma^*_a]\vartheta^*_a;\\
&\Omega^*_\xi = \Omega^*_{\xi 0} - \Omega^*_{\eta 0}\lambda^*_{arb}\\
&\Omega^*_\eta = \Omega^*_{\xi 0}\lambda^*_{arb} + \Omega^*_{\eta 0};\\
&\Omega^*_\zeta = \Omega^*_{\zeta 0}.
\end{aligned}
\tag{3.51}
$$

The solution of system (3.51) allows us to determine instrument parameter values (designated by an asterisk) where only systematic errors are present.

Basic data for the solution of system (3.51) are furnished by the true acceleration values $a_x(t)$, $a_y(t)$, $a_z(t)$ and by the initial values of the kinematic parameters desired.

In turn, accelerations $a_x(t)$, $a_y(t)$, $a_z(t)$ are determined in advance by means of solving the system (3.34) according to known values $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$.

INS systematic errors may be calculated according to equations (2.189) as the difference between the instrument and true parameter values in each moment of time under consideration.

The appropriate kinematic equations for medium-range objects have the following form:

$$
\begin{aligned}
&\dot{V}^*_\xi = a^*_\xi - \frac{V^*_\xi V^*_\eta}{R_{ea}} - 2(\Omega^*_\eta V^*_\zeta - \Omega^*_\zeta V^*_\eta);\\
&\dot{V}^*_\eta = a^*_\eta + \frac{V^{*2}_\xi}{R_{ea}} - 2(\Omega^*_\zeta V^*_\xi - \Omega^*_\xi V^*_\zeta) + g_\eta;\\
&\dot{V}^*_\zeta = a^*_\zeta - \frac{V^*_\eta V^*_\zeta}{R_{ea}} - 2(\Omega^*_\xi V^*_\eta - \Omega^*_\eta V^*_\xi);\\
&a^*_\xi = a_x \cos\psi^*_a \cos\vartheta^*_a + a_y \sin\vartheta^*_a - a_z \psi^*_a \cos\vartheta^*_a;\\
&a^*_\eta = a_x(\psi^*_a\gamma^*_a - \sin\vartheta^*_a) + a_y \cos\vartheta^*_a \cos\gamma^*_a + a_z(\gamma^*_a + \psi^*_a \sin\vartheta^*_a);\\
&a^*_\zeta = a_x(\psi^*_a + \gamma^*_a \sin\vartheta^*_a) - a_y\gamma^*_a \cos\vartheta^*_a + a_z \cos\psi^*_a \cos\gamma^*_a;\\
&\dot{\lambda}^*_{arb} = \frac{1}{R_{ea}+H} V^*_\xi;\\
&H^* = V^*_\eta;\\
&\dot{\varphi}^*_{arb} = -\frac{1}{R_{ea}} V^*_\zeta;\\
&\dot{\psi}^*_a = \frac{1}{\cos\vartheta^*_a}[\Omega^*_\eta \cos\gamma^*_a - (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\sin\gamma^*_a];\\
&\dot{\vartheta}^*_a = \Omega^*_\eta \sin\gamma^*_a + (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\cos\gamma^*_a;\\
&\dot{\gamma}^*_a = \Omega^*_\xi - \dot{\varphi}^*_{arb} - [\Omega^*_\eta \cos\gamma^*_a - (\Omega^*_\zeta - \dot{\lambda}^*_{arb})\sin\gamma^*_a]\tan\vartheta^*_a;\\
&\Omega^*_\xi = \Omega_{\xi 0}\cos\lambda^*_{arb} - \Omega_{\eta 0}\sin\lambda^*_{arb}\\
&\Omega^*_\eta = \Omega_{\xi 0}\sin\lambda^*_{arb} + \Omega_{\eta 0}\cos\lambda^*_{arb} - \Omega_{\zeta 0}\varphi^*_{arb};\\
&\Omega^*_\zeta = (\Omega_{\xi 0}\sin\lambda^*_{arb} + \Omega_{\eta 0}\cos\lambda^*_{arb})\varphi^*_{arb} + \Omega_{\zeta 0}.
\end{aligned}
\tag{3.52}
$$

The solution of system (3.52) and the determination of systematic errors are accomplished similarly to the solution of system (3.51). /143

We have the following system of equations for systematic errors for short-range objects after linearization (3.49):

$$
\begin{aligned}
&p\Delta V_{\xi_{sys}} + a_{1V_\zeta}\Delta V_{\zeta_{sys}} + a_{1a_\xi}\Delta a_{\xi_{sys}} = \delta_{sys}\dot{V}_\xi;\\
&a_{2V_\xi}\Delta V_{\xi_{sys}} + p\Delta V_{\zeta_{sys}} + a_{2a_\zeta}\Delta a_{\zeta_{sys}} + a_{2\Omega_{\eta_i}}\Delta\Omega_{\eta_{iM}} = \delta_{sys}\dot{V}_\zeta;\\
&a_{3a_\xi}\Delta a_{\xi_{sys}} + a_{3\psi}\Delta\psi_{a,sys} + a_{3\vartheta}\Delta\vartheta_{a,sys} = \delta_{sys}a_\xi;\\
&a_{4a_\zeta}\Delta a_{\zeta_{sys}} + a_{4\psi}\Delta\psi_{a,sys} + a_{4\vartheta}\Delta\vartheta_{a,sys} + a_{4\gamma}\Delta\gamma_{a,sys} = \delta_{sys}a_\zeta;\\
&a_{5V_\xi}\Delta V_{\xi_{sys}} + p\Delta\lambda_{arb,sys} = \delta_{sys}\dot{\lambda}_{arb};\\
&a_{6V_\zeta}\Delta V_{\zeta_{sys}} + p\Delta\varphi_{arb,sys} = \delta_{sys}\dot{\varphi}_{arb};\\
&a_{7\lambda}p\Delta\lambda_{arb,sys} + p\Delta\psi_{a,sys} + a_{7\gamma}\Delta\gamma_{a,sys} + a_{7\Omega_{\eta_i}}\Delta\Omega_{\eta_{isys}} + a_{7\Omega_\zeta}\Delta\Omega_{\zeta_{sys}} = \delta_{sys}\dot{\psi}_a;\\
&a_{8\lambda}p\Delta\lambda_{arb,sys} + p\Delta\vartheta_{a,sys} + a_{8\gamma}\Delta\gamma_{a,sys} + a_{8\Omega_{\eta_i}}\Delta\Omega_{\eta_{isys}} + a_{8\Omega_\zeta}\Delta\Omega_{\zeta_{sys}} = \delta_{sys}\dot{\vartheta}_a;\\
&a_{9\lambda}p\Delta\lambda_{arb,sys} + a_{9\varphi}p\Delta\varphi_{arb,sys} + a_{9\vartheta}\Delta\vartheta_{a,sys} + (p + a_{9\gamma})\Delta\gamma_{a,sys} + a_{9\Omega_\xi}\Delta\Omega_{\xi_M} +\\
&\qquad + a_{9\Omega_{\eta_i}}\Delta\Omega_{\eta_{isys}} + a_{9\Omega_\zeta}\Delta\Omega_{\zeta_{sys}} = \delta_{sys}\dot{\gamma}_a;\\
&a_{10\lambda}\Delta\lambda_{arb,sys} + a_{10\Omega_\xi}\Delta\Omega_{\xi_{sys}} = \delta_{sys}\Omega_\xi;\\
&a_{11\lambda}\Delta\lambda_{arb,sys} + a_{11\Omega_\eta}\Delta\Omega_{\eta_{sys}} = \delta_{sys}\Omega_\eta;\\
&a_{12\Omega_\zeta}\Delta\Omega_{\zeta_{sys}} = \delta_{sys}\Omega_\zeta,
\end{aligned}
\tag{3.53}
$$

where the coefficients of the equations

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$$
\begin{aligned}
&a_{1V_\zeta} = 2\Omega_{\eta_i 0};\\
&a_{1a_\xi} = -1;\\
&a_{2V_\xi} = -2\Omega_{\eta_i};\\
&a_{2\Omega_{\eta_i}} = -2V_\xi;\\
&a_{2a_\zeta} = -1;\\
&a_{3a_\xi} = 1;\\
&a_{3\psi} = a_z\cos\vartheta_a;\\
&a_{3\vartheta} = -(a_x - a_z\psi_a)\sin\vartheta_a + a_y\cos\vartheta_a,\\
&a_{4a_\zeta} = 1;\\
&a_{4\psi} = -a_x;\\
&a_{4\vartheta} = -\gamma_a(a_x\cos\vartheta_a + a_y\sin\vartheta_a);\\
&a_{4\gamma} = a_y\cos\vartheta_a - a_x\sin\vartheta_a;\\
&a_{5V_\xi} = -\frac{1}{R_{ea}};\\
&a_{6V_\zeta} = \frac{1}{R_{ea}};\\
&a_{9\Omega_\zeta} = -\gamma_a\vartheta_a;\\
&a_{10\lambda} = \Omega_{\eta_i 0};\\
&a_{10\Omega_\xi} = 1;
\end{aligned}
\qquad
\begin{aligned}
&a_{7\lambda} = -\gamma_a;\\
&a_{7\gamma} = \Omega_\zeta - \dot{\lambda}_{arb};\\
&a_{7\Omega_{\eta_i}} = -1;\\
&a_{7\Omega_\zeta} = \gamma_a;\\
&a_{8\lambda} = 1;\\
&a_{8\gamma} = -\Omega_\eta;\\
&a_{8\Omega_{\eta_i}} = -\gamma_a;\\
&a_{8\Omega_\zeta} = -1;\\
&a_{9\lambda} = \gamma_a\vartheta_a;\\
&a_{9\varphi} = 1;\\
&a_{9\vartheta} = \Omega_{\eta_i} - (\Omega_\zeta - \dot{\lambda}_{arb})\gamma_a;\\
&a_{9\gamma} = (\dot{\lambda}_{arb} - \Omega_\zeta)\vartheta_a;\\
&a_{9\Omega_\xi} = -1;\\
&a_{9\Omega_{\eta_i}} = \vartheta_a;\\
&a_{11\lambda} = -\Omega_{\xi 0};\\
&a_{11\Omega_{\eta_i}} = 1;\\
&a_{12\Omega_\zeta} = 1.
\end{aligned}
\tag{3.54}
$$

In turn, partial systematic errors in the determination of each parameter of the left sides of the equations (3.49) are determined as the difference between the right sides of the equations of systems (3.49) and (3.34).

If we consider that $H \ll R_{ea}$, then the expressions for partial systematic errors assume the form:

$$
\begin{aligned}
&\delta_{sys}\dot{V}_\xi = \frac{V_\xi V_\eta}{R_{ea}} + \frac{V_\xi V_\zeta}{R_{ea}}\tan\varphi_y - 2\Omega_\zeta V_\eta - 2V_\zeta(\Omega_{\eta 0} - \Omega_\eta);\\
&\delta_{sys}\dot{V}_\zeta = \frac{V_\eta V_\zeta}{R_{ea}} - \frac{V_\xi^2}{R_{ea}}\tan\varphi_y + 2\Omega_\xi V_\eta;\\
&\delta_{sys}a_\xi = a_x\cos\vartheta_a(1-\cos\psi_a) - a_z\cos\vartheta_a(\psi_a - \cos\psi_a);\\
&\delta_{sys}a_\zeta = a_x[(\psi_a - \sin\psi_a\cos\gamma_a) + \sin\vartheta_a(\gamma_a - \cos\psi_a\sin\gamma_a)] -\\
&\quad - a_y(\gamma_a - \sin\gamma_a)\cos\vartheta_a + a_z(1-\cos\psi_a\cos\gamma_a + \sin\psi_a\sin\vartheta_a\sin\gamma_a);\\
&\delta_{sys}\dot{\lambda}_{arb} = \frac{H}{R_3^2}V_\xi;\\
&\delta_{sys}\dot{\varphi}_{arb} = -\frac{H}{R_{ea}^2}V_\zeta;\\
&\delta_{sys}\dot{\psi}_a = \Omega_\eta\left(1 - \frac{\cos\gamma_a}{\cos\vartheta_a}\right) - \Omega_\zeta\left(\gamma_a - \frac{\sin\gamma_a}{\cos\vartheta_a}\right) + \dot{\lambda}_{arb}\left[\gamma_a - \frac{\sin(\varphi_{arb}\varphi_a)}{\cos\vartheta_a}\right];\\
&\delta_{sys}\dot{\vartheta}_a = \Omega_\eta(\gamma_a - \sin\gamma_a) + \Omega_\zeta(1-\cos\gamma_a) - \dot{\lambda}_{arb}[1-\cos(\varphi_{arb} + \gamma_a)];\\
&\delta_{sys}\dot{\gamma}_a = -\Omega_\eta(\vartheta_a - \cos\gamma_a\tan\vartheta_a) + \Omega_\zeta(\gamma_a\vartheta_a - \sin\gamma_a\tan\vartheta_a) - \dot{\lambda}_{arb}\times\\
&\qquad\qquad \times[\gamma_a\vartheta_a - \sin(\varphi_{arb} + \gamma_a)\tan\vartheta_a];\\
&\delta_{sys}\Omega_\xi = \Omega_{\xi 0}(1-\cos\lambda_{arb}) - \Omega_{\eta 0}(\lambda_{arb} - \sin\lambda_{arb});\\
&\delta_{sys}\Omega_\eta = \Omega_{\xi 0}(\lambda_{arb} - \sin\lambda_{arb}\cos\varphi_{arb}) + \Omega_{\eta 0}(1-\cos\lambda_{arb}\cos\varphi_{arb}) + \Omega_{\zeta 0}\sin\varphi_{arb};\\
&\delta_{sys}\Omega_\zeta = -(\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\sin\varphi_{arb} + \Omega_{\zeta 0}(1-\cos\varphi_{arb}).
\end{aligned}
\tag{3.55}
$$

The values of the kinematic parameters included in equations (3.54) and (3.55) are determined in advance by solving the system (3.34) according to known acceleration values $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$. /145

It should be noted that INS systematic errors for short-range objects ($L < 500$ km) may be calculated by solving equations (3.53) with constant coefficient values. The latter are calculated according to the average values of the true motion parameters just as in Section 2.5; the values of the errors obtained are increased somewhat.

Systematic error equations may be similarly derived for short-range objects with INS's constructed in accordance with the system of kinematic equations (3.50).

$$
\begin{aligned}
&(p+a_{1V_\xi})\Delta V_{\xi\,\mathrm{sys}}+a_{1V_\eta}\Delta V_{\eta\,\mathrm{sys}}+a_{1V_\zeta}\Delta V_{\zeta\,\mathrm{sys}}+a_{1a_\xi}\Delta a_{\xi\,\mathrm{sys}}+a_{1\Omega_\eta}\Delta\Omega_{\eta\,\mathrm{sys}}+\\
&\qquad+a_{1\Omega_\zeta}\Delta\Omega_{\zeta\,\mathrm{sys}}=\delta_{\mathrm{sys}}\dot{V}_\xi;\\
&a_{2V_\xi}\Delta V_{\xi\,\mathrm{sys}}+p\Delta V_{\eta\,\mathrm{sys}}+a_{2V_\zeta}\Delta V_{\zeta\,\mathrm{sys}}+a_{2a_\eta}\Delta a_{\eta\,\mathrm{sys}}+a_{2\Omega_\xi}\Delta\Omega_{\xi\,\mathrm{sys}}+\\
&\qquad+a_{2\Omega_\zeta}\Delta\Omega_{\zeta\,\mathrm{sys}}=\delta_{\mathrm{sys}}\dot{V}_\eta;\\
&a_{3V_\xi}\Delta V_{\xi\,\mathrm{sys}}+a_{3V_\eta}\Delta V_{\eta\,\mathrm{sys}}+(p+a_{3V_\zeta})\Delta V_{\zeta\,\mathrm{sys}}+a_{3a_\zeta}\Delta a_{\zeta\,\mathrm{sys}}+\\
&\qquad+a_{3\Omega_\xi}\Delta\Omega_{\xi\,\mathrm{sys}}+a_{3\Omega_\eta}\Delta\Omega_{\eta\,\mathrm{sys}}=\delta_{\mathrm{sys}}\dot{V}_\zeta;\\
&a_{4a_\xi}\Delta a_{\xi\,\mathrm{sys}}+a_{4\psi}\Delta\psi_{\mathrm{a,sys}}+a_{4\vartheta}\Delta\vartheta_{\mathrm{a,sys}}=\delta_{\mathrm{sys}}a_\xi;\\
&a_{5a_\eta}\Delta a_{\eta\,\mathrm{sys}}+a_{5\psi}\Delta\psi_{\mathrm{a,sys}}+a_{5\vartheta}\Delta\vartheta_{\mathrm{a,sys}}+a_{5\gamma}\Delta\gamma_{\mathrm{a,sys\,sys}}=\delta\,a_\eta;\\
&a_{6a_\zeta}\Delta a_{\zeta\,\mathrm{sys}}+a_{6\psi}\Delta\psi_{\mathrm{a,sys}}+a_{6\vartheta}\Delta\vartheta_{\mathrm{a,sys}}+a_{6\gamma}\Delta\gamma_{\mathrm{a,sys\,sys}}=\delta\,a_\zeta;\\
&a_{7V_\xi}\Delta V_{\xi\,\mathrm{sys}}+p\Delta\lambda_{\mathrm{arb,sys}}+a_{7H}\Delta H_{\mathrm{sys}}=\delta_{\mathrm{sys}}\dot{\lambda}_{\mathrm{arb}}\\
&a_{8V_\eta}\Delta V_{\eta\,\mathrm{sys}}+p\Delta H_{\mathrm{sys}}=\delta_{\mathrm{sys}}\dot{H};\\
&a_{9V_\zeta}\Delta V_{\zeta\,\mathrm{sys}}+p\Delta\varphi_{\mathrm{arb,sys}}=\delta_{\mathrm{sys}}\dot{\varphi}_{\mathrm{arb}}\\
&a_{10\lambda}p\Delta\lambda_{\mathrm{arb,sys}}+p\Delta\psi_{\mathrm{a,sys}}+a_{10\vartheta}\Delta\vartheta_{\mathrm{a,sys}}+a_{10\gamma}\Delta\gamma_{\mathrm{a,sys}}+a_{10\Omega_\eta}\Delta\Omega_{\eta_M\,\mathrm{sys}}+\\
&\qquad+a_{10\Omega_\zeta}\Delta\Omega_{\zeta_M\,\mathrm{sys}}=\delta_{\mathrm{sys}}\dot{\psi}_{\mathrm{a}};\\
&a_{11\lambda}p\Delta\lambda_{\mathrm{arb,sys}}+p\Delta\vartheta_{\mathrm{a,sys}}+a_{11\gamma}\Delta\gamma_{\mathrm{a,sys}}+a_{11\Omega_\eta}\Delta\Omega_{\eta\,\mathrm{sys}}+a_{11\Omega_\zeta}\Delta\Omega_{\zeta\,\mathrm{sys}}=\delta_{\mathrm{sys}}\dot{\vartheta}_{\mathrm{a}};\\
&a_{12\lambda}p\Delta\lambda_{\mathrm{arb,sys}}+p\Delta\varphi_{\mathrm{arb,sys}}+a_{12\vartheta}\Delta\vartheta_{\mathrm{a,sys}}+(p+a_{12\gamma})\Delta\gamma_{\mathrm{a,sys}}+a_{12\Omega_\xi}\Delta\Omega_{\xi\,\mathrm{sys}}+\\
&\qquad+a_{12\Omega_\eta}\Delta\Omega_{\eta\,\mathrm{sys}}+a_{12\Omega_\zeta}\Delta\Omega_{\zeta\,\mathrm{sys}}=\delta_{\mathrm{sys}}\dot{\gamma}_{\mathrm{a}};\\
&a_{13\lambda}\Delta\lambda_{\mathrm{arb,sys}}+a_{13\Omega_\xi}\Delta\Omega_{\xi\,\mathrm{sys}}=\delta_{\mathrm{sys}}\Omega_\xi;\\
&a_{14\lambda}\Delta\lambda_{\mathrm{arb,sys}}+a_{14\varphi}\Delta\varphi_{\mathrm{arb,sys}}+a_{14\Omega_\eta}\Delta\Omega_{\eta\,\mathrm{sys}}=\delta_{\mathrm{sys}}\Omega_\eta;\\
&a_{15\lambda}\Delta\lambda_{\mathrm{arb,sys}}+a_{15\varphi}\Delta\varphi_{\mathrm{arb,sys}}+a_{15\Omega_\zeta}\Delta\Omega_{\zeta\,\mathrm{sys}}=\delta_{\mathrm{sys}}\Omega_\zeta,
\end{aligned}
\tag{3.56}
$$

where the coefficient values are

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$$
\begin{aligned}
&a_{1V_\xi}=\frac{V_\eta}{R_{\mathrm{ea}}};\qquad && a_{2\Omega_\xi}=-2V_\zeta;\\
&a_{1V_\eta}=-\frac{V_\xi}{R_{\mathrm{ea}}}-2\Omega_\zeta; && a_{2\Omega_\zeta}=2V_\xi;\\
&a_{1V_\zeta}=2\Omega_\eta; && a_{3V_\xi}=-2\Omega_\eta;\\
&a_{1a_\xi}=-1; && a_{3V_\eta}=\frac{V_\zeta}{R_{\mathrm{ea}}}+2\Omega_\xi;\\
&a_{1\Omega_\eta}=2V_\zeta; && a_{3V_\zeta}=\frac{V_\eta}{R_{\mathrm{ea}}};\\
&a_{1\Omega_\zeta}=-2V_\eta; && a_{3a_\zeta}=-1;\\
&a_{2V_\xi}=2\Omega_\zeta-2\frac{V_\xi}{R_{\mathrm{ea}}}; && a_{3\Omega_\xi}=2V_\eta;
\end{aligned}
\tag{3.57}
$$

$$a_{2V_\zeta} = -2\Omega_\xi; \qquad a_{3\Omega_\eta} = -2V_\xi;$$

$$a_{2a_\eta} = -1; \qquad a_{4a_\xi} = 1;$$

$$a_{4\dot{\psi}} = (a_z + a_x \sin\psi_a)\cos\vartheta_a;$$

$$a_{4\vartheta} = (a_x \cos\psi_a - a_z\psi_a)\sin\vartheta_a - a_y\cos\vartheta_a;$$

$$a_{5a_\eta} = 1;$$

$$a_{5\dot{\psi}} = a_z \sin\vartheta_a - a_x\gamma_a;$$

$$a_{5\vartheta} = a_x\cos\vartheta_a + a_y\sin\vartheta_a\cos\gamma_a - a_z\psi_a\cos\vartheta_a;$$

$$a_{5\gamma} = -a_x\psi_a + a_y\cos\vartheta_a\sin\gamma_a - a_z;$$

$$a_{6a_\zeta} = 1;$$

$$a_{6\dot{\psi}} = -a_x + a_z\sin\psi_a\cos\gamma_a;$$

$$a_{6\vartheta} = -(a_x\cos\vartheta_a + a_y\sin\vartheta_a)\gamma_a;$$

$$a_{6\gamma} = -a_x\sin\vartheta_a + a_y\cos\vartheta_a + a_z\cos\psi_a\sin\gamma_a;$$

$$a_{7V_\xi} = -\frac{1}{R_{ea}+H};$$

$$a_{7H} = \frac{V_\xi}{(R_{ea}+H)^2};$$

$$a_{8V_\eta} = -1;$$

$$a_{9V_\zeta} = \frac{1}{R_{ea}};$$

$$a_{10\dot{\lambda}} = -\frac{\sin\gamma_a}{\cos\vartheta_a}; \qquad (3.57)$$

$$a_{10\vartheta} = -[\Omega_\eta\cos\gamma_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a]\frac{\sin\vartheta_a}{\cos^2\vartheta_a};$$

$$a_{10\gamma} = \frac{1}{\cos\vartheta_a}[\Omega_\eta\sin\gamma_a + (\Omega_\zeta - \dot{\lambda}_{arb})\cos\gamma_a];$$

$$a_{10\Omega_\eta} = -\frac{\cos\gamma_a}{\cos\vartheta_a};$$

$$a_{10\Omega_\zeta} = \frac{\sin\gamma_a}{\cos\vartheta_a};$$

$$a_{11\dot{\lambda}} = \cos\gamma_a;$$

$$a_{11\gamma} = (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a - \Omega_\eta\cos\gamma_a;$$

$$a_{11\Omega_\eta} = -\sin\gamma_a;$$

$$a_{11\Omega_\zeta} = -\cos\gamma_a;$$

$$a_{12\dot{\lambda}} = \sin\gamma_a\tan\vartheta_a;$$

$$a_{12\vartheta} = [\Omega_\eta\cos\gamma_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\gamma_a]\frac{1}{\cos^2\vartheta_a};$$

$$a_{12\gamma} = -[\Omega_\eta\sin\gamma_a + (\Omega_\zeta - \dot{\lambda}_{arb})\cos\gamma_a]\tan\vartheta_a;$$

$$a_{12\Omega_\xi} = -1;$$

$$a_{12\Omega_\eta} = \cos\gamma_a\tan\vartheta_a;$$

$$
\begin{aligned}
&a_{12\Omega_\zeta} = -\sin\gamma_a \tan\vartheta_a;\\
&a_{13\lambda} = \Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb}\\
&a_{13\Omega_\xi} = 1;\\
&a_{14\lambda} = -\Omega_{\xi 0}\cos\lambda_{arb} + \Omega_{\eta 0}\sin\lambda_{arb}\\
&a_{14\varphi} = \Omega_{\zeta 0};\quad a_{14\Omega_\eta} = 1;\\
&a_{15\lambda} = -(\Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\zeta 0}\sin\lambda_{arb})\varphi_y,\\
&a_{15\varphi} = -(\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb});\\
&a_{15\Omega_\zeta} = 1.
\end{aligned}
\qquad (3.57)
$$

Partial systematic errors included in the system (3.56) are determined as follows: /148

$$
\begin{aligned}
&\delta_{sys}\dot V_\xi = \frac{V_\xi V_\zeta}{R_{ea}+H}\tan\varphi_y + \frac{V_\xi V_\eta}{R_{ea}^2}H;\\
&\delta_{sys}\dot V_\eta = \frac{V_\xi^2}{R_{ea}^2}H - \frac{V_\xi^2}{R_{ea}+H};\\
&\delta_{sys}\dot V_\zeta = -\frac{V_\xi^2}{R_{ea}+H}\tan\vartheta_a - \frac{V_\eta V_\zeta}{R_{ea}^2}H;\\
&\delta_{sys}a_\xi = -a_z\cos\vartheta_a(\psi_a - \sin\psi_a);\\
&\delta_{sys}a_\eta = a_x[\psi_a\vartheta_a - \sin\psi_a - \sin\vartheta_a(1-\cos\psi_a\cos\gamma_a)] +\\
&\qquad + a_z[\gamma_a - \sin\gamma_a\cos\psi_a + \sin\vartheta_a(\psi_a - \sin\psi_a\cos\gamma_a)];\\
&\delta_{sys}a_\zeta = a_x[\psi_a - \sin\psi_a\cos\gamma_a + \sin\vartheta_a(\gamma_a - \sin\gamma_a\cos\psi_a)] -\\
&\qquad - a_y\cos\vartheta_a(\gamma_a - \sin\gamma_a) + a_z\sin\psi_a\sin\vartheta_a\sin\gamma_a;\\
&\delta_{sys}\dot\lambda_{arb} = \frac{V_\xi}{R_{ea}+H}\left(1 - \frac{1}{\cos\varphi_{arb}}\right);\\
&\delta_{sys}\dot\varphi_y = -\frac{V_\zeta}{R_{ea}^2}H;\quad \delta_{sys}\dot H = 0;\\
&\delta_{sys}\dot\psi_a = \frac{1}{\cos\vartheta_a}\dot\lambda_{arb}[\sin\gamma_a - \sin(\varphi_{arb}+\gamma_a)];\\
&\delta_{sys}\dot\vartheta_a = -\dot\lambda_{arb}\sin\varphi_{arb}\sin\gamma_a - \dot\lambda_{arb}\cos\gamma_a(1-\cos\varphi_{arb});\\
&\delta_{sys}\dot\gamma_a = -\dot\lambda_{arb}[\sin\gamma_a - \sin(\varphi_{arb}+\gamma_a)]\tan\vartheta_a;\\
&\delta_{sys}\dot\Omega_\xi = 0;\\
&\delta_{sys}\dot\Omega_\eta = (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})(1-\cos\varphi_{arb}) - \Omega_\zeta(\varphi_{arb} - \sin\varphi_{arb});\\
&\delta_{sys}\dot\Omega_\zeta = (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_\eta\cos\lambda_{arb})(\varphi_{arb} - \sin\varphi_{arb}) + \Omega_{\zeta 0}(1-\cos\varphi_{arb}).
\end{aligned}
\qquad (3.58)
$$

The true kinematic parameter values included in expressions for the coefficients and in the partial systematic errors are also determined by solving system (3.34) according to the previously established functions: $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$.

It should be noted that system (3.56), as well as system (2.199), is unstable as a result of systematic errors in the determination of vertical acceleration. /149

In order to eliminate this shortcoming of a three-component INS, the correction indicated in Section 2.5 must be employed.

§3.5. INS Instrument Error Equations

Instrument errors in INS's of the class under consideration, just as instrument errors in INS's with a platform oriented in the earth coordinate system, are determined by errors in accelerometers, gyroscopes and computers.

In constrast to systems with an inclined platform, the INS's under consideration have no errors as a result of platform inclination devices in the plane of the horizon and along the arbitrary equator, since these devices are lacking. But errors in INS's will occur in the determination of kinematic parameters; these errors are caused by errors in the computer devices which calculate accelerations a_x, a_y, a_z, measured by accelerometers on the $\xi\eta\zeta$ axes of the trihedron.

We note, however, in addition that as a result of a change in the position of the stabilized platform with respect to the direction of earth gravity, transient (and therefore difficult to compensate for) shifts may occur in the gyrostabilized platform.

The derivation of INS kinematic equations, allowing for instrument element errors, the derivation of error equations and the technique for solving these are similar to those outlined in Section 2.6.

The kinematic equations for short-range objects in INS's constructed according to kinematic equations (3.49), and with consideration for systematic and instrument errors, will have the following form:

$$\begin{aligned}
\dot{V}^*_\xi &= a^*_\xi - 2\Omega^*_{\tau,0}V^*_\zeta + \delta_{\text{in}}\dot{V}_\xi;\\
\dot{V}^*_\zeta &= a^*_\zeta + 2\Omega^*_\eta V^*_\xi + \delta_{\text{in}}\dot{V}_\zeta;\\
a^*_\xi &= a^*_x\cos\vartheta^*_a + a^*_y\sin\vartheta^*_a - a^*_z\gamma^*_a\cos\vartheta^*_a + \zeta_{\text{in}}a_\xi;\\
a^*_\zeta &= a^*_x(\psi^*_a + \gamma^*_a\sin\vartheta^*_a) - a^*_y\cos\vartheta^*_a\gamma^*_a + a^*_z + \delta_{\text{in}}a_\zeta;\\
\dot{\lambda}^*_{\text{arb}} &= \frac{1}{R_{\text{ea}}}V^*_\xi + \delta_{\text{in}}\dot{\lambda}_{\text{arb}}
\end{aligned} \qquad (3.60)$$

$$
\begin{aligned}
\dot{\varphi}_y^* &= -\frac{1}{R_{ea}} V_\zeta^* + \delta_{in}\dot{\varphi}_{arb}\\
\dot{\psi}_a^* &= \Omega_\eta^* - (\dot{\Omega}_\zeta^* - \dot{\lambda}_{arb}^*)\gamma_a^* + \delta_{in}\dot{\psi}_a;\\
\dot{\vartheta}_a^* &= \Omega_{\eta_i}^*\gamma_a^* + \Omega_\zeta^* - \dot{\lambda}_{arb}^* + \delta_{in}\dot{\vartheta}_a;\\
\dot{\gamma}_a^* &= \Omega_\xi^* - \dot{\varphi}_{arb}^* - [\Omega_\eta^* - (\Omega_\zeta^* - \dot{\lambda}_{arb}^*)\gamma_a^*]\vartheta_a^* + \delta_{in}\dot{\gamma}_a;\\
\Omega_\xi^* &= \Omega_{\xi 0}^* - \Omega_{\eta_i 0}^*\lambda_{arb}^* + \delta_{in}\Omega_\xi;\\
\Omega_{\eta_i}^* &= \Omega_{\xi 0}^*\lambda_{arb}^* + \Omega_{\eta_i 0}^* + \delta_{in}\Omega_{\eta_i};\\
\Omega_\zeta^* &= \Omega_{\zeta 0}^* + \delta_{in}\Omega_\zeta,
\end{aligned}
\tag{3.60}
$$

where

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$$
\left.
\begin{aligned}
\Omega_{\xi 0}^* &= \Omega\cos\varphi_0^*\cos\Psi_0^*;\\
\Omega_{\eta_i 0}^* &= \Omega\sin\varphi_0^*;\\
\Omega_{\zeta 0}^* &= -\Omega\cos\varphi_0^*\sin\Psi_0^*;\\
\varphi_0^* &= \varphi_0 + \Delta\varphi_0;\\
\Psi_0^* &= \Psi_0 + \Delta\Psi_0.
\end{aligned}
\right\}
\tag{3.61}
$$

Partial instrument error values may be calculated according to the following equations

$$
\begin{aligned}
\delta_{in}\dot{V}_\xi &= \delta_{co_{\dot{V}_\xi}}\dot{V}_\xi;\\
\delta_{in}\dot{V}_\zeta &= \delta_{co_{\dot{V}_\zeta}}\dot{V}_\zeta;\\
\delta_{in}a_\xi &= \delta_{co_{a_\xi}}a_\xi;\\
\delta_{in}a_\zeta &= \delta_{co_{a_\zeta}}a_\zeta;\\
\delta_{in}\dot{\lambda}_{arb} &= \delta_{co}\dot{\lambda}_{arb}\\
\delta_{in}\dot{\varphi}_{arb} &= \delta_{co}\dot{\varphi}_{arb}\\
\delta_{in}\dot{\psi}_a &= \delta_{co}\dot{\psi}_a + \Delta\dot{\psi}_{a0};\\
\delta_{in}\dot{\vartheta}_a &= \delta_{co}\dot{\vartheta}_a + \Delta\dot{\vartheta}_{a0};\\
\delta_{in}\dot{\gamma}_a &= \delta_{co}\dot{\gamma}_a + \Delta\dot{\gamma}_{a0},
\end{aligned}
\tag{3.62}
$$

where
$\delta_{co\dot{V}_\xi}$, $\delta_{co\dot{V}_\zeta}$, δ_{coa_ξ}, δ_{coa_ζ} are errors in the corresponding computer devices;
$\Delta\dot{\psi}_{a0}$, $\Delta\dot{\vartheta}_{a0}$, $\Delta\dot{\gamma}_{a0}$ are angular velocities of the inherent drift of the accelerometer platform in inertial space.

Instrument acceleration values a_x^*, a_y^*, a_z^*, which are obtained from the system accelerometers, and which are taken from equations (2.203), may be presented as

$$\left.\begin{aligned} a_x^* &= a_x + \Delta a_{x0} + \delta_{a_x} a_x; \\ a_y^* &= a_y + \Delta a_{y0} + \delta_{a_y} a_y; \\ a_z^* &= a_z + \Delta a_{z0} + \delta_{a_z} a_z. \end{aligned}\right\} \qquad (3.63)$$ /151

Equation (2.219) must be considered in solving system (3.50).

As a result of the solution to system (3.60), instrument values of the kinematic parameters are determined which include systematic and instrument system errors. The latter may be calculated according to equations (2.215) with allowance for the systematic errors previously found. The true parameter values are determined by solving system (3.34).

In turn, INS kinematic equations for medium-range objects, allowing for systematic and instrument error, will have the form

$$\begin{aligned}
\dot{V}_\xi^* &= a_\xi^* - \frac{\dot{V}_\xi^* V_\eta^*}{R_{ea}} - 2(\Omega_\eta^* V_\zeta^* - \Omega_\zeta^* V_\eta^*) + \delta_{in}\dot{V}_\xi; \\
\dot{V}_\eta^* &= a_\eta^* + \frac{V_\xi^{*2}}{R_{ea}} - 2(\Omega_\zeta^* V_\xi^* - \Omega_\xi^* V_\zeta^*) + g_\eta + \delta_{in}\dot{V}_\eta; \\
\dot{V}_\zeta^* &= a_\zeta^* - \frac{V_\eta^* V_\zeta^*}{R_{ea}} - 2(\Omega_\xi^* V_\eta^* - \Omega_\eta^* V_\xi^*) + \delta_{in}\dot{V}_\zeta; \\
a_x^* &= a_\xi^* \cos\psi_a^* \cos\vartheta_a^* + a_y^* \sin\vartheta_a^* - a_z^* \psi_a^* \cos\vartheta_a^* + \delta_{in} a_\xi; \\
a_\eta^* &= a_x^* (\psi_a^* \gamma_a^* - \sin\vartheta_a^*) + a_y^* \cos\vartheta_a^* \cos\gamma_a^* + a_z^* (\gamma_a^* + \psi_a^* \sin\vartheta_a^*) + \\
&\qquad + \delta_{in} a_\eta; \qquad (3.64) \\
a_\zeta^* &= a_x^* (\psi_a^* + \sin\vartheta_a^* \gamma_a^*) - a_y^* \gamma_a^* \cos\vartheta_a^* + a_z^* \cos\psi_a^* \cos\gamma_a^* + \delta_{in} a_\zeta; \\
\dot{\lambda}_{arb}^* &= \frac{1}{R_{ea} + H^*} V_\xi^* + \delta_{in}\dot{\lambda}_{arb} \\
\dot{H}^* &= V_\eta^* + \delta_{in}\dot{H}; \\
\dot{\varphi}_{arb}^* &= -\frac{1}{R_{ea}} V_\zeta^* + \delta_{in}\dot{\varphi}_{arb}
\end{aligned}$$

$$
\begin{aligned}
\dot{\psi}_a^* &= \frac{1}{\cos\vartheta_a^*}[\Omega_\eta^*\cos\gamma_a^* - (\Omega_\zeta^* - \dot{\lambda}_{arb}^*)\sin\gamma_a^*] + \delta_{in}\dot{\psi}_a;\\
\dot{\vartheta}_a^* &= \Omega_\eta^*\sin\gamma_a^* + (\Omega_\zeta^* - \dot{\lambda}_{arb}^*)\cos\gamma_a^* + \delta_{in}\vartheta_a;\\
\dot{\gamma}_a^* &= \Omega_\xi^* - \dot{\varphi}^* - [\Omega_\eta^*\cos\gamma_a^* - (\Omega_\zeta^* - \dot{\lambda}_{arb}^*)\sin\gamma_a^*]\tan\vartheta_a^* + \delta_{in}\dot{\gamma}_a;\\
\Omega_\xi^* &= \Omega_{\xi 0}^*\cos\lambda_{arb}^* - \Omega_{\eta 0}^*\sin\lambda_{arb}^* + \delta_{in}\Omega_\xi;\\
\Omega_\eta^* &= \Omega_{\xi 0}^*\sin\lambda_{arb}^* + \Omega_{\eta 0}^*\cos\lambda_{arb}^* - \Omega_{\zeta 0}^*\cos\varphi_{arb}^* + \delta_{in}\Omega_\eta;\\
\Omega_\zeta^* &= (\Omega_{\xi 0}^*\sin\lambda_{arb}^* + \Omega_{\eta 0}^*\cos\lambda_{arb}^*)\varphi_{arb}^* + \Omega_{\zeta 0}^* + \delta_{in}\Omega_\zeta.
\end{aligned}
\tag{3.64}
$$

The instrument values of the parameters obtained from the INS may be determined by the solution of system (3.64), allowing for equations (2.219). /152

The basic data for the solution of system (3.64) are furnished by the same parameters used for the solution of system (3.60).

It should be noted that INS instrument errors in the determination of vertical accelerations leads to an increase in errors not only in the determination of the vertical velocity and altitude, but also to errors in the determination of all other kinematic parameters.

For short-range objects equipped with INS's we have, after linearization of (3.49) and allowing for element errors:

$$
\begin{aligned}
&p\Delta V_{\xi in} + a_{1V_\zeta}\Delta V_{\zeta in} + a_{1a_\zeta}\Delta a_{\zeta in} = \delta_{in}\dot{V}_\xi;\\
&a_{2V_\xi}\Delta V_{\xi in} + p\Delta V_{\zeta in} + a_{2a_\zeta}\Delta a_{\zeta in} + a_{2\Omega_\eta}\Delta\Omega_{\eta in} = \delta_{in}\dot{V}_\zeta;\\
&a_{3a_\xi}\Delta a_{\xi in} + a_{3\psi}\Delta\psi_{a,in} + a_{3\vartheta}\Delta\vartheta_{a,in} = \delta_{in}a_\xi;\\
&a_{4a_\zeta}\Delta a_{\zeta in} + a_{4\psi}\Delta\psi_{a,in} + a_{4\vartheta}\Delta\vartheta_{a,in} + a_{4\gamma}\Delta\gamma_{a,in} = \delta_{in}a_\zeta;\\
&a_{5V_\xi}\Delta V_{\xi in} + p\Delta\lambda_{arb,in} = \delta_{in}\dot{\lambda}_{arb}\\
&a_{6V_\zeta}\Delta V_{\zeta in} + p\Delta\varphi_{arb,in} = \delta_{in}\dot{\varphi}_{arb}\\
&a_{7\lambda}p\Delta\lambda_{arb,in} + p\Delta\psi_{arb,in} + a_{7\gamma}\Delta\gamma_{arb,in} + a_{7\Omega_\eta}\Delta\Omega_{\eta in} + a_{7\Omega_\zeta}\Delta\Omega_{\zeta in} = \delta_{in}\dot{\psi}_a;\\
&a_{8\lambda}p\Delta\lambda_{arb,in} + p\Delta\vartheta_{arb,in} + a_{8\gamma}\Delta\gamma_{a,in} + a_{8\Omega_\eta}\Delta\Omega_{\eta in} + a_{8\Omega_\zeta}\Delta\Omega_{\zeta in} = \delta_{in}\dot{\vartheta}_a;\\
&a_{9\lambda}p\Delta\lambda_{arb,in} + a_{9\varphi}p\Delta\varphi_{arb,in} + a_{9\vartheta}\Delta\vartheta_{a,in} + (p + a_{9\gamma})\Delta\gamma_{a,in} + a_{9\Omega_\xi}\Delta\Omega_{\xi in} +\\
&\qquad + a_{9\Omega_\eta}\Delta\Omega_{\eta in} + a_{9\Omega_\zeta}\Delta\Omega_{\zeta in} = \delta_{in}\dot{\gamma}_a;\\
&a_{10\lambda}\Delta\lambda_{arb,in} + a_{10\Omega_\xi}\Delta\Omega_{\xi in} = \delta_{in}\Omega_\xi;\\
&a_{11\lambda}\Delta\lambda_{arb,in} + a_{11\Omega_\eta}\Delta\Omega_{\eta in} = \delta_{in}\Omega_\eta;\\
&a_{12\Omega_\zeta}\Delta\Omega_{\zeta in} = \delta_{in}\Omega_\zeta,
\end{aligned}
\tag{3.65}
$$

where the coefficient values are determined from (3.54) and the partial instrument values are determined from the following equations

$$
\begin{aligned}
&\delta_{in}\dot{V}_\xi = \delta_{co}\dot{V}_\xi;\\
&\delta_{in}\dot{V}_\zeta = \delta_{co}\dot{V}_\zeta;\\
&\delta_{in}a_\xi = \cos\vartheta_a(\Delta a_{x0}+\delta_{a_x}a_x)+\sin\vartheta_a(\Delta a_{y0}+\delta_{a_y}a_y)-\\
&\qquad -\psi_a\cos\vartheta_a(\Delta a_{z0}+\delta_{az}a_z)+\delta_{a_\zeta}a_\xi;\\
&\delta_{in}a_\zeta = (\psi_a+\gamma_a\sin\vartheta_a)(\Delta a_{x0}+\delta_{a_x}a_x)-\gamma_a\cos\vartheta_a(\Delta a_{y0}+\delta_{a_y}a_y)+\\
&\qquad +\Delta a_{z0}+\delta_{az}a_z+\delta_{ba_\zeta}a_\zeta;\\
&\\
&\delta_{in.arb}\dot{\lambda} = \delta_{co}\dot{\lambda}_{arb}\\
&\delta_{in}\dot{\varphi}_{arb} = \delta_{co}\dot{\varphi}_{arb}\\
&\delta_{in}\dot{\psi}_a = \delta_{co}\dot{\psi}_a+\Delta\dot{\psi}_{a0};\\
&\delta_{in}\dot{\vartheta}_a = \delta_{co}\dot{\vartheta}_a+\Delta\dot{\vartheta}_{a0};\\
&\delta_{in}\dot{\gamma}_a = \delta_{co}\dot{\gamma}_a+\Delta\dot{\gamma}_{a0};\\
&\delta_{in}\Omega_\xi = \delta_{co}\Omega_\xi-\Omega(\sin\varphi_0\cos\Psi_0\Delta\varphi_0+\cos\varphi_0\sin\Psi_0\Delta\Psi_0)-\Omega\lambda_{arb}\cos\varphi_0\Delta\varphi_0;\\
&\delta_{in}\Omega_\eta = \delta_{co}\Omega_\eta+\Omega\cos\varphi_0\Delta\varphi_0-\Omega(\sin\varphi_0\cos\Psi_0\Delta\varphi_0+\\
&+\cos\varphi_0\sin\Psi_0\Delta\Psi_0)\lambda_{arb}-\Omega(\sin\varphi_0\sin\Psi_0\Delta\varphi_0-\cos\varphi_0\cos\Psi_0\Delta\Psi_0)\varphi_y;\\
&\delta_{in}\Omega_\zeta = \delta_{co}\Omega_\zeta-\Omega(\sin\varphi_0\sin\Psi_0\Delta\varphi_0+\cos\varphi_0\cos\Psi_0\Delta\Psi_0)\cdot\varphi_{arb}+\\
&\qquad +\Omega\cos\varphi_0\Delta\varphi_0\cdot\varphi_{arb}+\Omega(\sin\varphi_0\sin\Psi_0\Delta\varphi_0-\cos\varphi_0\cos\Psi_0\Delta\Psi_0),
\end{aligned}
\tag{3.66}
$$

in which $\Delta\phi_0$, $\Delta\Psi_0$ are errors in the determination and in the introduction into the system of initial geographic latitude and course angle values. /153

The basic data for the solution of system (3.65) are furnished by the true values of the kinematic parameters included in the coefficients of the equations, and also errors in the separate elements of the system.

In addition, equations (2.219) must be considered in solving equations (3.65).

INS instrument error equations for medium-range objects are derived similarly:

$$
\begin{aligned}
&(p+a_{1V_\xi})\Delta V_{\xi in}+a_{1V_\eta}\Delta V_{\eta in}+a_{1V_\zeta}\Delta V_{\zeta in}+a_{3a_\zeta}\Delta a_{\xi in}+a_{1\Omega_\eta}\Delta\Omega_{\eta in}+\\
&\qquad +a_{1\Omega_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\dot{V}_\xi;\\
&a_{2V_\xi}\Delta V_{\xi in}+p\Delta V_{\eta in}+a_{2V_\zeta}\Delta V_{\zeta in}+a_{3a_\eta}\Delta a_{\eta in}+a_{2\Omega_\xi}\Delta\Omega_{\xi in}+\\
&\qquad +a_{2\Omega_\zeta}\Delta\Omega_{\zeta in}=\delta_{in}\dot{V}_\eta;\\
&a_{3V_\xi}\Delta V_{\xi in}+a_{3V_\eta}\Delta V_{\eta in}+(p+a_{3V_\zeta})\Delta V_{\zeta in}+a_{3a_\zeta}\Delta a_{\zeta in}+\\
&\qquad +a_{3\Omega_\xi}\Delta\Omega_{\xi in}+a_{3\Omega_\eta}\Delta\Omega_{\eta in}=\delta_{in}\dot{V}_\zeta,
\end{aligned}
\tag{3.67}
$$

$$
\begin{aligned}
&a_{4a_\xi}\Delta a_{\xi\,\text{in}}+a_{4\psi}\Delta\psi_{a,\text{in}}+a_{4\vartheta}\Delta\vartheta_{a,\text{in}}=\delta_{\text{in}}a_\xi;\\
&a_{5a_\eta}\Delta a_{\eta\,\text{in}}+a_{5\psi}\Delta\psi_{a,\text{in}}+a_{5\vartheta}\Delta\vartheta_{a,\text{in}}+a_{5\gamma}\Delta\gamma_{a,\text{in}}=\delta_{\text{in}}a_\eta;\\
&a_{6a_\zeta}\Delta a_{\zeta\,\text{in}}+a_{6\psi}\Delta\psi_{a,\text{in}}+a_{6\vartheta}\Delta\vartheta_{a,\text{in}}+a_{6\gamma}\Delta\gamma_{a,\text{in}}=\delta_{\text{in}}a_\zeta;\\
&a_{7V_\xi}\Delta V_{\xi\,\text{in}}+p\Delta\lambda_{\text{arb},\text{in}}+a_{7H}\Delta H_{\text{in}}=\delta_{\text{in}}\dot\lambda_y;\\
&a_{8V_\eta}\Delta V_{\eta\,\text{in}}+p\Delta H_{\text{in}}=\delta_{\text{in}}\dot H;\\
&a_{9V_\zeta}\Delta V_{\zeta\,\text{in}}+p\Delta\varphi_{\text{arb},\text{in}}=\delta_{\text{in}}\dot\varphi_y;\\
&a_{10\lambda}p\Delta\lambda_{\text{arb},\text{in}}+p\Delta\psi_{a,\text{in}}+a_{10\vartheta}\Delta\vartheta_{a,\text{in}}+a_{10\gamma}\Delta\gamma_{a,\text{in}}+a_{10\Omega_\eta}\Delta\Omega_{\eta\,\text{in}}+\\
&\qquad+a_{10\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}}=\delta_{\text{in}}\dot\psi_a;\\
&a_{11\lambda}p\Delta\lambda_{\text{arb},\text{in}}+p\Delta\vartheta_{a,\text{in}}+a_{11\gamma}\Delta\gamma_{a,\text{in}}+a_{11\Omega_\eta}\Delta\Omega_{\eta\,\text{in}}+a_{11\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}}=\delta_{\text{in}}\dot\vartheta_a;\\
&a_{12\lambda}p\Delta\lambda_{\text{arb},\text{in}}+p\Delta\varphi_{\text{arb},\text{in}}+a_{12\vartheta}\Delta\vartheta_{a,\text{in}}+(p+a_{12\gamma})\Delta\gamma_{a,\text{in}}+a_{12\Omega_\xi}\Delta\Omega_{\xi\,\text{in}}+\\
&\qquad+a_{12\Omega_\eta}\Delta\Omega_{\eta\,\text{in}}+a_{12\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}}=\delta_{\text{in}}\dot\gamma_a;\\
&a_{13\lambda}\Delta\lambda_{\text{arb},\text{in}}+a_{13\Omega_\xi}\Delta\Omega_{\xi\,\text{in}}=\delta_{\text{in}}\Omega_\xi;\\
&a_{14\lambda}\Delta\lambda_{\text{arb},\text{in}}+a_{14\varphi}\Delta\varphi_{\text{arb},\text{in}}+a_{14\Omega_\eta}\Delta\Omega_{\eta\,\text{in}}=\delta_{\text{in}}\Omega_\eta;\\
&a_{15\lambda}\Delta\lambda_{\text{arb},\text{in}}+a_{15\varphi}\Delta\varphi_{\text{arb},\text{in}}+a_{15\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}}=\delta_{\text{in}}\Omega_\zeta,
\end{aligned}
\tag{3.67}
$$

where

$$
\begin{aligned}
&\delta_{\text{in}}\dot V_\xi=\delta_{\text{co}}\dot V_\xi;\\
&\delta_{\text{in}}\dot V_\eta=\delta_{\text{co}}\dot V_\eta;\\
&\delta_{\text{in}}\dot V_\zeta=\delta_{\text{co}}\dot V_\zeta;\\
&\delta_{\text{in}}a_\xi=\cos\psi_a\cos\vartheta_a(\Delta a_{x0}+\delta_a a_x)+\sin\vartheta_a(\Delta a_{y0}+\delta_a a_y)-\\
&\qquad-\psi_a\cos\vartheta_a(\Delta a_{z0}+\delta_a a_z);\\
&\delta_{\text{in}}a_\eta=(\psi_a\gamma_a-\sin\vartheta_a)(\Delta a_{x0}+\delta_a a_x)+\cos\vartheta_a\cos\gamma_a(\Delta a_{y0}+\delta_a a_y)+\\
&\qquad+(\gamma_a+\psi_a\sin\vartheta_a)(\Delta a_{z0}+\delta_a a_z);\\
&\delta_{\text{in}}a_\zeta=(\psi_a+\gamma_a\sin\vartheta_a)(\Delta a_{x0}+\delta_a a_x)-\gamma_a\cos\vartheta_a(\Delta a_{y0}+\delta_a a_y)+\\
&\qquad+\cos\psi_a\cos\gamma_a(\Delta a_{z0}+\delta_a a_z);\\
&\delta_{\text{in arb}}\dot\lambda=\delta_{\text{co arb}}\dot\lambda;\\
&\delta_{\text{in}}\dot H=\delta_{\text{co}}\dot H;\\
&\delta_{\text{in arb}}\dot\varphi=\delta_{\text{co arb}}\dot\varphi;\\
&\delta_{\text{in}}\dot\psi_a=\delta_{\text{co}}\dot\psi_a+\Delta\dot\psi_{a0};\\
&\delta_{\text{in}}\dot\vartheta_a=\delta_{\text{co}}\dot\vartheta_a+\Delta\dot\vartheta_{a0};\\
&\delta_{\text{in}}\dot\gamma_a=\delta_{\text{co}}\dot\gamma_a+\Delta\dot\gamma_{a0};\\
&\delta_{\text{in}}\Omega_\xi=\delta_{\text{co}}\Omega_\xi-\Omega(\sin\varphi_0\cos\Psi_0\Delta\varphi_0+\cos\varphi_0\sin\Psi_0\Delta\Psi_0)\times\\
&\qquad\times\cos\lambda_{\text{arb}}-\Omega\sin\lambda_{\text{arb}}\cos\varphi_0\Delta\varphi_0;
\end{aligned}
\tag{3.68}
$$

$$\delta_{in}\Omega_\eta = \delta_{co}\Omega_\eta + \Omega\cos\varphi_0\cos\lambda_{arb}\Delta\varphi_0 - \Omega(\sin\varphi_0\cos\Psi_0\Delta\varphi_0 + \cos\varphi_0\sin\Psi_0\Delta\Psi_0)\sin\lambda_{arb}$$

$$\delta_{in}\Omega_\zeta = \delta_{co}\Omega_\zeta - \Omega(\sin\varphi_0\cos\Psi_0\Delta\varphi_0 + \cos\varphi_0\cos\Psi_0\Delta\Psi_0)\varphi_{arb} + \Omega\cos\varphi_0\Delta\varphi_0\varphi_{arb} + \Omega(\sin\varphi_0\sin\Psi_0\Delta\varphi_0 - \cos\varphi_0\cos\Psi_0\Delta\Psi_0). \quad (3.68)$$

The values of the equation coefficients (3.67) are determined by the relationships (3.57). /154

§3.6. INS Systematic and Instrument Errors /155

In order to calculate systematic and instrument errors in an INS with a fixed platform, the same object and motion program were taken which were used during the investigation of the inclined platform system in Section 2.7.

In agreement with the conclusions reached in Section 3.3 the equation system (3.49) is recommended for the INS in the present case.

It has already been established that an INS constructed in accordance with the kinematic equation system (3.49) may be a two-component system, since in the first two equations the vertical velocity component V_η is lacking; however, for a determination of the true parameters values a_ξ, a_ζ, it is necessary to have three accelerometers in the system.

The INS block diagram for the given class of objects (Figure 3.13) consists of platform P which maintains its arbitrary position in inertial space unchanged. Accelerometers A_x, A_y, A_z, the xyz measurement axes of which are mutually perpendicular, are stabilized in inertial space along the ξ_1, η_1, ζ_1 axes of the coordinate trihedron. Acceleration components a_x, a_y, a_z, measured by the accelerometers, are delivered to the a_ξ and a_ζ parameter computer.

The horizontal vector componets a_ξ, a_ζ of absolute acceleration, in turn, are applied to the computer for accelerations $\dot{V}_\xi$, $\dot{V}_\zeta$, the values of which are subsequently integrated twice in the appropriate computers for the V_ξ, V_ζ and λ_{arb}, ϕ_{arb} parameters.

In addition, in order to determine the angles ψ_a, ϑ_a, γ_a, which are included /156 in equations for a_ξ, a_ζ, $\Omega_{\xi 0}$, $\Omega_{\xi 0}$, $\Omega_{\zeta 0}$, Ω_ξ, Ω_η, Ω_ζ parameter computers are provided, through which feedback is accomplished along the parameters ϕ_{arb}, γ_{arb}, and $\dot{\phi}_{arb}$, $\dot{\gamma}_{arb}$, just as in the INS with a platform stabilized along the $\xi\eta\zeta$ axes of the arbitrary coordinate trihedron.

It is apparent from the block diagram shown that in contrast to an INS with platform inclination, which physically models the position of the $\xi\eta\zeta$ coordinate trihedron associated with the arbitrary coordinate system, in an INS with a platform which is "fixed" in inertial space the position of the $\xi\eta\zeta$ coordinate trihedron is modelled mathematically (by calculating the parameters ψ_a, ϑ_a, γ_a and a_ξ, a_η, a_ζ).

Thus in the INS under consideration the instantaneous values of the vector projections of absolute acceleration (a_ξ, a_η, a_ζ) on the axes of the arbitrary trihedron are not directly measured by the accelerometers, but are continuously calculated in accordance with data (a_x, a_y, a_z) obtained from the accelerometers and by the instantaneous values of angles ψ_a, γ_a, ϑ_a, which are also determined analytically.

The computer section of the system is constructed of continuous calculation elements.

The technique for the preparation of initial data for the determination of system errors is set forth in section 3.4.

The problem of the preparation of initial data involves the determination of the true values of the kinematic parameters of object motion, including the vector components of absolute acceleration a_x, a_y, a_z, which must be measured by the corresponding system accelerometers. These accelerations are basic for the solution of the INS kinematic equation system together with the geographic latitude ϕ_0 of the point of take-off and the initial course angle values Ψ_0 of the arbitrary equator.

In agreement with the technique described and in accordance with known parameter values $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$, ϕ_0, ψ_0, true acceleration values $a_\xi(t)$, $a_\eta(t)$, $a_\zeta(t)$, $a_x(t)$, $a_y(t)$, $a_z(t)$, values of angles $\psi_a(t)$, $\vartheta_a(t)$, $\gamma_a(t)$ and the values of other parameters included in system (3.34) were determined through the solution of system (2.50).

The INS elements are exactly the same as for the system with an inclined platform; error values are given in Table 2.1, Section 2.7. The kinematic equation system for the INS under investigation, with consideration for element errors, is shown in Section 3.5 [see equations (3.60)].

Systematic errors in the INS under consideration are determined in accordance with the technique outlined in Section 2.5 and 3.4.

In agreement with the technique indicated, the values of the these errors may be determined through the solution of the corresponding system of INS kinematic equations (3.51).

Instantaneous acceleration values a_x, a_y, a_z, initial kinematic parameter values $V_{\xi 0}$, $V_{\zeta 0}$, ϕ_{arb0}, γ_{arb0} of object motion, the geographic latitude ϕ_0 of the starting point and the initial course angle value Ψ_0 of the arbitrary equator to the programmed orthodrome all serve as initial data for the solution of this system. /157

INS systematic error values in the determination of parameters V_ξ, V_ζ, λ_{arb}, ϕ_{arb} ($\Delta V_{\xi\ sys}$, $\Delta V_{\zeta\ sys}$, $\Delta\lambda_{arb\ sys}$, $\Delta\phi_{arb\ sys}$) are determined in accordance with expressions (2.189) where the parameter values with asterisks V^*_ξ, V^*_ζ, λ^*_{arb}, ϕ^*_{arb} are obtained as a result of the solution of system (3.51).

It is known that in the values of the parameters with asterisks, where instrument errors in system elements are lacking, only systematic errors remain which are caused by kinematic equation simplifications.

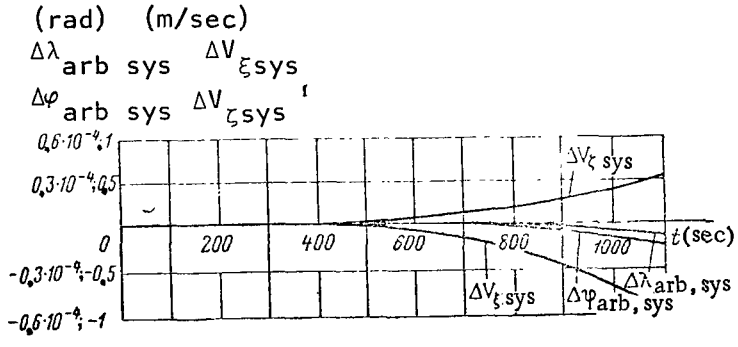


Figure 3.14. Graphs of ISN Systematic Error Change.

Flight time dependencies of the required INS systematic errors $\Delta V_{\xi\ sys}$, $\Delta V_{\zeta\ sys}$, $\Delta\lambda_{arb\ sys}$, $\Delta\varphi_{arb\ sys}$ are shown in Figure 3.14.

It is apparent from the graph that INS systematic errors increase with time and at the end of the controlled section of the flight trajectory will attain the following values

$$\Delta V_{\xi sys} \leqslant 0{,}520 \text{ m/sec}; \quad \Delta V_{\zeta sys} \leqslant 0{,}530 \text{ m/sec};$$
$$\Delta\lambda_{arb,sys} \leqslant 0{,}103\cdot10^{-4} \text{ rad} \equiv 2''; \quad \Delta\varphi_{arb,sys} \leqslant -0{,}1544\cdot10^{-4} \text{ rad} \simeq 3''.$$

We may state on the basis of results obtained that the technique of INS kinematic equation simplifications outlined in Section 2.5 is fully applicable, and that the kinematic equations recommended for INS's of the given object class completely satisfy accuracy requirements.

In order to clarify the influence of separate INS element errors on operational accuracy, the system (3.60) was solved for a number of variants which differed from each other by the values of partial INS instrument errors.

A summary table (Table 3.1) has been prepared in accordance with the results of these solutions showing the values of INS instrument error in the determination of the corresponding kinematic parameters of object motion at the end of the trajectory.

TABLE 3.1. INSTRUMENT ERROR IN INS'S OF THE II CLASS.

Name of Elements and Errors	Element Error	Solution Variant	INS Instrument Error			
			ΔV_ξ in	ΔV_ζ in	$\Delta\varphi$ arb	$\Delta\lambda$ arb
			m/sec		rad	
Accelerometer error in the measurement of a_ξ	$\Delta a_{\xi 0}=0,5\cdot 10^{-4}g$ $\delta_a=10^{-3}$	1	0,691	0,0827	$-0,0769^{-4}$ ($\approx 15''$)	$1,083\cdot 10^{-4}$ ($\approx 0,3'$)
Platform (systematic drift in the longitudinal plane)	$\Delta\dot{\vartheta}_{a0}=0,5\cdot 10^{-6}$ 1/sec	2	2,957	0,0544	$-0,003\cdot 10^{-4}$ ($\approx 0,5''$)	$1,821\cdot 10^{-4}$ ($\approx 0,6'$)
Platform (systematic platform drift in the lateral plane)	$\Delta\dot{\gamma}_{a0}=0,5\cdot 10^{-6}$ 1/sec	3	0,000	$-2,9033$	$1,3\cdot 10^{-4}$ ($\approx 0,6'$)	$0,056\cdot 10^{-4}$ ($\approx 1'$)
Platform (initial display in the longitudinal plane at the starting pt)	$\Delta\vartheta_{a0}=0,5\cdot 10^{-3}$	4	3,916	0,415	$-0,26\cdot 10^{-4}$ ($\approx 0,1'$)	$3,99\cdot 10^{-4}$ ($\approx 1,3'$)
Platform (initial display error in the lateral plane)	$\Delta\gamma_{a0}=0,5\cdot 10^{-3}$	5	0,425	$-3,86$	$3,97\cdot 10^{-4}$ ($\approx 1,2'$)	$0,215\cdot 10^{-4}$ (0,1')

Tr. Note: Commas indicate decimal points.

Error in the V_ξ integrator and in the introduction of initial values $V_{\xi 0}$	$\Delta V_{\xi 0}$=0,3 m/sec δ_{co}=0,0001	6	0,470	0,005	10^{-6} ($\approx$0,1″)	$0{,}328\cdot10^{-4}$ (0,12′)
Errors in the $\dot\lambda_{arb}$ integrator and in the introduction of initial values λ_{arb0}	$\Delta\lambda_{arb0} = 5\cdot10^{-5}$ $\delta_{co} = 10^{-4}$	7	—0,043	0,3709	$0{,}48\cdot10^{-4}$	$-0{,}014\cdot10^{-4}$
Platform (azimuth) display error at the starting point and errors in the ψ_a integrator	$\Delta\psi_{a0}=5{,}8\cdot10^{-4}$	8	—0,043	0,3706	$0{,}47\cdot10^{-4}$ ($\approx$0,15)	$0{,}014\cdot10^{-4}$ ($\approx$0,2″)
Accelerometer errors in the measurement of a_ζ	$\Delta a_{\zeta 0}=0{,}5\cdot10^{-4}$ g $\delta_a=10^{-3}$	9	<0,074	<0,2917	$0{,}334\cdot10^{-4}$ (<0,11′)	$0{,}153\cdot10^{-4}$ (<0,05′)
Platform (systematic azimuth drift)	$\Delta\dot\psi_u=0{,}5\cdot10^{-6}$ 1/sec (0,18 deg/hr) $\delta_{\dot\psi a}=7\cdot10^{-8}$ 1/sec	10	0,190	1,91	$0{,}58\cdot10^{-4}$ (<0,2′)	$0{,}0055\cdot10^{-4}$ (<1″)
Errors in the $\dot V_\zeta$ integrator and in the introduction of initial values $V_{\zeta 0}$.	$\Delta V_{\zeta 0}$=0,125 m/sec δ_{co} = 0.001	11	0,014	0,0312	$0{,}52\cdot10^{-4}$ (<0,18′)	$2{,}15\cdot10^{-4}$ (<0,7′)
Errors in the $\dot\phi_{arb}$ integrator and in the introduction of initial conditions ϕ_{arb0}	$\Delta\phi_{arb0} = 5\cdot10^{-5}$ $\delta_{co} = 10^{-4}$	12	0,450	0,0312	$0{,}52\cdot10^{-4}$ (0,16′)	$2{,}15\cdot10^{-4}$ (<0,7′)

Tr. Note: Commas indicate decimal points.

Figures 3.15-3.18 show time dependencies of INS instrument errors resulting from errors in each element. /160

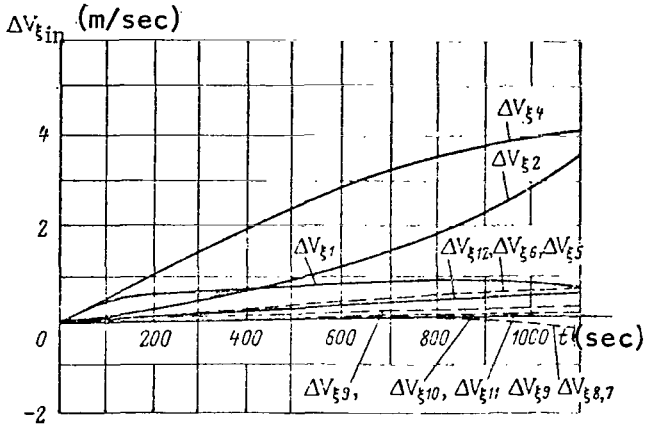


Figure 3.15. Graphs of Instrument Errors $\Delta V_{\xi in}(t)$.

We may make the following conclusions from the results of the instrument error calculations mentioned:

1. Maximum influence on the operational accuracy of the INS under consideration is exerted by accelerometer errors in the measurement of the longitudinal acceleration component a_ξ, the integrators of longitudinal acceleration $\dot{V}_\xi$ and velocity V_ξ, and also by initial display errors and platform stabilization errors in the longitudinal and lateral planes. The values of the INS errors indicated toward the end of the controlled trajectory segment will not exceed

$$\Delta V_{\xi in} \leqslant 4 \text{ m/sec}; \quad \Delta V_{\zeta in} \leqslant 3{,}9 \text{ m/sec};$$
$$\Delta\lambda_{arb,in} \leqslant 1{,}3' \text{ m/sec}; \quad \Delta\varphi_{arb,in} \leqslant 1{,}2' \text{ m/sec}.$$

2. Accelerometer errors in the measurement of the lateral absolute acceleration component, systematic platform drift in azimuth, and also errors in the integrators of lateral acceleration $\dot{V}_\zeta$ and velocity V_ζ exert less influence on INS operational accuracy (by one order of magnitude) in comparison with the factors indicated in paragraph 1. The values of these instrument errors do not exceed

$$\Delta V_{\xi in} < 0{,}5 \text{ m/sec}; \quad \Delta V_{\zeta in} < 0{,}45 \text{ m/sec};$$
$$\Delta\lambda_{arb,in} \leqslant 0{,}2' \text{ m/sec}; \quad \Delta\varphi_{arb,in} \leqslant 0{,}2' \text{ m/sec}.$$

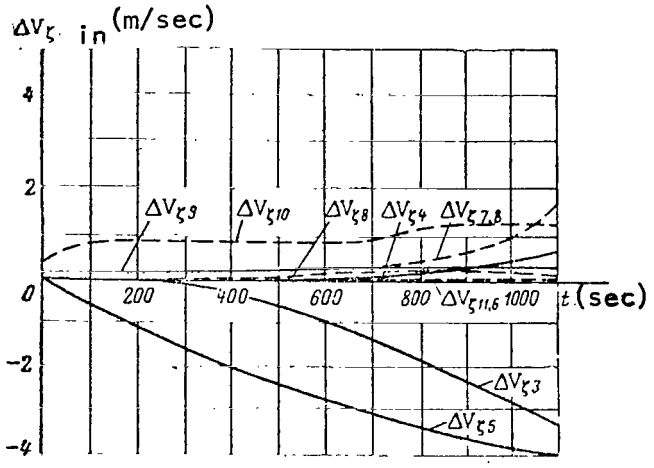


Figure 3.16. Graphs of Instrument Error $\Delta V_{\zeta\ in}(t)$.

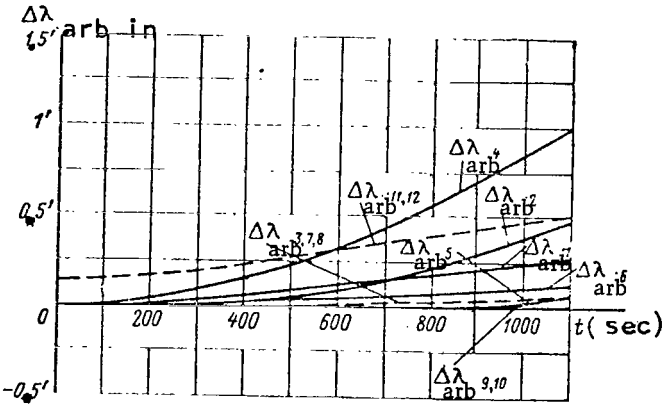


Figure 3.17. Graphs of Instrument Error $\Delta\lambda_{arb\ in}(t)$ /161

3. In view of the fact that the maximum influence on INS operational accuracy is exerted by accelerometer and platform stabilization errors, it is feasible to conduct the initial coarse calculation for INS operational accuracy by considering these errors alone. /162

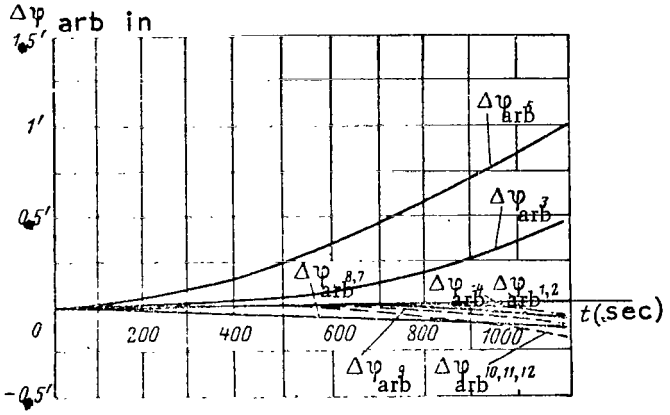


Figure 3.18. Graphs of Instrument Error $\Delta\varphi_{arb\ in}(t)$

CHAPTER IV

THE KINEMATICS OF AN INERTIAL NAVIGATION SYSTEM WITHOUT A STABILIZED PLATFORM.

§4.1. The Fundamental Possibility of Constructing an INS Without a Stabilized Platform.

Errors in the systems examined are in many cases caused by errors in platform stabilization along the axes of the earth or inertial coordinate trihedrons. /163
The development of precision platforms is a problem of great technical difficulty. Their operational accuracy is limited by technological production capabilities.

In addition, gyrostabilized INS platforms have a comparatively long starting time, significant weight and dimensions, and possess limited prospects for an increase in their reliability.

Therefore research in INS designs which would not require gyrostabilized platforms for their operation presents specific practical interest.

Theoretical investigations [27-31], [37], [38] reveal that it is quite possible to construct an INS without a platform, in which platform motion is modelled mathematically in the computer part of the system. Here the accuracy of the modelling indicated is determined by the operational accuracy of the latter and, naturally, by the accuracy of the primary sensors of navigational information.

If a discrete calculator is employed in an INS without a platform, the accuracy and reliability of INS operations may be increased with a significant simplification in production technology, and a reduction in system dimensions and weight.

We shall demonstrate the fundamental possiblity of constructing such systems by using a single-component system as an example.

In order to simplify the problem we shall introduce the same assumptions which were made in Section 3.1.

A single-component navigation system must contain only two accelerometers A_x and A_y, the x,y measurement axes of which (Figure 4.1) at the starting point are aligned along the $\xi_0\eta_0$ axes of the coordinate dihedron, which is linked with the earth's arbitrary coordinate system.

The starting position of the accelerometer measurement axes is fixed on the object during the entire time of object motion.

In order to determine accelerations measured by the accelerometers which are /164
tightly bound to the object, we shall examine the moment of time t, when the object is located at point B, which is removed from point A by a distance corresponding to arbitrary coordinate λ_{arb}.

It is apparent that the angular position of the $\xi\eta$ arbitrary coordinate dihedron at point B with respect to $\xi_0\eta_0$ dihedron is also determined by arbitrary coordinate $\lambda_{arb.}$ The position of the x, y accelerometer measurement axes with respect to the $\xi_0\eta_0$ and $\xi\eta$ coordinate dihedrons is determined by the angles ϑ' and ϑ_a, respectively. /165

The INS accelerometers (Figure 4.2) will measure the non-gravitational vector components of absolute acceleration (a_x, a_y) of the motion of point O of the xy dihedron:

$$\left.\begin{aligned} a_x &= a_{ax} + g_{0x}; \\ a_y &= a_{ay} + g_{0y}. \end{aligned}\right\} \tag{4.1}$$

Here

$$\left.\begin{aligned} g_{0x} &= g_0 \sin \vartheta_a; \\ g_{0y} &= g_0 \cos \vartheta_a. \end{aligned}\right\} \tag{4.2}$$

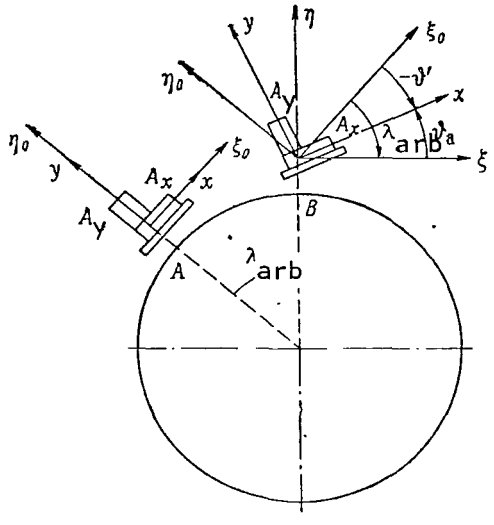


Figure 4.1.

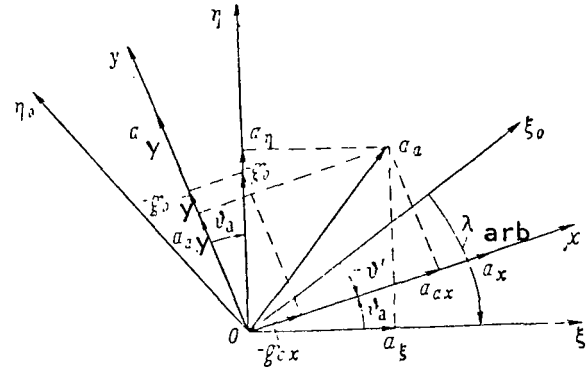


Figure 4.2.

In order to determine the kinematic parameters which characterize object motion with respect to the surface of the earth, it is necessary to know the instantaneous values a_ξ, a_η of the vector projections of acceleration of the object center of mass on the axes of the arbitrary coordinate dihedron:

$$\left.\begin{aligned} a_\xi &= a_{ax}\cos\vartheta_a - a_{ay}\sin\vartheta_a; \\ a_\eta &= a_{ax}\sin\vartheta_a + a_{ay}\cos\vartheta_a. \end{aligned}\right\} \qquad (4.3)$$

By substituting the values a_x and a_y, found from equations (4.1), in equations (4.3) and considering (4.2), we obtain

$$\left.\begin{aligned} a_\xi &= a_x\cos\vartheta_a - a_y\sin\vartheta_a; \\ a_\eta &= a_x\sin\vartheta_a + a_y\cos\vartheta_a - g_0. \end{aligned}\right\} \qquad (4.4.)$$

It is apparent from Figure 4.2 that

$$\vartheta_a = -(\lambda_{arb} - \vartheta'), \qquad (4.5)$$

then

$$\left.\begin{aligned} a_\xi &= a_x\cos(\lambda_{arb} - \vartheta') + a_y\sin(\lambda_{arb} - \vartheta'); \\ a_\eta &= -a_x\sin(\lambda_{arb} - \vartheta') + a_y\cos(\lambda_{arb} - \vartheta') - g_0. \end{aligned}\right\} \qquad (4.6)$$

In turn, the instantaneous values of the angle of deviation of the accelerometer measurement axes with respect to the $\xi_0\eta_0$ coordinate dihedron may be determined both by direct measurement (by means of a free gyroscope), as well as by measurement of the instantaneous values of angular acceleration $\ddot{\vartheta}'$ or velocity $\dot{\vartheta}'$ with subsequent integration of these values

$$\left.\begin{aligned} \vartheta' &= \int_0^t \left(\int_0^t \ddot{\vartheta}'\,dt + \dot{\vartheta}'_0 \right) dt; \\ \vartheta' &= \int_0^t \dot{\vartheta}'\,dt, \end{aligned}\right\} \qquad (4.7)$$

where $\dot{\vartheta}_0$ is the initial angular velocity value.

Thus, depending on the method of determining the angular position of the object in inertial space, the following basic variants of an INS design of this class are possible: /166

The first version (Figure 4.3, a) presupposes the presence in the INS of accelerometers (A'_x, A''_x, A'_y, A''_y), distributed along the object axes at distance $2l$, and a computing device (VU):

The second variant (Figure 4.3 b) includes two linear accelerometers A_x, A_y and a gauge for the measurement of angular velocity (IUS) of object motion with respect to the center of mass, installed in the center of mass of the object, and also computer device (VU);

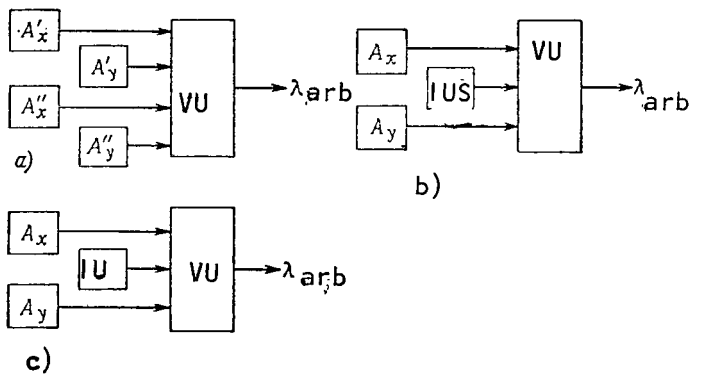


Figure 4.3. Block Diagram of a Single-Component INS.

The third version (Figure 4.3 c) provides for the presence of two linear accelerometers (A_x, A_y), one gauge (IU) for the angular position of the object in inertial space installed at the center of mass of the object, and a computer device (VU).

In order to determine the instantaneous values of the horizontal velocity component V_ξ of object motion and of arbitrary coordinate λ_{arb} by means of an INS of the first design variant, it is necessary to solve the following kinematic equation systems

$$
\begin{aligned}
a_\xi &= a_x \cos(\lambda_{arb} - \vartheta') + a_y \sin(\lambda_{arb} - \vartheta');\\
V_\xi &= V_{\xi 0} + \int_0^t a_\xi dt;\\
\dot{\lambda}_{arb} &= \frac{1}{R_{ea}} V_\xi;\\
\vartheta' &= \int_0^t \left(\int_0^t \ddot{\vartheta}' dt + \dot{\vartheta}'_0 \right) dt.
\end{aligned}
\tag{4.8}
$$

After transformations the system (4.8) may be rewritten in the form /167

$$
\begin{aligned}
\ddot{\lambda}_{arb} &= \frac{1}{R_{ea}} [a_x \cos(\lambda_{arb} - \vartheta') + a_y \sin(\lambda_{arb} - \vartheta')];\\
\vartheta' &= \int_0^t \left(\int_0^t \ddot{\vartheta}' dt + \dot{\vartheta}'_0 \right) dt.
\end{aligned}
\tag{4.9}
$$

In turn,

$$
\left.
\begin{aligned}
a_x &= \frac{a'_x + a''_x}{2};\\
a_y &= \frac{a'_y + a''_y}{2};\\
\ddot{\vartheta}' &= \frac{a'_y - a''_y}{l},
\end{aligned}
\right\}
\tag{4.10}
$$

where
a'_x, a'_y, a''_x, a''_y are indications of the respective system accelerometers.

Similarly the kinematic equations will be:
for the second design variant

$$
\begin{aligned}
\ddot{\lambda}_{arb} &= \frac{1}{R_{ea}} [a_x \cos(\lambda_{arb} - \vartheta') + a_y \sin(\lambda_{arb} - \vartheta')];\\
\vartheta' &= \int_0^t \dot{\vartheta}' dt;
\end{aligned}
\tag{4.11}
$$

for the third variant:

$$\ddot{\lambda}_{arb} = \frac{1}{R_{ea}}[a_x \cos(\lambda_{arb} - \vartheta') + a_y \sin(\lambda_{arb} - \vartheta'). \tag{4.12}$$

Basic data for the solution of system (4.9), considering (4.10), include accelerometer indications a_x', a_x'', a_y', a_y'' and initial values of object angular velocity $\dot{\vartheta}_0'$ at the moment of start.

In order to solve system (4.11) it is necessary to have the instantaneous acceleration values a_x, a_y and angular velocity value $\dot{\vartheta}'$ of objection motion with respect to the center of mass.

In turn, basic data for the solution of system (4.12) are provided by accelerometer indications a_x, a_y and by the gauge for object angular position in inertial space ϑ'.

In addition, in solving the systems (4.9), (4.11) and (4.12) the initial values of the angular coordinate ϑ_0' and of linear velocity $V_{\xi 0}$ of the object center of mass with respect to the surface of the earth must be known. /168

As an example Figure 4.4 shows the block diagram of a continuous-action computer device for the second design variant of the INS, from which it is apparent that the system is closed with respect to the λ_{arb} coordinate just as in the INS's of the I and II classes.

We note that the INS without a platform permits us to determine not only the parameters which characterize the motion of the object's center of mass with respect to the surface of the earth, but also the parameters which characterize the angular position of the object in the plane of the horizon. For example, in order to determine object pitch angle ϑ, it is necessary that the computer solve equation (4.5), allowing for the angle ϑ_{a0} established by the accelerometer axes at the moment of start

$$\vartheta = \vartheta_a + \vartheta_{a0}.$$

INS instrument errors are determined by accelerometer errors, errors in angle or angular velocity gauges, and also by errors in the computer. It is apparent that with the utilization of a computer of relatively high accuracy, INS errors will be determined mainly by errors in the primary measurment devices included in the system.

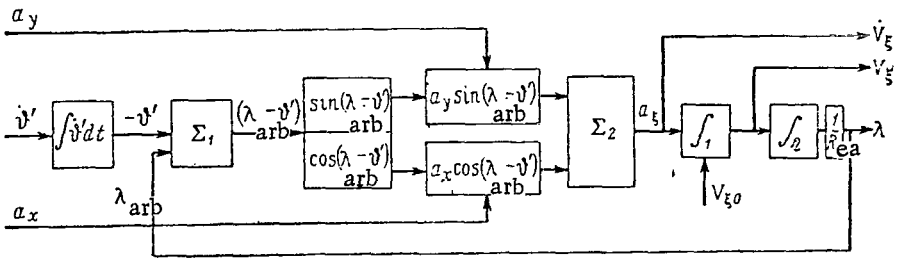


Figure 4.4. Block Diagram of a Computer Installation for a Single-Component INS.

If the INS accelerometers measure accelerations a_x and a_y with errors Δa_x and Δa_y, then it is apparent that this leads to error in the determination of the $\Delta\lambda_{arb}$ coordinate.

Instrument values of the parameters indicated (with the mark "*") will be

$$\begin{aligned} &a_\xi^* = a_\xi + \Delta a_\xi; \quad a_x^* = a_x + \Delta a_x; \quad a_y^* = a_y + \Delta a_y; \\ &\dot{\lambda}_{arb}^* = \dot{\lambda}_{arb} + \Delta\dot{\lambda}_{arb} \quad \lambda_y^* = \lambda_{arb} + \Delta\lambda_{arb} \quad \ddot{\vartheta}'^* = \ddot{\vartheta}' + \Delta\ddot{\vartheta}'; \\ &\dot{\vartheta}'^* = \dot{\vartheta}' + \Delta\dot{\vartheta}'; \quad \vartheta'^* = \vartheta' + \Delta\vartheta'. \end{aligned} \tag{4.13}$$

Having substituted the values of these parameters in the first equations of systems (4.9), (4.11) and (4.12) and having carried out appropriate transformations while considering the second equation of system (4.6), we obtain the following error equations for all three INS design variants: /169

$$\Delta\ddot{\lambda}_{arb} + \frac{(a_\eta + g_0)}{R_{ea}}\Delta\lambda_{arb} = \frac{1}{R_{ea}}[\Delta a_x\cos(\lambda_{arb} - \vartheta') + \Delta a_y\sin(\lambda_{arb} - \vartheta')]. \tag{4.14}$$

It is apparent that the left side of equation (4.14) is (where $a_\eta = 0$) a Schuler pendulum equation, and the right side has a perturbation effect.

The λ_{arb} coordinate and the angle ϑ' changes continuously in the process of motion, and therefore the right side of equation (4.14) will also vary with time.

In considering equation (4.5) and the fact that with automatic motion control the object angle of deviation from the plane of the horizon is relatively small, and also considering that

$$\Delta a_x = \Delta a_y = \Delta a,$$

we obtain as a first approximation

$$\Delta\ddot{\lambda}_{arb} + \frac{1}{R_{ea}}(a_\eta + g_0)\Delta\lambda_{arb} \cong \frac{\Delta a}{R_{ea}}. \tag{4.15}$$

Where $a_\eta = 0$, $\Delta a = \text{const}$, the solution of equation (4.15) will be as follows:

$$\Delta\lambda_{arb} \cong \frac{\Delta a}{g_0}\left(1 - \cos\sqrt{\frac{g_0}{R_{ea}}}\cdot t\right). \tag{4.16}$$

It is apparent from equation (4.16) that INS error in the determination of the λ_{arb} coordinate, caused by accelerometer error, will have both constant and variable components.

The maximum value of the error will not exceed

$$\Delta\lambda_{arb} \leqslant 2\frac{\Delta a}{g_0}.$$

for example, where $\Delta a = 10^{-4} g_0$, $\Delta\lambda_{arb\ m} \leqslant 0.7'$.

A graph of $\Delta\lambda_{arb}(t)$ function, determined in accordance with equation (4.16), is shown in Figure 4.5 a.

The error function $\Delta\lambda_{arb}(t)$ for a single-component INS, caused by constant accelerometer error will be similar to the function (2.9) derived previously.

If the object angular velocity gauge has error $\Delta\dot{\vartheta}'$, then the instrument value of angular velocity is

$$\dot{\vartheta}'^* = \dot{\vartheta}' - \Delta\dot{\vartheta}'.$$

In this connection it is apparent that errors will occur in the determination of other parameters.

In substituting parameter values ϑ'^* and λ^*_{arb} in equation (4.11), after transformations and allowing for the second equation of system (4.6), we obtain /170

$$\Delta\ddot{\lambda}_{arb} + \frac{a_\eta + g_0}{R_{ea}}\Delta\lambda_{arb} = -\frac{a_\eta + g_0}{R_{ea}}\Delta\vartheta'. \tag{4.17}$$

It is apparent that the left side of equation (4.17) in this case also (where $a_\eta = 0$) represents a Schuler pendulum equation, and the right side represents a perturbation factor, caused by errors in the measurement of angle ϑ'.

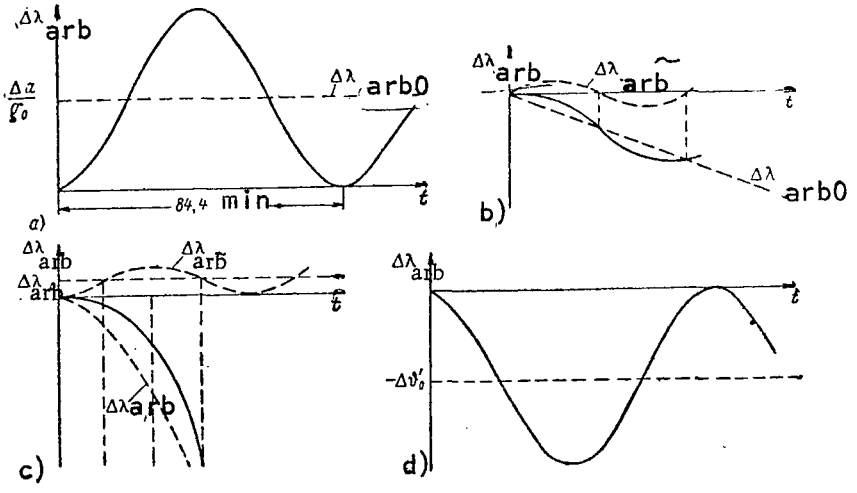


Figure 4.5. Graph of Error $\Delta\lambda_{arb}(t)$.

If we assume the error

$$\Delta\dot{\vartheta}' = \Delta\dot{\vartheta}'_0 = \text{const},$$

then

$$\Delta\vartheta' = \Delta\dot{\vartheta}'_0 t,$$

then here the solution of equation (4.17) will be (if $a_\eta = 0$) as follows:

$$\Delta\lambda_{arb} = \Delta\dot{\vartheta}'_0\left(\sqrt{\frac{R_{ea}}{g_0}}\sin\sqrt{\frac{g_0}{R_{ea}}}\cdot t - t\right). \tag{4.18}$$

It is apparent from equation (4.18) that error in the determination of the λ_{arb} coordinate, caused by constant error in the angular velocity gauge has in a first approximation two components (Figure 4.5 b), one of which increases in proportion to flight time.

$$\Delta\lambda_{arb_0} = \Delta\dot{\vartheta}'_0 t,$$

and the other varies with the Schuler pendulum. /171

$$\Delta\lambda_{arb}\sim = \Delta\dot{\vartheta}'_0 \sqrt{\frac{R_{ea}}{g_0}} \sin \sqrt{\frac{g_0}{R_{ea}}} t. \qquad (4.19)$$

For example, where $\Delta\dot{\vartheta}'_0 = 5\cdot10^{-7}$ 1/sec during flight time $t \cong 60$ min

$$\Delta\lambda_{arb} \simeq 3'.$$

The relationship (4.18) is similar to relationships (2.12) and (3.29), i.e., error in the angular velocity gauge in an INS with no platform causes the same errors as gyroscope platform drift in an INS of the I and II classes.

Where errors are present in the measurement of angular accelerations in the first design variant, we note that

$$\ddot{\vartheta}'^{*} = \ddot{\vartheta}' + \Delta\ddot{\vartheta}',$$

where $\ddot{\vartheta}'^{*}$ is the instrument value of angular acceleration.

It is apparent that these errors cause, in turn, errors in the measurement of angle ϑ' and coordinate λ_{arb}.

The INS error equation in this case will have the form (4.17).

If we consider that

$$\Delta\ddot{\vartheta}' = \Delta\ddot{\vartheta}_0 = \text{const},$$

then

$$\Delta\vartheta' = \frac{1}{2}\,\Delta\ddot{\vartheta}_0' t^2.$$

where $a_\eta = 0$ with consideration for the last expression, equation (4.17) will have the following solution:

$$\Delta\lambda_{arb} = \Delta\ddot{\vartheta}_0 \left[\frac{R_{ea}}{g_0} \left(1 - \cos \sqrt{\frac{g_0}{R_{ea}}} \cdot t \right) - \frac{1}{2} t^2 \right]. \qquad (4.20)$$

It is apparent from equation (4.20) that in a single-component INS of the first design variant, error in the determination of the coordinate which is caused by constant error in the measurement of angular accelerations $\Delta\ddot{\vartheta}'_0$, has three components (see Figure 4.5 c):
one of these is a constant

$$\Delta\lambda_{arb}{}^0 = \Delta\ddot{\vartheta}_0 \frac{R_{ea}}{g_0},$$

and the other increases in proportion to the square of the duration of motion:

$$\Delta\lambda_{arb}{}^{av} = -\Delta\ddot{\vartheta}_0 \frac{R_{ea}}{g_0} t^2,$$

and the third varies with the Schuler pendulum period:

$$\Delta\lambda_{arb}{}^{\sim} = -\Delta\ddot{\vartheta}_0 \frac{R_{ea}}{g_0} \cos \sqrt{\frac{g_0}{R_{ea}}} \cdot t.$$

It is clear from this that this variant of the system may be employed only with sufficiently accurate accelerometers and for objects which accomplish short-duration flights. /172

If the free gyroscope of an INS of the third design variant measures angular object position in inertial space with error $\Delta\vartheta'$ so that the instrument value of the angle is

$$\vartheta' = \vartheta + \Delta\vartheta',$$

then the INS error equation in the determination of the angular coordinate will have the form (4.17).

Where systematic gyroscope drift in inertial space is present with constant angular velocity

$$\Delta\dot{\vartheta} = \Delta\dot{\vartheta}_0 = \text{const}$$

the solution of equation (4.17) has the form (4.18).

If constant error is present in the measurement of the angle

$$\Delta\vartheta' = \Delta\vartheta_0 = \text{const},$$

then the solution of equation (4.17) will be as follows:

$$\Delta\lambda_{\text{arb}} = \Delta\vartheta_0\left(\cos\sqrt{\frac{g_0}{R_{\text{ea}}}}\cdot t - 1\right). \tag{4.21}$$

It is obvious from Figure 4.5 d and equation (4.21) that in this case INS error fluctuates around average value

$$\Delta\lambda_{\text{arb}0} = \Delta\vartheta_0$$

with Schuler pendulum period and amplitude

$$\Delta\lambda_{\text{arb}m} = \Delta\vartheta_0.$$

Here the maximum INS error does not exceed the dual value of angle gauge error

$$(\Delta\lambda_{\text{arb}})_m \leqslant 2\Delta\vartheta_0.$$

It is obvious that this design variant may be employed for objects which are in flight for a relatively extended period of time.

It is apparent from what has already been presented that the construction of an INS without a platform is possible in principle; therefore the systems

without platforms of the second and third design variants possess all the basic properties of the I and II class systems (within the meaning of characteristic error dependence on time). The systems indicated represent mathematical models of the kinematics of the Schuler pendulum in contrast to INSs with inclined platforms, which model the kinematics of this pendulum physically.

§4.2. INS Kinematic Equations.

A three-component INS without a platform, just as the single-component INS, may have the following basic design variants: /173

I Variant - the measurement part of the system consists of six linear accelerometers, distributed in pairs at specific distances with respect to a certain point along the three orthogonal object axes (Figure 4.6 a, 4.7 a);

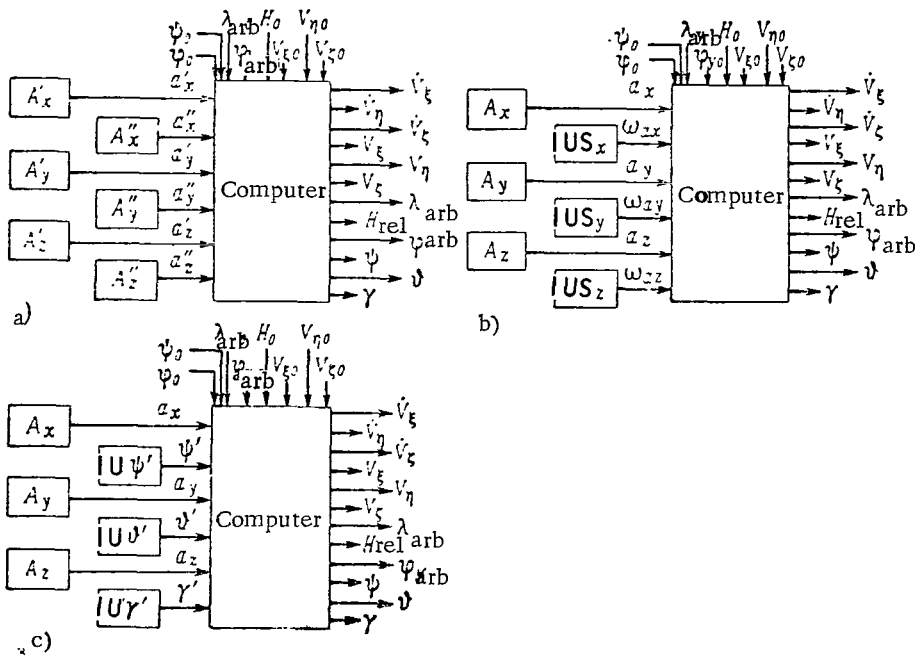


Figure 4.6. The Block Diagram of an INS without a Stabilized Platform.

II Variant - the measurement instruments include three linear accelerometers and three angular velocity gauges, [IUS], installed at a certain point on the object (Figure 4.6 b, 4.7 b);

III Variant - the system includes three accelerometers and three gauges [IU] for object angular position in inertial space (Figures 4.6 c, 4.7 c).

In all of the non-platform variants of the navigation system the presence of a computer is assumed for the solution of the corresponding systems of kinematic equations. The measurement axes of the system instruments are mutually

perpendicular and their position with respect to the arbitrary trihedron at the starting point must be known. Before the start it is feasible to orient the responsive axes of the instruments along the $\xi\eta\zeta$ axes of the coordinate trihedron which is linked with the earth's coordinate system in which object navigation will be accomplished. At the moment the INS is switched into operation, the position of the measurement axes are fixed on the objects, and subsequently these axes are coupled with the object. /174

Let us assume that an INS without a platform, constructed in accordance with any of the variants enumerated, is located in an object with varying motion parameters.

The point on the surface of the earth which corresponds to the instantaneous location of the object will be determined in the arbitrary coordinate system.

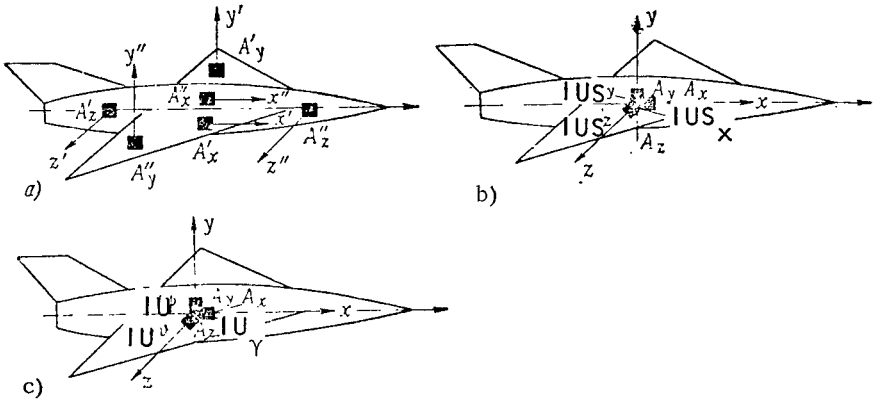


Figure 4.7. Diagram of the Location of INS Gauges on an Aircraft.

Let us assume that the object is in motion in the region of the arbitrary equator with arbitrarily variable flight altitude H(t).

We shall introduce into the examination the $\xi_1\eta_1\zeta_1$ inertial, the $\xi\eta\zeta$ arbitrary and the xyz accelerometer trihedrons. The first two coordinate trihedrons are oriented just as in Section 2.4.

In turn, the accelerometer trihedron is tightly bound to the object and reproduces all of its motions.

The formulation of the INS kinematic equations is reduced to the derivation of the motion equations of the xyz accelerometer trihedron with respect to the surface of the earth. This procedure is similar to the derivation of the kinematic equations for an INS with a platform oriented in the earth's coordinate system.

In describing the motion of the accelerometer trihedron with respect to the surface of the earth, it is possible to employ the kinematic equations previously derived (Section 2.4) for the arbitrary coordinate trihedron by supplementing these with equations which define the position of the accelerometer trihedron with respect to the inertial and arbitrary trihedrons.

In fact, in an INS without a platform the position of the accelerometer measurement axes with respect to the arbitrary trihedron axes is not measured directly, but the instantaneous position of the xyz accelerometer trihedron in inertial space relative to the initial position may be determined by the measurement of angles, angular velocities, or angular accelerations of object motion in inertial space. /175

This circumstance makes it possible to calculate the projections of the accelerations a_x, a_y, a_z, measured by the accelerometers, on the $a_{\xi 1}$, $a_{\eta 1}$, $a_{\zeta 1}$ axes of the inertial trihedron.

Further, the problem of determining the kinematic parameters may be solved just as it was solved for the INS with a platform fixed in inertial space.

Another method for the derivation of the kinematic equations for an INS without a platform provides for the direct calculation of accelerations a_x, a_y, a_z, measured by the accelerometers, on the ξ, η, ζ axes. For this purpose the accelerometer trihedron deviation angles with respect to the ψ_a, ϑ_a, γ_a arbitrary trihedron are determined.

The values of these angles are determined uniquely by the object deviation angles in inertial space, measured by the corresponding instruments, and by the arbitrary coordinates of object location.

The kinematic equations for an INS of the second design variant may be advantageously derived by the second method, which makes it possible to employ the kinematic equation system for arbitrary trihedron motion (2.50).

The instantaneous values of accelerations a_ξ, a_η, a_ζ, included in the right sides of this system, may be calculated (Figure 4.8) from known accelerations a_x, a_y, a_z:

$$\left.\begin{aligned} a_\xi &= a_x \cos\psi_a \cos\vartheta_a + a_y(\sin\psi_a \sin\gamma_a - \cos\psi_a \sin\vartheta_a \cos\gamma_a) + \\ &\quad + a_z(\sin\psi_a \cos\gamma_a + \cos\psi_a \sin\vartheta_a \sin\gamma_a); \\ a_\eta &= a_x \sin\vartheta_a + a_y \cos\vartheta_a \cos\gamma_a - a_z \cos\vartheta_a \sin\gamma_a; \\ a_\zeta &= -a_x \sin\psi_a \cos\vartheta_a + a_y(\cos\psi_a \sin\gamma_a + \sin\psi_a \sin\vartheta_a \cos\gamma_a) + \\ &\quad + a_z(\cos\psi_a \cos\gamma_a - \sin\psi_a \sin\vartheta_a \sin\gamma_a), \end{aligned}\right\} \qquad (4.22)$$

where ψ_a, ϑ_a, γ_a are deviation angles of the xyz accelerometer trihedron with respect to the $\xi\eta\zeta$ arbitrary trihedron.

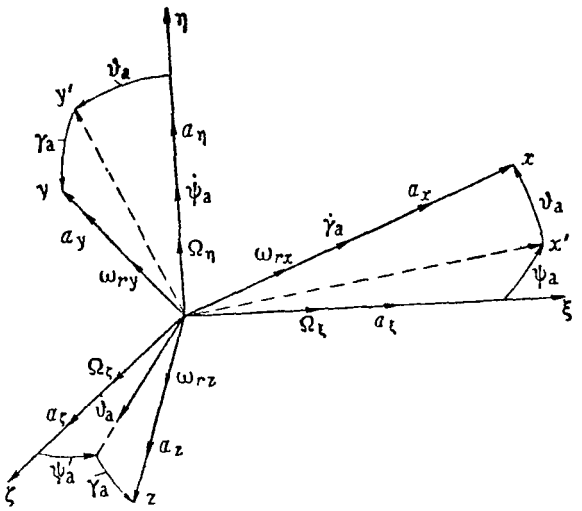


Figure 4.8.

The values of angles ψ_a, ϑ_a, γ_a are uniquely calculated (Figure 4.8) by solving the known equation system

$$\left.\begin{aligned}\dot{\psi}_a &= \frac{1}{\cos\vartheta_a}(\omega_{ry}\cos\gamma_a - \omega_{rz}\sin\gamma_a);\\ \dot{\vartheta}_a &= \omega_{ry}\sin\gamma_a + \omega_{rz}\cos\gamma_a;\\ \dot{\gamma}_a &= \omega_{rx} - (\omega_{ry}\cos\gamma_a - \omega_{rz}\sin\gamma_a)\tan\vartheta_a,\end{aligned}\right\} \quad (4.23)$$

where ω_{rx}, ω_{ry}, ω_{rz} are vector projections of the angular velocity of the rotation of the xyz accelerometer trihedron with respect to the arbitrary trihedron on the xyz axes.

In turn, the vector projections ω_{rx}, ω_{ry}, ω_{rz} of the relative angular velocity are determined as the difference between the vector projections of absolute ω_{ax}, ω_{ay}, ω_{az} and migratory ω_{ex}, ω_{ey}, ω_{ez} angular velocities. /176

$$\left.\begin{aligned}\omega_{rx} &= \omega_{ax} - \omega_{ex};\\ \omega_{ry} &= \omega_{ay} - \omega_{ey};\\ \omega_{rz} &= \omega_{az} - \omega_{ez}.\end{aligned}\right\} \quad (4.24)$$

The instantaneous values ω_{ax}, ω_{ay}, ω_{az} of the vector projections of absolute angular velocity may be directly measured by the appropriate instruments.

In the given case we shall call the angular velocity of the rotation of the $\xi\eta\zeta$ arbitrary trihedron in inertial space the migratory angular velocity.

The instantaneous values of the projections of this angular velocity (Figure 4.9) on the xyz axes of the accelerometer trihedron equal, respectively,

$$
\left.\begin{aligned}
\omega_{ex} &= \omega_\xi \cos\psi_a \cos\vartheta_a + \omega_\eta \sin\vartheta_a - \omega_\zeta \sin\psi_a \cos\vartheta_a;\\
\omega_{ey} &= \omega_\xi (\sin\psi_a \sin\gamma_a - \cos\psi_a \sin\vartheta_a \cos\gamma_a) + \omega_\eta \cos\vartheta_a \cos\gamma_a +\\
&\quad + \omega_\zeta (\sin\psi_a \sin\vartheta_a \cos\gamma_a + \cos\psi_a \sin\gamma_a);\\
\omega_{ez} &= \omega_\xi (\sin\psi_a \cos\gamma_a + \cos\psi_a \sin\vartheta_a \sin\gamma_a) - \omega_\eta \cos\vartheta_a \sin\gamma_a +\\
&\quad + \omega_\zeta (\cos\psi_a \cos\gamma_a - \sin\psi_a \sin\vartheta_a \sin\gamma_a),
\end{aligned}\right\} \qquad (4.25)
$$

where the vector projections of the angular velocity ω_ξ, ω_η, ω_ζ of the rotation of the $\xi\eta\zeta$ arbitrary trihedron in inertial space on the axes of this trihedron are determined as /177

$$
\left.\begin{aligned}
\omega_\xi &= \Omega_\xi - \dot{\varphi}_{arb}\\
\omega_\eta &= \Omega_\eta + \dot{\lambda}_{arb} \sin\varphi_{arb};\\
\omega_\zeta &= \Omega_\zeta - \dot{\lambda}_{arb} \cos\varphi_{arb}
\end{aligned}\right\} \qquad (4.26)
$$

The Ω_ξ, Ω_η, Ω_ζ projections are calculated by solving equations (2.47) with consideration for (2.46).

Further, in considering (2.45), (2.50) and (4.22), we obtain the kinematic equations for an INS without a platform for the second design variant in the following form:

$$
\left.\begin{aligned}
\dot{V}_\xi &= a_\xi - \frac{V_\xi V_\eta \cos\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} - \frac{V_\xi V_\zeta \cos\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} -\\
&\quad - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta) + g_\xi;\\
\dot{V}_\eta &= a_\eta + \frac{V_\xi^2 \cos\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} + \frac{V_\zeta^2}{R_{arb,m} + H} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta;\\
\dot{V}_\zeta &= a_\zeta + \frac{V_\xi \sin\varphi_{arb}}{R_{arb,p} + H\cos\varphi_{arb}} - \frac{V_\eta V_\zeta}{R_{arb,sys} + H} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi) + g_\zeta;\\
a_\xi &= a_x \cos\psi_a \cos\vartheta_a + a_y (\sin\psi_a \sin\gamma_a - \cos\psi_a \sin\vartheta_a \cos\gamma_a) +\\
&\quad + a_z (\sin\psi_a \cos\gamma_a + \cos\psi_a \sin\vartheta_a \sin\gamma_a);\\
a_\eta &= a_x \sin\vartheta_a + a_y \cos\vartheta_a \cos\gamma_a - a_z \cos\vartheta_a \sin\gamma_a;\\
a_\zeta &= -a_x \sin\psi_a \cos\vartheta_a + a_y (\cos\psi_a \sin\gamma_a + \sin\psi_a \sin\vartheta_a \cos\gamma_a) +\\
&\quad + a_z (\cos\psi_a \cos\gamma_a - \sin\psi_a \sin\vartheta_a \sin\gamma_a);\\
\dot{\lambda}_{arb} &= \frac{V_\xi}{R_{arb,p} + H\cos\varphi_{arb}};\\
\dot{H} &= V_\eta;
\end{aligned}\right\} \qquad (4.27)
$$

$$\dot{\varphi}_{arb} = -\frac{V_\zeta}{R_{arb,m} + H};$$

$$\dot{\psi}_a = \frac{1}{\cos\vartheta_a}(\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a + \omega_\xi\cos\psi_a\sin\vartheta_a -$$

$$- \omega_\eta\cos\vartheta_a - \omega_\zeta\sin\psi_a\sin\vartheta_a);$$

$$\dot{\vartheta}_a = \omega_{ay}\sin\gamma_a + \omega_{az}\cos\gamma_a - \omega_\xi\sin\psi_a - \omega_\zeta\cos\psi_a;$$

$$\dot{\gamma}_a = \omega_{ax} - (\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a)\tan\vartheta_a - \omega_\xi\cos\psi_a\sec\vartheta_a +$$

$$+ \omega_\zeta\sin\psi_a\sec\vartheta_a; \qquad (4.27)$$

$$\omega_\xi = \Omega_\xi - \dot{\varphi}_{arb}$$

$$\omega_\eta = \Omega_\eta + \dot{\lambda}_{arb}\sin\varphi_{arb};$$

$$\omega_\zeta = \Omega_\zeta - \dot{\lambda}_{arb}\cos\varphi_{arb};$$

$$\Omega_\xi = \Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0}\sin\lambda_{arb};$$

$$\Omega_\eta = (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\cos\varphi_{arb} - \Omega_{\zeta 0}\sin\varphi_{arb};$$

$$\Omega_\zeta = (\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\sin\varphi_{arb} + \Omega_{\zeta 0}\cos\varphi_{arb}$$

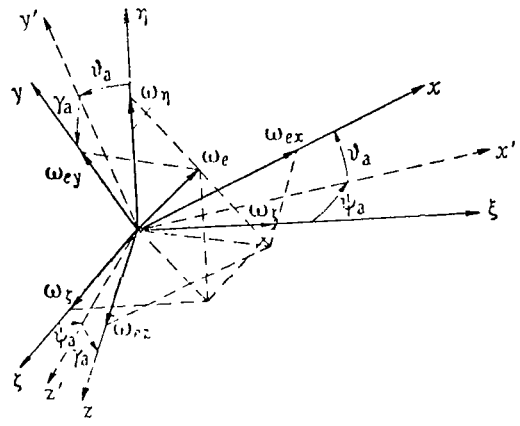


Figure 4.9.

where $\Omega_{\xi 0}$, $\Omega_{\eta 0}$, $\Omega_{\zeta 0}$ are determined from equations (2.46). /178

It is apparent that the system of kinematic equations for an INS without a platform (4.27) differs substantially from the system previously derived in that it contains a number of supplementary equations, linked with the determination of angles ψ_a, ϑ_a, γ_a.

This system is nonlinear with cross links along the velocity vector projections V_ξ, V_η, V_ζ of object motion with respect to the surface of the earth, with respect to angular velocity components Ω_ξ, Ω_η, Ω_ζ of the rotation of the earth, with respect to acceleration components a_x, a_y, a_z measured by the accelerometers and with respect to angular velocities ω_ξ, ω_η, ω_ζ.

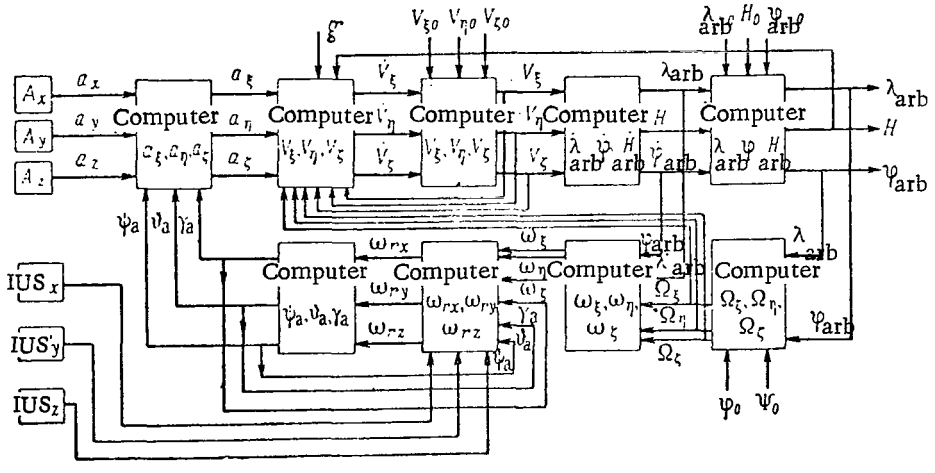
Therefore, the system under consideration must be a three-component system.

Figure 4.10 shows the block diagram of an INS which is constructed in accordance with equations (4.27), from which it is obvious that the system is

closed with respect to parameters λ_{arb}, ϕ_{arb}, $\dot{\lambda}_{arb}$, $\dot{\phi}_{arb}$ through the sections which, in calculating ψ_a, ϑ_a, γ_a, mathematically model the motion of the accelerometer and arbitrary $\xi\eta\zeta$ trihedrons. /179

The solution of the kinematic equation system (4.27) permits us to determine the instantaneous values of all of the kinematic parameters which are defined by the INS's previously examined, including also object deviation angles ψ, ϑ, γ with respect to the surface of the earth, if the accelerometer adjustment angles (ψ_{a0}, ϑ_{a0}, γ_{a0}) are known:

$$\left.\begin{aligned}\psi&=\psi_a-\psi_{a0};\\ \vartheta&=\vartheta_a-\vartheta_{a0};\\ \gamma&=\gamma_a-\gamma_{a0}.\end{aligned}\right\} \qquad (4.28)$$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measured directly, but are calculated in accordance with data obtained from the accelerometers with subsequent integration of these data.

It is easy to see that acceleration values a_x, a_y, a_z, which are on the right sides of equations pertaining to a_ξ, a_η, a_ζ of the system (4.27) may be calculated in accordance with the equations: /180

$$\left.\begin{aligned} a_x &= \frac{a'_x + a''_x}{2}; \\ a_y &= \frac{a'_y + a''_y}{2}; \\ a_z &= \frac{a'_z + a''_z}{2}, \end{aligned}\right\} \qquad (4.29)$$

where a'_x, a''_x, a'_y, a''_y, a'_z, a''_z are indications of the corresponding system accelerometers A'_x, A''_x, A'_y, A''_y, A'_z, A''_z.

In turn, angular velocity values ω_{ax}, ω_{ay}, ω_{az} are determined from the following equation system:

$$\left.\begin{aligned} \omega_{ax} &= \int_0^t \left(\frac{a'_y - a''_y}{l} - \omega_{ay}\omega_{az} \right) dt; \\ \omega_{ay} &= \int_0^t \left(\frac{a'_z - a''_z}{l} - \omega_{az}\omega_{ax} \right) dt; \\ \omega_{az} &= \int_0^t \left(\frac{a'_x - a''_x}{l} - \omega_{ax}\omega_{ay} \right) dt, \end{aligned}\right\} \qquad (4.30)$$

where l is half of the distance between the installation points of the corresponding accelerometers.

Thus the kinematic equation system for a non-platform INS constructed in accordance with the first design variant will differ from the equation system (4.27) of the second variant by supplemental equations (4.29) and (4.30), which are required to determine parameters a_x, a_y, a_z, ω_{ay}, ω_{ax}, ω_{az}.

In an INS constructed in accordance with the third design variant, in contrast to the first two variants, the object deviation angles ψ', ϑ', γ' in inertial space (with respect to the $\xi_1\eta_1\zeta_1$ axes of the inertial trihedron) are measured directly by appropriate gyroscope devices.

Therefore (Figure 4.11) accelerometer indications a_x, a_y, a_z are easily re- /181
calculated on the axes of the inertial trihedron

$$\left.\begin{aligned} a_{\xi 1} &= a_x \cos\psi' \cos\vartheta' + a_y(\sin\psi' \sin\gamma' - \cos\psi' \sin\vartheta' \cos\gamma') + \\ &\quad + a_z(\sin\psi' \cos\gamma' + \cos\psi' \sin\vartheta' \sin\gamma'); \\ a_{\eta 1} &= a_x \sin\vartheta' + a_y \cos\vartheta \cos\gamma' - a_z \cos\vartheta' \sin\gamma'; \\ a_{\zeta 1} &= -a_x \sin\psi' \cos\vartheta' + a_y(\cos\psi' \sin\gamma' + \sin\psi' \sin\vartheta' \cos\gamma') + \\ &\quad + a_z(\cos\psi' \cos\gamma' - \sin\psi' \sin\vartheta' \sin\gamma'). \end{aligned}\right\} \qquad (4.31)$$

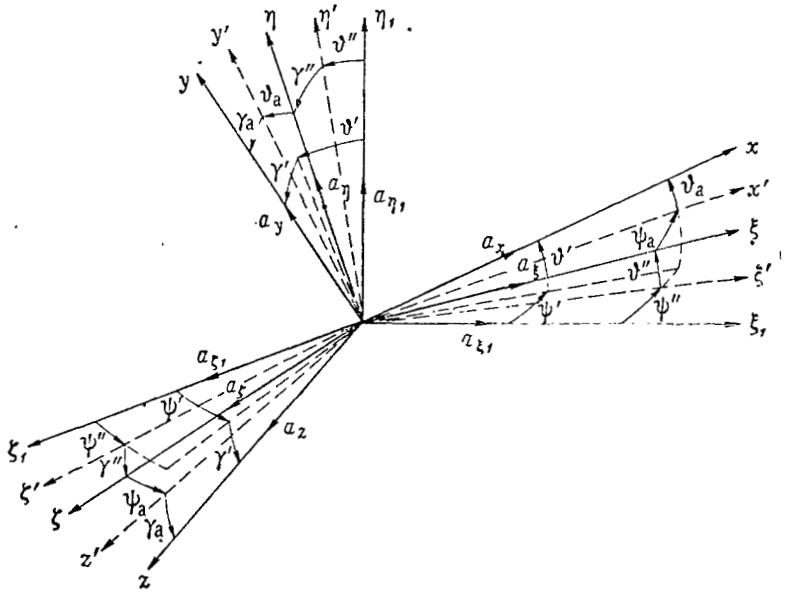

Figure 4.11.

Further, it is possible to determine a_ξ, a_η, a_ζ exactly as calculated for the INS with a "fixed" platform

$$
\left.
\begin{aligned}
a_\xi &= a_{\xi_1}\cos\psi''\cos\vartheta'' + a_{\eta_1}\sin\vartheta'' - a_{\zeta_1}\sin\psi''\cos\vartheta'';\\
a_\eta &= a_{\xi_1}(\sin\psi''\sin\gamma'' - \cos\psi''\sin\vartheta''\cos\gamma'') + a_{\eta_1}\cos\vartheta''\cos\gamma'' +\\
&\quad + a_{\zeta_1}(\sin\psi''\sin\vartheta''\cos\gamma'' + \cos\psi''\sin\gamma'');\\
a_\zeta &= a_{\xi_1}(\sin\psi''\cos\gamma'' + \cos\psi''\sin\vartheta''\sin\gamma'') - a_{\eta_1}\cos\vartheta''\sin\gamma'' +\\
&\quad + a_{\zeta_1}(\cos\psi''\cos\gamma'' - \sin\psi''\sin\vartheta''\sin\gamma''),
\end{aligned}
\right\} \qquad (4.32)
$$

where ψ'', ϑ'', γ'' are the angles of rotation of the $\xi\eta\zeta$ axes with respect to the $\xi_1\eta_1\zeta_1$ inertial trihedron.

The instantaneous values of the angles ψ'', ϑ'', γ'' may be determined through the solution of the following differential equation system /182

$$
\begin{aligned}
\dot\psi'' &= \frac{1}{\cos\vartheta''}(\omega_\eta\cos\gamma'' - \omega_\zeta\sin\gamma'');\\
\dot\vartheta'' &= \omega_\eta\sin\gamma'' + \omega_\zeta\cos\gamma'';\\
\dot\gamma'' &= \omega_\xi - (\omega_\eta\cos\gamma'' - \omega_\zeta\sin\gamma'')\tan\vartheta'',
\end{aligned}
\qquad (4.33)
$$

in which the vector projections of the angular velocity of the rotation of the arbitrary trihedron (ω_ξ, ω_η, ω_ζ) in inertial space on the axes of this trihedron are determined in agreement with (4.26), where the angular velocity projections of the rotation of the earth are determined from (2.47).

Allowing for equations (2.47), (2.50), (4.31), (4.33), we obtain the following equation system:

$$
\begin{aligned}
\dot V_\xi &= a_\xi - \frac{V_\xi V_\eta\cos\varphi_{arb}}{R_{arb,\,p} + H\cos\varphi_{arb}} - \frac{V_\xi V_\zeta\sin\varphi_{arb}}{R_{arb,\,p} + H\cos\varphi_{arb}} -\\
&\quad - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta) + g_\xi(\varphi_y,\ \lambda_y,\ H);\\
\dot V_\eta &= a_\eta + \frac{V_\xi^2\cos\varphi_{arb}}{R_{arb,\,p} + H\cos\varphi_{arb}} + \frac{V_\zeta^2}{R_{arb,\,p} + H} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) +\\
&\quad + g_\eta(\varphi_{arb},\ \lambda_{arb},\ H);\\
\dot V_\zeta &= a_\zeta - \frac{V_\eta V_\zeta}{R_{arb,\,m} + H} + \frac{V_\xi^2\sin\varphi_{arb}}{R_{arb,\,p} + H\cos\varphi_{arb}} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi) +\\
&\quad + g_\zeta(\varphi_{arb},\ \lambda_{arb},\ H);\\
a_\xi &= a_{\xi_1}\cos\psi''\cos\vartheta'' + a_{\eta_1}\sin\vartheta'' - a_{\zeta_1}\sin\psi''\cos\vartheta'';\\
a_\eta &= a_{\xi_1}(\sin\psi''\sin\gamma'' - \cos\psi''\sin\vartheta''\cos\gamma'') + a_{\eta_1}\cos\vartheta''\cos\gamma'' +\\
&\quad + a_{\zeta_1}(\sin\psi''\sin\vartheta''\cos\gamma'' + \cos\psi''\sin\gamma'');
\end{aligned}
\qquad (4.34)
$$

$$
\begin{aligned}
a_\zeta &= a_{\xi_1}(\sin\psi''\cos\gamma'' + \cos\psi''\sin\vartheta''\sin\gamma'') - a_{\eta_1}\cos\vartheta''\sin\gamma'' + \\
&\quad + a_{\zeta_1}(\cos\psi''\cos\gamma'' - \sin\psi''\sin\vartheta''\sin\gamma''); \\
a_{\xi_1} &= a_x\cos\psi'\cos\vartheta' + a_y(\sin\psi'\sin\gamma' - \cos\psi'\sin\vartheta'\cos\gamma') + \\
&\quad + a_z(\sin\psi'\cos\gamma' + \cos\psi'\sin\vartheta'\sin\gamma'); \\
a_{\eta_1} &= a_x\sin\vartheta' + a_y\cos\vartheta'\cos\gamma' - a_z\cos\vartheta'\sin\gamma'; \\
a_{\zeta_1} &= -a_x\sin\psi'\cos\vartheta' + a_y(\cos\psi'\sin\gamma' + \sin\psi'\sin\vartheta'\cos\gamma') + \\
&\quad + a_z(\cos\psi'\cos\gamma' - \sin\psi'\sin\vartheta'\sin\gamma'); \\
\dot\lambda_{arb} &= \frac{V_\xi}{R_{arb,p} + H\cos\varphi_{arb}}; \\
\dot H &= V_\eta; \\
\dot\varphi_{arb} &= -\frac{V_\zeta}{R_{arb,m} + H}; \\
\dot\psi'' &= \frac{1}{\cos\vartheta''}[\Omega_\eta\cos\gamma'' - \Omega_\zeta\sin\gamma'' + \dot\lambda_{arb}\sin(\varphi_{arb} - \gamma'')]; \\
\dot\vartheta'' &= (\Omega_\eta + \dot\lambda_{arb}\sin\varphi_{arb})\sin\gamma'' + (\Omega_\zeta - \dot\lambda_{arb}\cos\varphi_{arb})\cos\gamma''; \\
\dot\gamma'' &= \Omega_\xi - \dot\varphi_{arb} - [\Omega_\eta\cos\gamma'' - \Omega_\zeta\sin\gamma'' + \dot\lambda_{arb}\sin(\varphi_{arb} + \gamma'')]\tan\vartheta''; \\
\Omega_\xi &= \Omega_{\xi_0}\cos\lambda_{arb} - \Omega_{\eta_0}\sin\lambda_{arb}; \\
\Omega_\eta &= (\Omega_{\xi_0}\sin\lambda_{arb} + \Omega_{\eta_0}\cos\lambda_{arb})\cos\varphi_{arb} - \Omega_{\zeta_0}\sin\varphi_{arb}; \\
\Omega_\zeta &= (\Omega_{\xi_0}\sin\lambda_{arb} + \Omega_{\eta_0}\cos\lambda_{arb})\sin\varphi_{arb} + \Omega_{\zeta_0}\cos\varphi_{arb}
\end{aligned}
\qquad (4.34)
$$

where the values Ω_{ξ_0}, Ω_{η_0}, Ω_{ζ_0} are determined from equations (2.46) in accord- /183
and with known values ϕ_0 and Ψ_0.

The kinematic equation system of an INS of the third design variant differs essentially from the kinematic equations of the first two INS variants by equations which define accelerations a_ξ, a_η, a_ζ.

This system, just as the first two systems, has cross links along practically all parameters, and therefore the INS constructed in accordance with equations (4.34) in a general case must be a three-component system.

As a result of the solution of system (4.34) a whole complex of kinematic parameters is defined, which characterizes object motion with respect to the surface of the earth.

Specific practical interest is presented by the operation of the navigational system in the inertial coordinate system with the origin at the center of the earth, the Z_i axis of which is directed along the daily rotational axis of the latter, and the X_i and Y_i axes lie in the plane of the earth equator.

In the coordinate system indicated, the equations for an INS without a stabilized platform, constructed in accordance with the first design variant, have the form:

$$
\begin{aligned}
&\ddot{X}_i = a_X + g_X;\\
&\ddot{Y}_i = a_Y + g_Y;\\
&\ddot{Z}_i = a_Z + g_Z;\\
&a_X = a_x l_{xX} + a_y l_{yX} + a_z l_{zX};\\
&a_Y = a_x l_{xY} + a_y l_{yY} + a_z l_{zY};\\
&a_Z = a_x l_{xZ} + a_y l_{yZ} + a_z l_{zZ};\\
&\dot{l}_{xX} = \omega_{az} l_{yX} - \omega_{ay} l_{zX};\\
&\dot{l}_{yX} = \omega_{ax} l_{zX} - \omega_{az} l_{xX};\\
&\dot{l}_{zX} = \omega_{ay} l_{xX} - \omega_{ax} l_{yX};\\
&\dot{l}_{xY} = \omega_{az} l_{yY} - \omega_{ay} l_{zY};\\
&\dot{l}_{yY} = \omega_{ax} l_{zY} - \omega_{az} l_{xY};\\
&\dot{l}_{zY} = \omega_{ay} l_{xY} - \omega_{ax} l_{yY};\\
&l^2_{xZ} = l_{yX} l_{zY} - l_{zX} l_{yY};\\
&l^2_{yZ} = l_{zX} l_{xY} - l_{xX} l_{zY};\\
&l^2_{zZ} = l_{xX} l_{yY} - l_{yX} l_{xY};
\end{aligned}
\qquad (4.35)
$$

$$
\omega_{ax} = \omega_{ax}(0) + \int_0^t \left(\frac{a'_y - a''_y}{l_z} - \omega_{ay}\omega_{az} \right) dt;
$$

$$
\omega_{ay} = \omega_{ay}(0) + \int_0^t \left(\frac{a'_z - a''_z}{l_x} - \omega_{az}\omega_{ax} \right) dt;
$$

$$
\omega_{az} = \omega_{az}(0) + \int_0^t \left(\frac{a'_x - a''_x}{l_y} - \omega_{ax}\omega_{ay} \right) dt;
$$

$$
a_x = \frac{a'_x + a''_x}{2};
$$

$$
a_y = \frac{a'_y + a''_y}{2};
$$

$$
a_z = \frac{a'_z + a''_z}{2},
$$

where $\ddot{X}_i$, $\ddot{Y}_i$, $\ddot{Z}_i$ are second time derivatives of the inertial coordinates of the center of mass of the aircraft; /184

a_X, a_Y, a_Z, g_X, g_Y, g_Z are vector projections of the accelerations of motion of the center of mass of the aircraft under the effect of forces of a non-gravitational and a gravitational nature, respectively, on the $X_iY_iZ_i$ axes of the inertial coordinate system;

a_x, a_y, a_z are vector projections of the acceleration of motion of the center of mass of an object under the influence of forces of a non-gravitational nature on the xyz coherent axes of the aircraft;

l_{xX}, ... , l_{zZ} are direction cosines which determine the position of the coherent xyz axes with respect to the $X_iY_iZ_i$ inertial axes;

ω_{ax}, ω_{ay}, ω_{az} are vector projections of the absolute angular velocity of the rotation of the aircraft with respect to the center of mass on the xyz bound axes; /185

a'_x, a''_x, a'_y, a''_y, a'_z, a''_z are indications of corresponding ideally operating accelerometers;

l_x, l_y, l_z are distances between corresponding accelerometers, situated in pairs along the object axes.

As a result of solution of the kinematic equation system with known initial conditions [$X_i(0)$, $Y_i(0)$, $Z_i(0)$, $\dot{X}_i(0)$, $\dot{Y}_i(0)$, $\dot{Z}_i(0)$, $\omega_{ax}(0)$, $\omega_{ay}(0)$, $\omega_{az}(0)$, $l_{xX}(0)$, ... $l_{zZ}(0)$] instantaneous values may be determined for coordinate $X_i(t)$, $Y_i(t)$, $Z_i(t)$, for velocities $\dot{X}_i(t)$, $\dot{Y}_i(t)$, $\dot{Z}_i(t)$ and for accelerations $\ddot{X}_i(t)$, $\ddot{Y}_i(t)$, $\ddot{Z}_i(t)$ of the motion of the center of mass of the aircraft, and also for the instantaneous values of the direction cosines $l_{xX}(t)$, ... $l_{zZ}(t)$. When required the inertial coordinates may be inverted into geographic or orthodromic coordinates, and the direction cosines into yaw, pitch and bank angles.

The values $g_X(X_i, Y_i, Z_i)$, $g_Y(X_i, Y_i, Z_i)$, $g_Z(X_i, Y_i, Z_i)$, which are included in the INS kinematic equation system, may be calculated in advance in accordance with known programmed values $X_{i0}(t)$, $Y_{i0}(t)$, $Z_{i0}(t)$ of the coordinate motion of the aircraft and introduced into the memory of the BVM [large computer]. This will simplify the solution of the inertial navigation problem on board without a significant reduction in accuracy.

For the second INS design variant, the kinematic equations subject to solution by the BVM will not contain the latter six expressions pertaining to ω_{ax}, ω_{ay}, ω_{az}, a_x, a_y, a_z, since these parameters are measured directly by the corresponding angular velocity gauges and accelerometers.

In the system of kinematic equations for the third design variant of the INS, expressions for ω_{ax}, ω_{ay}, ω_{az}, a_x, a_y, a_z and, in addition, nine differential equations for direction cosines will not be present.

In this INS design variant, the values a_X, a_Y, a_Z are calculated in accordance with the equations

$$\left.\begin{aligned}
a_X &= a_x\cos\psi'\cos\vartheta' + a_y\sin\vartheta' - a_z\sin\psi'\cos\vartheta';\\
a_Y &= a_x(\sin\psi'\sin\gamma' - \cos\psi'\sin\vartheta'\cos\gamma') + a_y\cos\vartheta'\cos\gamma_a + \\
&\quad + a_z(\cos\psi'\sin\gamma' + \sin\psi'\sin\vartheta'\cos\gamma');\\
a_Z &= a_x(\sin\psi'\cos\gamma' + \cos\psi'\sin\vartheta'\sin\gamma') - \\
&\quad - a_y\cos\vartheta'\sin\gamma' + a_z(\cos\psi'\cos\gamma' - \sin\psi'\sin\vartheta'\sin\gamma').
\end{aligned}\right\} \qquad (4.36)$$

The instantaneous values of angles ψ', ϑ', γ', which are included in equations (4.36), are measured with the corresponding object angular position gauges ($I\psi'$, $I\vartheta'$, $I\gamma'$) in the $X_iY_iZ_i$ inertial coordinate system.

The kinematic equation system for an INS without a gyroplatform is additionally simplified if in place of the angle gauges $I\psi'$, $I\vartheta'$, $I\gamma'$ we employ direction cosine guages (for example, two gyroscopes with an electrostatic suspension); here only the first six equations pertaining to X_i, Y_i, Z_i, a_X, a_Y, a_Z will be included in the system. /186

§4.3. INS Kinematic Equation Simplifications

The technique of kinematic equation simplification for INS's without platforms is similar to that applied to systems (2.50) and (3.34).

Considering the fact that the kinematic equation system for an INS without a platform (4.27) and (4.34) contains equations, the simplification of which was previously analyzed, we shall further simplify only those equations pertaining to a_ξ, a_η, a_ζ, a_{ξ_1}, a_{η_1}, a_{ζ_1}, $\dot\psi''$, $\dot\vartheta''$, $\dot\gamma''$.

The simplification of the equation for accelerations $a_\xi a_\eta a_\zeta$ is required, based on the ranges of the change in programmed values of the angles of pitch, bank and yaw, and from object stabilization accuracy on the programmed values of these parameters.

It is apparent that the lower the range of change of yaw angle ψ, pitch angle ϑ, and bank angle γ and the more accurately the object is stabilized in accordance with these parameters, the smaller will be the range of change in the deviation angles ϑ_a, γ_a of the accelerometer measurement axes (tightly bound to

the object) with respect to the plane of the horizon and in azimuth ψ_a (4.23) and thus the equation system (4.27) may be simplified to a greater degree with respect to a_ξ, a_η, a_ζ.

Thus, for example, for objects in motion where angles ψ_a, ϑ_a, γ_a are relatively small, equations for a_ξ, a_η, a_ζ may be simplified to the form

$$\left.\begin{aligned} a_\xi &= a_x + a_y(\psi_a\gamma_a - \vartheta_a) + a_z\psi_a; \\ a_\eta &= a_x\vartheta_a + a_y - a_z\gamma_a; \\ a_\zeta &= -a_x\psi_a + a_y(\gamma_a + \psi_a\vartheta_a) + a_z. \end{aligned}\right\} \tag{4.37}$$

Here the following systematic errors will occur:

$$\left.\begin{aligned} \delta a_\xi &= a_x(1 - \cos\psi_a\cos\vartheta_a) + a_y(\psi_a\gamma_a - \vartheta_a - \sin\psi_a\sin\gamma_a + \\ &\quad + \cos\psi_a\sin\vartheta_a\cos\gamma_a) + a_z(\psi_a - \sin\psi_a\cos\gamma_a - \\ &\quad - \cos\psi_a\sin\vartheta_a\sin\gamma_a); \\ \delta a_\eta &= a_x(\vartheta_a - \sin\vartheta_a) + a_y(1 - \cos\vartheta_a\cos\gamma_a) - \\ &\quad - a_z(\gamma_a - \cos\vartheta_a\sin\gamma_a); \\ \delta a_\zeta &= -a_x(\psi_a - \sin\psi_a\cos\vartheta_a) + a_y(\gamma_a + \psi_a\vartheta_a - \cos\psi_a\sin\gamma_a - \\ &\quad - \sin\psi_a\sin\vartheta_a\cos\gamma_a) + a_z(1 - \cos\psi_a\cos\gamma_a + \\ &\quad + \sin\psi_a\sin\vartheta_a\sin\gamma_a). \end{aligned}\right\} \tag{4.38}$$

It is not difficult to verify that for objects under automatic control in which during the process of motion on the basic segment of the trajectory the average value of the parameter is. /187

$$|a_x| < 0{,}1g, \quad |a_y| \cong g, \quad a_z \leqslant 0{,}1g;$$

and residual deviations are

$$|\psi_a|_{res} \leqslant 1^\circ; \quad |\vartheta_a|_{res} \leqslant 1^\circ; \quad |\gamma_a|_{res} < 1^\circ,$$

the systematic errors noted fall within the limits of the dead zones of contemporary accelerometers.

For objects in which the system of automatic control on the basic segment of the trajectory does not provide the required accuracy in the stabilization of the angular position with respect to the surface of the earth

($\psi_{a\,av} > 1°$; $\vartheta_{a\,av} > 1°$; $\gamma_{a\,av} > 1°$), it is feasible to employ the kinematic equations pertaining to a_ξ, a_η, a_ζ in the form (4.27).

The equations pertaining to $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$, which are included in the INS kinematic equation system for the second design variant (4.27), may also be simplified if the values of angles ψ_a, ϑ_a, γ_a are sufficiently small:

$$\begin{aligned}
\dot{\psi}_a &= \frac{1}{\cos\vartheta_a}(\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a) + \omega_\xi\vartheta_a - \omega_\eta - \omega_\zeta\psi_a\vartheta_a;\\
\dot{\vartheta}_a &= \omega_{ay}\sin\gamma_a + \omega_{az}\cos\gamma_a - \omega_\xi\psi_a - \omega_\zeta;\\
\dot{\gamma}_a &= \omega_{ax} - (\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a)\tan\vartheta_a - \omega_\xi + \omega_\zeta\psi_a.
\end{aligned} \tag{4.39}$$

In equations (4.39) the trigonometric functions standing near ω_{ay}, ω_{az} are not replaced by arguments since these parameters may obtain significant values in comparison with ω_ξ, ω_η, ω_ζ.

Partial systematic INS errors in the determination of $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$ may be determined as the difference between the right sides of the corresponding equation of the systems (4.39) and (4.27):

$$\begin{aligned}
\delta_{sys}\dot{\psi}_a &= \omega_\xi(\vartheta_a - \cos\psi_a\tan\vartheta_a) - \omega_\zeta(\psi_a\vartheta_a - \sin\psi_a\tan\vartheta_a);\\
\delta_{sys}\dot{\vartheta}_a &= -\omega_\xi(\psi_a - \sin\psi_a) - \omega_\zeta(1 - \cos\psi_a);\\
\delta_{sys}\dot{\gamma}_a &= -\omega_\xi(1 - \cos\psi_a\sec\vartheta_a) + \omega_\zeta(\psi_a - \sin\psi_a\sec\vartheta_a).
\end{aligned} \tag{4.40}$$

If we carry out an evaluation of errors for objects in flight with velocity $V < 500$ m/sec, for which in the process of flight $|\vartheta_a| < 1°$, $|\gamma_a| < 1°$, $|\vartheta_a| < 1°$, then it is possible to verify that the values of these errors do not exceed the dead zones of contemporary angular velocity gauges.

It is feasible to employ the full equations pertaining to $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$ (4.27) for INS's in objects in which the kinematic parameters exceed the limits indicated above.

In considering the discussion in Section 2.4 and 3.3 for short-range INS objects in motion near the arbitrary equator with angular parameters stabilization errors $|\psi_a|_{av} < 1°$, $|\vartheta_a|_{av} < 1°$, $|\gamma_a|_{av} < 1°$, it is possible to recommend the following kinematic equation system: /188 on

$$
\begin{aligned}
&\dot{V}_\xi = a_\xi - 2\Omega_{\eta_0} V_\zeta;\\
&\dot{V}_\zeta = a_\zeta + 2V_\xi(\Omega_{\xi_0}\lambda_{\text{arb}} + \Omega_{\eta_0});\\
&a_\xi = a_x + a_y(\psi_a\gamma_a - \vartheta_a) + a_z\psi_a;\\
&a_\zeta = -a_x\psi_a + a_y(\gamma_a + \psi_a\vartheta_a) + a_z;\\
&\dot{\lambda}_{\text{arb}} = \frac{V_\xi}{R_{ea}};\\
&\dot{\varphi}_{\text{arb}} = -\frac{V_\zeta}{R_{ea}};\\
&\dot{\psi}_a = \frac{1}{\cos\vartheta_a}(\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a) + \Omega_{\xi_0}(\vartheta_a - \lambda_{\text{arb}}) - \Omega_{\eta_0};\\
&\dot{\vartheta}_a = \omega_{ay}\sin\gamma_a + \omega_{az}\cos\gamma_a - \Omega_{\xi_0}\psi_a - \Omega_{\zeta_0} + \dot{\lambda}_{\text{arb}};\\
&\dot{\gamma}_a = \omega_{ax} - (\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a)\tan\vartheta_a - \Omega_{\xi_0} + \Omega_{\eta_0}\lambda_{\text{arb}}
\end{aligned}
\qquad (4.41)
$$

in which the values Ω_{ξ_0}, Ω_{η_0}, Ω_{ζ_0} are determined from (2.46) and the values a_x, a_y, a_z and ω_{ax}, ω_{ay}, ω_{az} are measured by the accelerometers and angular velocity gauges, respectively.

If the values $|\psi_a|>1°$, $|\vartheta_a|>1°$, $|\gamma_a|>1°$, then the following is recommended for INS short-range objects:

$$
\begin{aligned}
&\dot{V}_\xi = a_\xi - 2\Omega_{\eta_0} V_\zeta;\\
&\dot{V}_\zeta = a_\zeta + 2V_\xi(\Omega_{\xi_0}\lambda_{\text{arb}} + \Omega_{\eta_0});\\
&a_\xi = a_x\cos\psi_a\cos\vartheta_a + a_y(\sin\psi_a\sin\gamma_a - \cos\psi_a\sin\vartheta_a\cos\gamma_a) +\\
&\quad + a_z(\sin\psi_a\cos\gamma_a + \cos\psi_a\sin\vartheta_a\sin\gamma_a);\\
&a_\zeta = -a_x\sin\psi_a\cos\vartheta_a + a_y(\cos\psi_a\sin\gamma_a + \sin\psi_a\sin\vartheta_a\cos\gamma_a) +\\
&\quad + a_z(\cos\psi_a\cos\gamma_a - \sin\psi_a\sin\vartheta_a\sin\gamma_a);\\
&\dot{\lambda}_{\text{arb}} = \frac{V_\xi}{R_{ea}};\\
&\dot{\varphi}_{\text{arb}} = -\frac{V_\zeta}{R_{ea}};\\
&\dot{\psi}_a = \frac{1}{\cos\vartheta_a}(\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a + \omega_\xi\cos\psi_a\sin\vartheta_a -\\
&\quad - \omega_\eta\cos\vartheta_a - \omega_\zeta\sin\psi_a\sin\vartheta_a);
\end{aligned}
\qquad (4.42)
$$

$$
\begin{aligned}
&\dot{\vartheta}_a = \omega_{ay}\sin\gamma_a + \omega_{az}\cos\gamma_a - \omega_\xi \sin\psi_a - \omega_\zeta\cos\psi_a;\\
&\dot{\gamma}_a = \omega_{ax} - (\omega_{ay}\cos\gamma_a + \omega_{az}\sin\gamma_a)\tan\vartheta_a - \omega_\xi\cos\psi_a\sec\vartheta_a +\\
&\quad + \omega_\zeta\sin\psi_a\sec\vartheta_a;\\
&\omega_\xi = \Omega_{\xi_0} - \Omega_{\eta_0}\lambda_{\text{arb}} - \dot{\varphi}_{\text{arb}};\\
&\omega_{\eta_i} = \Omega_{\xi_0}\lambda_{\text{arb}} + \Omega_{\eta_0};\\
&\omega_\zeta = \Omega_{\zeta_0} - \dot{\lambda}_{\text{arb}}
\end{aligned}
\qquad (4.42)
$$

It is apparent from equations (4.41) and (4.42) that an INS constructed in accordance with these equations may be a two-component system, since in the first two equations the vertical velocity component V_η is lacking; however, in order to determine accelerations a_ξ and a_ζ the INS must be equipped with three accelerometers. /189

We must employ the following kinematic equation systems for INS-equipped objects with a medium radius of action, which are in motion near the arbitrary equator:

$$
\begin{aligned}
&\dot{V}_\xi = a_\xi - \frac{V_\xi V_\eta}{R_{\text{ea}}} - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta);\\
&\dot{V}_\eta = a_\eta + \frac{V_\xi^2}{R_{\text{ea}}} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta;\\
&\dot{V}_\zeta = a_\zeta - \frac{V_\eta V_\zeta}{R_{\text{ea}}} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi);\\
&a_\xi = a_x\cos\psi_a\cos\vartheta_a + a_y(\sin\psi_a\sin\gamma_a - \cos\psi_a\sin\vartheta_a\cos\gamma_a) +\\
&\quad + a_z(\sin\psi_a\cos\gamma_a + \cos\psi_a\sin\vartheta_a\sin\gamma_a);\\
&a_\eta = a_x\sin\vartheta_a + a_y\cos\vartheta_a\cos\gamma_a - a_z\cos\vartheta_a\sin\gamma_a;\\
&a_\zeta = -a_x\sin\psi_a\cos\vartheta_a + a_y(\cos\psi_a\sin\gamma_a + \sin\psi_a\sin\vartheta_a\times\\
&\quad \times\cos\gamma_a) + a_z(\cos\psi_a\cos\gamma_a - \sin\psi_a\sin\vartheta_a\sin\gamma_a);\\
&\dot{\lambda}_{\text{arb}} = \frac{1}{R_{\text{ea}} + H}V_\xi;\\
&\dot{H} = V_\eta;\\
&\dot{\varphi}_{\text{arb}} = -\frac{1}{R_{\text{ea}}}V_\zeta;\\
&\dot{\psi}_a = \frac{1}{\cos\vartheta_a}[\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a + (\Omega_\xi - \dot{\varphi}_{\text{arb}})\cos\psi_a\sin\vartheta_a -\\
&\quad - \Omega_\eta\cos\vartheta_a - (\Omega_\zeta - \dot{\lambda}_{\text{arb}})\sin\psi_a\sin\vartheta_a];\\
&\dot{\vartheta}_a = \omega_{ay}\sin\gamma_a + \omega_{az}\cos\gamma_a - (\Omega_\xi - \dot{\varphi}_{\text{arb}})\sin\psi_a - (\Omega_\zeta - \dot{\lambda}_{\text{arb}})\cos\psi_a;\\
&\dot{\gamma}_a = \omega_{ax} - (\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a)\tan\vartheta_a - (\Omega_\xi - \dot{\varphi}_{\text{arb}})\cos\psi_a\sec\vartheta_a +
\end{aligned}
\qquad (4.43)
$$

$$
\begin{aligned}
&+(\Omega_\zeta-\dot{\lambda}_{arb})\sin\psi_a\sec\vartheta_a;\\
\Omega_\xi&=\Omega_{\xi_0}\cos\lambda_{arb}-\Omega_{\eta_0}\sin\lambda_{arb};\\
\Omega_\eta&=\Omega_{\xi_0}\sin\lambda_{arb}+\Omega_{\eta_0}\cos\lambda_{arb}-\Omega_{\zeta_0}\varphi_{arb};\\
\Omega_\zeta&=(\Omega_{\xi_0}\sin\lambda_{arb}+\Omega_{\eta_0}\cos\lambda_{arb})\varphi_{arb}+\Omega_{\zeta_0},
\end{aligned}
\qquad (4.43)
$$

where Ω_{ξ_0}, Ω_{η_0}, Ω_{ζ_0} are determined from equations (2.46) in accordance with known values ϕ_0 and Ψ_0. /190

It is apparent from the equation system (4.43) that an INS for objects of this class must be constructed in accordance with the three-component design, since the first three equations have cross links with the vertical velocity components.

The kinematic equation system for an INS of the first design variant with no platform differs from the equation system of the second variant only by supplemental equations (4.29) and (4.30) for the calculation of accelerations a_x, a_y, a_z and angular velocities ω_{ax}, ω_{ay}, ω_{az}. Therefore the kinematic equation simplifications (4.27), presented in the preceding section, are also correct in the present case: the simplified kinematic equations for an INS of the first variant are the same for corresponding objects as the simplified equation system of the second variant; however, in solving the latter it is necessary to take into consideration equations (4.29) and (4.30).

Kinematic equations for an INS of the third design variant (4.34) differ from the kinematic equations for an INS of the second variant in that they contain several different equations for a_ξ, a_η, a_ζ and equations bound with them pertaining to accelerations recalculated on the axes of the inertial trihedron (a_{ξ_1}, a_{η_1}, a_{ζ_1}), as well as equations pertaining to angles of rotation ψ'', ϑ'', γ'' of the arbitrary trihedron in inertial space.

Therefore we shall examine the possibility of simplifying the equations which pertain only to the parameters noted.

For INS-equipped objects in flight during the course of relatively small time intervals and under conditions where angles ψ'', ϑ'', γ'' are comparatively small, equations (4.32) may be simplified to the form

$$
\left.
\begin{aligned}
a_\xi&=a_{\xi_1}+a_{\eta_1}\vartheta''-a_{\zeta_1}\psi'';\\
a_\eta&=-a_{\xi_1}\vartheta''+a_{\eta_1}+a_{\zeta_1}\gamma'';\\
a_\zeta&=a_{\xi_1}\psi''-a_{\eta_1}\gamma''+a_{\zeta_1}.
\end{aligned}
\right\}
\qquad (4.44)
$$

Here the following partial systematic errors will occur:

$$\begin{aligned}
\overset{\mathrm{I}}{\delta}_{\mathrm{sys}} a_\xi &= a_{\xi_1}(1-\cos\psi''\cos\vartheta'')+a_{\eta_1}(\vartheta''-\sin\vartheta'')-a_{\zeta_1}(\psi''-\\
&\quad-\sin\psi''\cos\vartheta'');\\
\overset{\mathrm{I}}{\delta}_{\mathrm{sys}} a_\eta &= -a_{\xi_1}(\vartheta''-\sin\psi''\sin\gamma''+\cos\psi''\sin\vartheta''\cos\gamma'')+a_{\eta_1}(1-\\
&\quad-\cos\vartheta''\cos\gamma'')+a_{\zeta_1}(\gamma''-\cos\psi''\sin\gamma''-\sin\psi''\times\\
&\quad\times\sin\vartheta''\cos\gamma'');\\
\overset{\mathrm{I}}{\delta}_{\mathrm{sys}} a_\zeta &= a_{\xi_1}(\psi''-\sin\psi''\cos\gamma''+\cos\psi''\sin\vartheta''\sin\gamma'')-a_{\eta_1}(\gamma''-\\
&\quad-\cos\vartheta''\sin\gamma'')-a_{\zeta_1}(1-\cos\psi''\cos\gamma''+\sin\psi''\sin\vartheta''\sin\gamma'').
\end{aligned} \tag{4.45}$$

For an approximate evaluation of these errors we shall consider that /191

$$\begin{aligned}
\dot\psi''&\cong\Omega_\eta;\\
\dot\vartheta''&\cong\Omega_\zeta-\dot\lambda_{\mathrm{arb}}\\
\dot\gamma''&=\Omega_\xi-\dot\varphi_{\mathrm{arb}}
\end{aligned} \tag{4.46}$$

Further, in considering the small value of ϕ_{arb} and the fact that the following disparity is always correct

$$\Omega_\eta\leqslant\Omega;\quad\Omega_\zeta\leqslant\Omega;\quad\Omega_\xi\leqslant\Omega,$$

we obtain approximately

$$\begin{aligned}
\psi''&\leqslant\Omega t;\\
\vartheta''&\leqslant\Omega t-\lambda_{\mathrm{arb}}\\
\gamma''&\leqslant\Omega t.
\end{aligned} \tag{4.47}$$

If on the basic segment of the trajectory of object motion

$$a_{\xi_1}\leqslant 0{,}1\,g;\quad a_{\eta_1}\cong g;\quad a_{\zeta_1}\cong 0{,}1g,$$

then considering (4.47), we may write

$$\begin{aligned}
\overset{\circ}{\underset{sys}{\delta}} a_\xi &\leqslant g\,[\Omega t - \lambda_{\underset{arb}{}} - \sin(\Omega t - \lambda_{\underset{arb}{}})];\\
\overset{\circ}{\underset{sys}{\delta}} a_\eta &\leqslant g\,[1 - \cos(\Omega t - \lambda_{\underset{arb}{}})\cos\Omega t];\\
\overset{\circ}{\underset{sys}{\delta}} a_\zeta &\leqslant g\,[\Omega t - \cos(\Omega t - \lambda_{\underset{arb}{}})\sin\Omega t].
\end{aligned} \tag{4.48}$$

An analysis of the errors cited in accordance with equations (4.48) reveals that for objects in motion near the arbitrary equator at distance L < < 300 km during time t < 1,000 sec, the values of these errors will not exceed the dead zones of comtemporary accelerometers.

For objects in flight at distances 300 km < L < 500 km during time t < < 1,000 sec, the values of angles ψ'', ϑ'', γ'' will be such that it is feasible to assume that

$$\left.\begin{aligned}
a_{\xi_1} &= g\sin\vartheta''; \quad a_{\eta_1} = g\cos\vartheta''; \quad a_{\zeta_1} = g\sin\gamma'';\\
\cos\psi'' &\cong 1; \qquad \cos\gamma'' \cong 1; \qquad \cos\vartheta'' \neq 1;\\
\sin\psi'' &\cong \psi''; \qquad \sin\gamma'' \cong \gamma''; \qquad \sin\vartheta'' \cong \vartheta'',
\end{aligned}\right\} \tag{4.49}$$

then the equations pertaining to a_ξ, a_η, a_ζ will have the form: /192

$$\left.\begin{aligned}
a_\xi &= a_{\xi_1}\cos\vartheta'' + a_{\eta_1}\sin\vartheta'' - a_{\zeta_1}\dot{\psi}''\cos\vartheta'';\\
a_\eta &= a_{\xi_1}(\dot{\psi}''\gamma'' - \sin\vartheta'') + a_{\eta_1}\cos\vartheta'' + a_{\zeta_1}(\gamma'' + \dot{\psi}''\sin\vartheta'');\\
a_\zeta &= a_{\xi_1}(\dot{\psi}'' + \gamma''\sin\vartheta'') - a_{\eta_1}\gamma''\cos\vartheta'' + a_{\zeta_1}.
\end{aligned}\right\} \tag{4.50}$$

Allowing here for (4.47), we obtain the following expressions for systematic errors

$$\begin{aligned}
\delta^{II}_{a_\xi} &\cong \frac{1}{2}\,g\,[\sin 2(\Omega t - \lambda_{\underset{arb}{}})(1 - \cos\Omega t) - 2\sin\Omega t\cos(\Omega t - \\
&\quad - \lambda_{\underset{arb}{}})(\Omega t - \sin\Omega t)];\\
\delta^{II}_{a_\eta} &\cong g\sin(\Omega t - \lambda_{\underset{arb}{}})\{\Omega^2 t^2 - \sin^2\Omega t\,[1 + \sin(\Omega t - \lambda_{\underset{arb}{}})]\} +\\
&\quad + g\sin\Omega t\left(\Omega t - \frac{1}{2}\sin 2\Omega t\right)\times\\
&\quad \times[1 + \sin(\Omega t - \lambda_{\underset{arb}{}})] + g\cos^2(\Omega t - \lambda_{\underset{arb}{}})(1 - \cos\Omega t);\\
\delta^{II}_{a_\zeta} &\cong g\left\{\sin(\Omega t - \lambda_{\underset{arb}{}})\left(\Omega t - \frac{1}{2}\sin 2\Omega t\right)[1 + \sin(\Omega t - \lambda_{\underset{arb}{}})] -\right.\\
&\quad \left. - \cos^2(\Omega t - \lambda_{\underset{arb}{}})(\Omega t - \sin\Omega t)] + \sin^3\Omega t\,[1 + \sin(\Omega t - \lambda_{\underset{arb}{}})]\right\}.
\end{aligned} \tag{4.51}$$

It is not difficult to verify that for short-range INS-equipped objects in flight near the arbitrary equator, the values of the systematic errors examined lie within the limits of the dead zones of contemporary accelerometers; therefor equations (4.50) may be employed for such objects.

In turn, it is possible to show that for medium-range INS-equipped objects it is feasible to determine values a_ξ, a_η, a_ζ in accordance with the following equations

$$\left.\begin{aligned} a_\xi &= a_{\xi_1}\cos\psi''\cos\vartheta'' + a_{\eta_1}\sin\vartheta'' - a_{\zeta_1}\psi''\cos\vartheta'';\\ a_\eta &= a_{\xi_1}(\psi''\gamma'' - \sin\vartheta'') + a_{\eta_1}\cos\vartheta''\cos\gamma'' + a_{\zeta_1}(\gamma'' + \psi''\sin\vartheta'');\\ a_\zeta &= a_{\xi_1}(\psi'' + \gamma''\sin\vartheta'') - a_{\eta_1}\gamma''\cos\vartheta'' + a_{\zeta_1}\cos\psi''\cos\gamma''. \end{aligned}\right\} \qquad (4.52)$$

The kinematic equations pertaining to a_{ξ_1}, a_{η_1}, a_{ζ_1} may be simplified for short-range objects with a sufficiently refined stabilization system with respect to the plane of the horizon and in azimuth. Where $\psi < 1°$, $\vartheta < 1°$, $\gamma < 1°$, we may assume that

$$\left.\begin{aligned} a_{\xi_1} &= a_x\cos\vartheta' + a_y(\psi'\gamma' - \sin\vartheta') + a_z(\psi' + \gamma'\sin\vartheta');\\ a_{\eta_1} &= a_x\sin\vartheta' + a_y\cos\vartheta' - a_z\gamma'\cos\vartheta';\\ a_{\zeta_1} &= -a_x\psi'\cos\vartheta' + a_y(\gamma' + \psi'\sin\vartheta') + a_z. \end{aligned}\right\} \qquad (4.53)$$

For objects with a medium action radius where ($\psi < 1°$, $\vartheta < 1°$; $\gamma < 1°$) acceleration values may be determined in accordance with the following equations /193

$$\left.\begin{aligned} a_{\xi_1} &= a_x\cos\psi'\cos\vartheta' + a_y(\sin\psi'\sin\gamma' - \cos\psi'\cos\gamma'\sin\vartheta') + \\ &\quad + a_z(\psi' + \gamma'\sin\vartheta');\\ a_{\eta_1} &= a_x\sin\vartheta' + a_y\cos\vartheta'\cos\gamma' - a_z\gamma'\cos\vartheta';\\ a_{\zeta_1} &= -a_x\psi'\cos\vartheta' + a_y(\cos\psi'\sin\gamma' + \sin\psi'\sin\vartheta'\cos\gamma') + \\ &\quad + a_z\cos\psi'\cos\gamma'. \end{aligned}\right\} \qquad (4.54)$$

Based on the foregoing, it is possible to recommend the following INS kinematic equation system for objects in motion near the arbitrary equator with sufficiently refined angular position stabilization on the basic segment of the trajectory:

1. For short-range objects

$$
\begin{aligned}
&\dot{V}_\xi = a_\xi - 2\Omega_{\eta_0} V_\zeta;\\
&\dot{V}_\zeta = a_\zeta + 2V_\xi(\Omega_{\eta_0} + \Omega_{\xi_0}\dot{\lambda}_{\text{arb}});\\
&a_\xi = a_{\xi_1}\cos\dot{\vartheta}'' + a_{\eta_1}\sin\vartheta'' - a_{\zeta_1}\dot{\psi}''\cos\vartheta'';\\
&a_\zeta = a_{\xi_1}(\dot{\psi}'' + \gamma''\sin\vartheta'') - a_{\eta_1}\gamma''\cos\vartheta'' + a_{\zeta_1};\\
&a_{\xi_1} = a_x\cos\vartheta' + a_y(\dot{\psi}'\gamma' - \sin\vartheta) + a_z(\dot{\psi}' + \gamma'\sin\vartheta');\\
&a_{\eta_1} = a_x\sin\vartheta' + a_y\cos\vartheta - a_z\gamma'\cos\vartheta';\\
&a_{\zeta_1} = -a_x\dot{\psi}'\cos\vartheta' + a_y(\gamma' + \psi\sin\vartheta') + a_z;\\
&\dot{\psi}'' = \frac{1}{\cos\vartheta''}[(\Omega_{\xi_0}\dot{\lambda}_{\text{arb}} + \Omega_{\eta_0})\cos\gamma'' - (\Omega_{\zeta_0} - \dot{\lambda}_{\text{arb}})\sin\gamma''];\\
&\dot{\vartheta}'' = (\Omega_{\xi_0}\dot{\lambda}_{\text{arb}} + \Omega_{\eta_0})\sin\gamma'' + (\Omega_{\zeta_0} - \dot{\lambda}_{\text{arb}})\cos\gamma'';\\
&\dot{\gamma}'' = \Omega_{\zeta_0} - \Omega_{\eta_0}\dot{\lambda}_{\text{arb}} - \dot{\varphi}_{\text{arb}} - [(\Omega_{\xi_0}\dot{\lambda}_{\text{arb}} + \Omega_{\eta_0})\cos\gamma'' - (\Omega_{\xi_0} - \dot{\lambda}_{\text{arb}})\times\\
&\quad\times\sin\gamma'']\tan\vartheta''
\end{aligned}
\tag{4.55}
$$

2. For medium-range objects

$$
\begin{aligned}
&\dot{V}_\xi = a_\xi - \frac{V_\xi V_\eta}{R_{\text{ea}}} - 2(\Omega_\eta V_\zeta - \Omega_\zeta V_\eta);\\
&\dot{V}_\eta = a_\eta + \frac{V_\xi^2}{R_{\text{ea}}} - 2(\Omega_\zeta V_\xi - \Omega_\xi V_\zeta) + g_\eta;\\
&\dot{V}_\zeta = a_\zeta - \frac{V_\eta V_\zeta}{R_{\text{ea}}} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi);\\
&a_\xi = a_{\xi_1}\cos\dot{\psi}''\cos\vartheta'' + a_{\eta_1}\sin\vartheta'' - a_{\zeta_1}\dot{\psi}''\cos\vartheta'';\\
&a_\eta = a_{\xi_1}(\dot{\psi}''\gamma'' - \sin\vartheta'') + a_{\eta_1}\cos\vartheta\cos\gamma'' + a_{\zeta_1}(\gamma'' + \dot{\psi}''\sin\vartheta'');\\
&a_\zeta = a_{\xi_1}(\dot{\psi}'' + \gamma''\sin\vartheta'') - a_{\eta_1}\gamma''\cos\vartheta'' + a_{\zeta_1}\cos\dot{\psi}''\cos\gamma'';\\
&a_{\xi_1} = a_x\cos\dot{\psi}'\cos\vartheta' + a_y(\sin\dot{\psi}'\sin\gamma' - \cos\dot{\psi}'\cos\gamma'\sin\vartheta') +\\
&\qquad + a_z(\dot{\psi}' + \dot{\gamma}'\sin\vartheta');\\
&a_{\eta_1} = a_x\sin\vartheta' + a_y\cos\vartheta'\cos\gamma' - a_z\gamma'\cos\vartheta';\\
&a_{\zeta_1} = -a_x\dot{\psi}'\cos\vartheta' + a_y(\cos\dot{\psi}'\sin\gamma' + \sin\dot{\psi}'\sin\vartheta'\cos\gamma') +\\
&\qquad + a_z\cos\dot{\psi}'\cos\gamma';\\
&\dot{\lambda}_{\text{arb}} = \frac{1}{R_{\text{ea}} + H}V_\xi;\\
&\dot{H} = V_\eta;\\
&\dot{\varphi}_{\text{arb}} = -\frac{1}{R_{\text{ea}}}V_\zeta;\\
&\dot{\psi}'' = \frac{1}{\cos\vartheta''}[\Omega_\eta\cos\gamma'' - \Omega_\zeta\sin\gamma'' + \dot{\lambda}_{\text{arb}}\sin(\varphi_{\text{arb}} + \gamma'')];
\end{aligned}
\tag{4.56}
$$

$$
\begin{aligned}
&\dot{\vartheta}'' = \Omega_\eta \sin\gamma'' + (\Omega_\zeta - \dot{\lambda}_{arb}) \cos\gamma'';\\
&\dot{\gamma}'' = \Omega_\xi - \dot{\varphi}_{arb} - [\Omega_\eta \cos\gamma'' - \Omega_\zeta \sin\gamma'' + \lambda_{arb} \sin(\varphi_{arb} + \gamma'')] \operatorname{tan}\vartheta'';\\
&\Omega_\xi = \Omega_{\xi_0} \cos\lambda_{arb} - \Omega_{\eta_0} \sin\lambda_{arb};\\
&\Omega_\eta = \Omega_{\xi_0} \sin\lambda_{arb} + \Omega_{\eta_0} \cos\lambda_{arb} - \Omega_{\zeta_0}\varphi_{arb};\\
&\Omega_\zeta = (\Omega_{\xi_0} \sin\lambda_{arb} + \Omega_{\eta_0} \cos\lambda_{arb})\varphi_{arb} + \Omega_{\zeta_0}.
\end{aligned}
\tag{4.56}
$$

It is obvious that an INS constructed in accordance with kinematic equations (4.55) may be a two-component system for the determination of velocity and acceleration components, since the first two equations of the system do not contain a vertical velocity component. /194

In the remaining equations cross links occur along all parameters; therefore there must be three accelerometers and three angle gauges in the system.

An INS constructed in accordance with kinematic equations (4.56) must be a three-component system, since all of the equations have cross links with the corresponding parameters.

§4.4. INS Systematic Error Equations.

Let us examine the systematic errors in all three INS variants without a platform, caused by kinematic equation simplifications.

Systematic errors may be calculated both by the solution of the simplified nonlinear INS kinematic equation system, as well as by the solution of the systematic error equation systems which are derived by linearization of the former. /195

The derivation of the equations indicated as well as the technique for the calculation of errors are similar to the procedures in Section 2.5.

The kinematic equations of a non-platform INS of the second design variant for short-range objects, with consideration for systematic errors, are derived from equations (4.42) and have the following form:

$$
\begin{aligned}
&\dot{V}^*_\xi = a^*_\xi - 2\Omega_{\eta_0} V^*_\zeta;\\
&\dot{V}^*_\zeta = a_\zeta + 2V^*_\xi(\Omega_{\xi_0}\lambda^*_{arb} + \Omega_{\eta_0});\\
&a^*_\xi = a_x \cos\psi^*_a \cos\vartheta^*_a + a_y(\sin\psi^*_a \sin\gamma^*_a - \cos\psi^*_a \sin\vartheta^*_a \times\\
&\quad \times \cos\gamma^*_a) + a_z(\sin\psi^*_a \cos\gamma^*_a + \cos\psi^*_a \sin\vartheta^*_a \sin\gamma^*_a);\\
&a^*_\zeta = -a_x \sin\psi^*_a \cos\vartheta^*_a + a_y(\cos\psi^*_a \sin\gamma^*_a + \sin\psi^*_a \sin\vartheta^*_a \times\\
&\quad \times \cos\gamma^*_a) + a_z(\cos\psi^*_a \cos\gamma^*_a - \sin\psi^*_a \sin\vartheta^*_a \sin\gamma^*_a);
\end{aligned}
\tag{4.57}
$$

$$
\begin{aligned}
&\dot{\lambda}^*_{\text{arb}} = \frac{V^*_\xi}{R_{\text{ea}}};\\
&\dot{\varphi}^*_{\text{arb}} = -\frac{V^*_\zeta}{R_{\text{ea}}};\\
&\dot{\psi}^*_a = \frac{1}{\cos\vartheta^*_a}(\omega_{ay}\cos\gamma^*_a - \omega_{az}\sin\gamma^*_a + \omega^*_\xi\cos\psi^*_a\sin\vartheta^*_a -\\
&\quad - \omega^*_\eta\cos\vartheta^*_a - \omega^*_\zeta\sin\psi^*_a\sin\vartheta^*_a);\\
&\dot{\vartheta}^*_a = \omega_{ay}\sin\gamma^*_a + \omega_{az}\cos\gamma^*_a - \omega^*_\xi\sin\psi^*_a - \omega^*_\zeta\cos\psi^*_a;\\
&\dot{\gamma}^*_a = \omega_{ax} - (\omega_{ay}\cos\gamma^*_a - \omega_{az}\sin\gamma^*_a)\tan\vartheta^*_a -\\
&\quad - \omega^*_\xi\cos\psi^*_a\sec\vartheta^*_a + \omega^*_\zeta\sin\psi^*_a\sec\vartheta^*_a;\\
&\omega^*_\xi = \Omega_{\xi_0} - \Omega_{\eta_0}\lambda^*_{\text{arb}} - \dot{\varphi}^*_{\text{arb}};\\
&\omega^*_\eta = \Omega_{\xi_0}\lambda^*_{\text{arb}} + \Omega_{\eta_0};\\
&\omega^*_\zeta = \Omega_{\zeta_0} - \dot{\lambda}^*_{\text{arb}}
\end{aligned}
\qquad (4.57)
$$

The solution of system (4.57) determines kinematic parameter instrument values of object motion.

The basic data for the solution of system (4.57) are furnished by the true acceleration values a_x, a_y, a_z and the true values of angular velocities ω_{ax}, ω_{ay}, ω_{az}, measured by the corresponding system instruments, and also by the initial values of the kinematic parameters desired.

The time dependencies of accelerations a_x, a_y, a_z and angular velocities ω_{ax}, ω_{ay}, ω_{az} are preliminarily determined by solving the kinematic equation system (4.27) in accordance with known values $\dot{\psi}_a(t)$, $\dot{\vartheta}_a(t)$, $\dot{\gamma}_a(t)$, $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$, established by the kinematics of the object programmed trajectory. /196

The kinematic equations of the INS under consideration, intended for medium range objects, and considering systematic errors, may be derived from the system (4.43)

$$
\begin{aligned}
&\dot{V}^*_\xi = a^*_\xi - \frac{V^*_\xi V^*_\eta}{R_{\text{ea}}} - 2(\Omega^*_\eta V^*_\zeta - \Omega^*_\zeta V^*_\eta);\\
&\dot{V}^*_\eta = a^*_\eta + \frac{V^{*2}_\xi}{R_{\text{ea}}} - 2(\Omega^*_\zeta V^*_\xi - \Omega^*_\xi V^*_\zeta) + g_\eta;\\
&\dot{V}^*_\zeta = a^*_\zeta - \frac{V^*_\eta V^*_\xi}{R_{\text{ea}}} - 2(\Omega^*_\xi V^*_\eta - \Omega^*_\eta V^*_\xi);
\end{aligned}
\qquad (4.58)
$$

$$
\begin{aligned}
a^*_\xi &= a_x \cos\psi^*_a \cos\vartheta^*_a + a_y(\sin\psi^*_a \sin\gamma^*_a - \cos\psi^*_a \sin\vartheta^*_a \cos\gamma^*_a) + \\
&+ a_z(\sin\psi^*_a \cos\gamma^*_a - \cos\psi^*_a \sin\vartheta^*_a \cos\gamma^*_a); \\
a^*_\eta &= a_x \sin\vartheta^*_a + a_y \cos\vartheta^*_a \cos\gamma^*_a - a_z \cos\vartheta^*_a \sin\gamma^*_a; \\
a^*_\zeta &= -a_x \sin\psi^*_a \cos\vartheta^*_a + a_y(\cos\psi^*_a \sin\gamma^*_a + \sin\psi^*_a \sin\vartheta^*_a \cos\gamma^*_a) + \\
&+ a_z(\cos\psi^*_a \cos\gamma^*_a - \sin\psi^*_a \sin\vartheta^*_a \sin\gamma^*_a); \\
\dot\lambda^*_{arb} &= \frac{1}{R_{ea} + H^*} V^*_\xi; \\
\dot H^* &= V^*_\eta; \\
\dot\varphi^*_{arb} &= -\frac{1}{R_{ea}} V^*_\zeta; \\
\dot\psi^*_a &= \frac{1}{\cos\vartheta^*_a}[\omega_{ay}\cos\gamma^*_a - \omega_{az}\sin\gamma^*_a + (\Omega^*_\xi - \dot\varphi^*_{arb})\cos\psi^*_a \sin\vartheta^*_a - \\
&- \Omega^*_\eta \cos\vartheta^*_a - (\Omega^*_\zeta - \dot\lambda^*_{arb})\sin\psi^*_a \sin\vartheta^*_a]; \\
\dot\vartheta^*_a &= \omega_{ay}\sin\gamma^*_a + \omega_{az}\cos\gamma^*_a - (\Omega^*_\xi - \dot\varphi^*_{arb})\sin\psi^*_a - \\
&- (\Omega^*_\zeta - \dot\lambda^*_{arb})\cos\psi^*_a; \\
\dot\gamma^*_a &= \omega_{ax} - (\omega_{ay}\cos\gamma^*_a - \omega_{az}\sin\gamma^*_a)\tan\vartheta^*_a - \\
&- (\Omega^*_\xi - \dot\varphi^*_{arb})\cos\psi^*_a \sec\vartheta^*_a + (\Omega^*_\zeta - \dot\lambda^*_{arb})\sin\psi^*_a \sec\vartheta^*_a; \\
\Omega^*_\xi &= \Omega_{\xi 0}\cos\lambda^*_{arb} - \Omega_{\eta 0}\sin\lambda^*_{arb}; \\
\Omega^*_\eta &= \Omega_{\xi 0}\sin\lambda^*_{arb} + \Omega_{\eta 0}\cos\lambda^*_{arb} - \Omega_{\zeta 0}\varphi^*_{arb}; \\
\Omega^*_\zeta &= (\Omega_{\xi 0}\sin\lambda^*_{arb} + \Omega_{\eta 0}\cos\lambda^*_{arb})\varphi^*_{arb} + \Omega_{\zeta 0}.
\end{aligned}
\tag{4.58}
$$

The calculation of systematic errors for an INS which is constructed in accordance with the kinematic equation system (4.43) is similar to the calculation of these errors for an INS which utilizes the system (4.42). /197

The kinematic equations for an INS of the first design variant, with consideration for systematic errors, will be exactly the same as the kinematic equations for the second variant. However, in calculating parameter instrument values in accordance with these equations we must determine in advance the instantaneous acceleration values a'_x, a''_x, a'_y, a''_y, a'_z, a''_z, in terms of the kinematics of object motion, and then, utilizing equations (4.29) and (4.30), we find the initial values a_x, a_y, a_z, ω_{ax}, ω_{ay}, ω_{az}.

The kinematic equations of an INS of the third design variant may be derived as in (4.58). These equations, as applied to short-range objects, will have the form:

$$
\begin{aligned}
&\dot V^*_\xi = a^*_\xi - 2\Omega_{\eta 0} V^*_\zeta;\\
&\dot V^*_\zeta = a^*_\zeta + 2V^*_\xi(\Omega_{\eta 0} + \Omega_{\xi 0}\lambda^*_y);\\
&a^*_\xi = a_{\xi_1}\cos\vartheta''^* + a_{\eta_1}\sin\vartheta''^* - a_{\zeta_1}\dot\psi''^*\cos\vartheta''^*;\\
&a^*_\zeta = a_{\xi_1}(\dot\psi''^* + \gamma''^*\sin\vartheta''^*) - a_{\eta_1}\gamma''^*\cos\vartheta''^* + a_{\zeta_1};\\
&a^*_{\xi_1} = a_x\cos\vartheta'^* + a_y(\dot\psi'^*\gamma'^* - \sin\vartheta'^*) + a_z(\dot\psi'^* + \gamma'^*\sin\vartheta'^*);\\
&a^*_{\eta_1} = a_x\sin\vartheta'^* + a_y\cos\vartheta'^* - a_z\gamma'^*\cos\vartheta'^*;\\
&a^*_{\zeta_1} = -a_x\dot\psi'^*\cos\vartheta'^* + a_y(\gamma'^* + \dot\psi'^*\sin\vartheta'^*) + a_z;\\
&\dot\psi''^* = \frac{1}{\cos\vartheta''^*}\left[(\Omega_{\xi 0}\lambda^*_{\text{arb}} + \Omega_{\eta 0})\cos\gamma''^* - (\Omega_{\zeta 0} - \dot\lambda^*_{\text{arb}})\sin\gamma''^*\right];\\
&\dot\vartheta''^* = (\Omega_{\xi 0}\lambda^*_{\text{arb}} + \Omega_{\eta 0})\sin\gamma''^* + (\Omega_{\zeta 0} - \dot\lambda^*_{\text{arb}})\cos\gamma''^*;\\
&\dot\gamma''^* = \Omega_{\xi 0} - \Omega_{\eta 0}\lambda^*_{\text{arb}} - \dot\varphi^*_{\text{arb}}\left[(\Omega_{\xi 0}\lambda^*_{\text{arb}} + \Omega_{\eta 0})\cos\gamma''^* - \right.\\
&\quad \left. - (\Omega_{\zeta 0} - \dot\lambda^*_{\text{arb}})\sin\gamma''^*\right]\tan\vartheta''^*,
\end{aligned}
\tag{4.59}
$$

where Ω_{ξ_0}, Ω_{η_0}, Ω_{ζ_0} are determined from equations (2.46).

In turn, the kinematic equations of an INS of the third design variant, intended for medium-range objects (4.56), will have the form:

$$
\begin{aligned}
&\dot V^*_\xi = a^*_\xi - \frac{V^*_\xi V^*_\eta}{R_{\text{ea}}} - 2(\Omega^*_\eta V^*_\zeta - \Omega^*_\zeta V^*_\eta);\\
&\dot V^*_\eta = a^*_\eta + \frac{V^{*2}_\xi}{R_{\text{ea}}} - 2(\Omega^*_\zeta V^*_\xi - \Omega^*_\xi V^*_\zeta) + g_\eta;\\
&\dot V^*_\zeta = a^*_\zeta - \frac{V^*_\eta V^*_\zeta}{R_{\text{ea}}} - 2(\Omega^*_\xi V^*_\eta - \Omega^*_\eta V^*_\xi);\\
&a^*_\xi = a^*_{\xi_1}\cos\dot\psi''^*\cos\vartheta''^* + a^*_{\eta_1}\sin\vartheta''^* - a^*_{\zeta_1}\dot\psi''^*\cos\vartheta''^*;\\
&a^*_\eta = a^*_{\xi_1}(\dot\psi''^*\gamma''^* - \sin\vartheta''^*) + a^*_{\eta_1}\cos\vartheta''^*\cos\gamma''^* +\\
&\quad + a^*_{\zeta_1}(\gamma''^* + \dot\psi''^*\sin\vartheta''^*);\\
&a^*_\zeta = a^*_{\xi_1}(\dot\psi''^* + \gamma''^*\sin\vartheta''^*) - a^*_{\eta_1}\gamma''^*\cos\vartheta''^* + a^*_{\zeta_1}\cos\dot\psi''^*\cos\gamma''^*;\\
&a^*_{\xi_1} = a_x\cos\dot\psi'^*\cos\vartheta'^* + a_y(\sin\dot\psi'^*\sin\gamma'^* -\\
&\quad - \cos\dot\psi'^*\cos\gamma'^*\sin\vartheta'^*) + a_z(\dot\psi'^* + \gamma'^*\sin\vartheta'^*);\\
&a^*_{\eta_1} = a_x\sin\vartheta'^* + a_y\cos\vartheta'^*\cos\gamma'^* - a_z\gamma'^*\cos\vartheta'^*;\\
&a^*_{\zeta_1} = -a_x\dot\psi'^*\cos\vartheta'^* + a_y(\cos\dot\psi'^*\sin\gamma'^* +\\
&\quad + \sin\dot\psi'^*\sin\vartheta'^*\cos\gamma'^*) + a_z\cos\dot\psi'^*\cos\gamma'^*;
\end{aligned}
\tag{4.60}
$$

$$
\begin{aligned}
&\dot{\lambda}^*_{\text{arb}} = \frac{1}{R_{\text{ea}} + H^*} V^*_\xi;\\
&\dot{H}^* = V^*_\eta;\\
&\dot{\varphi}^*_{\text{arb}} = -\frac{V^*_\zeta}{R_{\text{ea}}};\\
&\dot{\psi}''^* = \frac{1}{\cos\vartheta''^*}[\Omega^*_{\eta_1}\cos\gamma''^* - \Omega^*_\zeta \sin\gamma''^* + \dot{\lambda}^*_{\text{arb}}\sin(\varphi^*_{\text{arb}} + \gamma''^*)];\\
&\dot{\vartheta}''^* = \Omega^*_{\eta_1}\sin\gamma''^* + (\Omega^*_\zeta - \dot{\lambda}^*_{\text{arb}})\cos\gamma''^*;\\
&\dot{\gamma}''^* = \Omega^*_\xi - \dot{\varphi}^*_{\text{arb}}[\Omega^*_{\eta_1}\cos\gamma''^* - \Omega^*_\zeta\sin\gamma''^* +\\
&\quad + \dot{\lambda}^*_{\text{arb}}\sin(\varphi^*_{\text{arb}} + \gamma''^*)]\tan\vartheta''^*;\\
&\Omega^*_\zeta = \Omega_{\xi 0}\cos\lambda^*_{\text{arb}} - \Omega_{\eta 0}\sin\lambda^*_{\text{arb}};\\
&\Omega^*_\eta = \Omega_{\xi 0}\sin\lambda^*_{\text{arb}} + \Omega_{\eta 0}\cos\lambda^*_{\text{arb}} - \Omega_{\zeta 0}\varphi^*_{\text{arb}};\\
&\Omega^*_\zeta = (\Omega_{\xi 0}\sin\lambda^*_{\text{arb}} + \Omega_{\eta 0}\cos\lambda^*_{\text{arb}})\varphi^*_{\text{arb}} + \Omega_{\zeta 0}.
\end{aligned}
\tag{4.60}
$$

The basic data for the solution of INS kinematic equation systems (4.59) and (4.60) are accelerations a_x, a_y, a_z and angles ψ', ϑ', γ', as measured by appropriate instruments. /198

The true values of these accelerations and angles are determined by solving the complete INS kinematic equation system (4.34).

Systematic error equations for an INS constructed in accordance with the second design variant, as applied to short-range objects, are derived through the linearization of system (4.42) and have the following form:

$$
\begin{aligned}
&p\Delta V_{\xi\,\text{sys}} + a_{1V_\zeta}\Delta V_{\zeta\,\text{sys}} + a_{1a_\zeta}\Delta V_{\xi\,\text{sys}} = \delta\dot{V}_{\xi\,\text{sys}};\\
&a_{2V_\xi}\Delta V_{\xi\,\text{sys}} + p\Delta V_{\zeta\,\text{sys}} + a_{2\lambda_{\text{arb}}}\Delta\lambda_{\text{arb, sys}} + a_{2a_\zeta}\Delta a_{\zeta\,\text{sys}} = \delta\dot{V}_{\zeta\,\text{sys}};\\
&a_{3a_\xi}\Delta a_{\xi\,\text{sys}} + a_{3\psi}\Delta\psi_{\text{a, sys}} + a_{3\vartheta}\Delta\vartheta_{\text{a, sys}} + a_{3\gamma}\Delta\gamma_{\text{a, sys}} = \delta\dot{a}_{\xi\,\text{sys}};\\
&a_{4a_\zeta}\Delta a_{\zeta\,\text{sys}} + a_{4\psi}\Delta\psi_{\text{a, sys}} + a_{4\vartheta}\Delta\vartheta_{\text{a, sys}} + a_{4\gamma}\Delta\gamma_{\text{a, sys}} = \delta\dot{a}_{\zeta\,\text{sys}};\\
&a_{5V_\xi}\Delta V_{\xi\,\text{sys}} + p\Delta\lambda_{\text{arb, sys}} = \delta\dot{\lambda}_{\text{sys arb}};\\
&a_{6V_\zeta}\Delta V_{\zeta\,\text{sys}} + p\Delta\varphi_{\text{arb, sys}} = \delta\dot{\varphi}_{\text{sys arb}};\\
&(p + a_{7\psi})\Delta\psi_{\text{a, sys}} + a_{7\vartheta}\Delta\vartheta_{\text{a, sys}} + a_{7\gamma}\Delta\gamma_{\text{a, sys}} + a_{7\omega_\xi}\Delta\omega_{\xi\,\text{sys}} +\\
&\quad + a_{7\omega_{\eta_1}}\Delta\omega_{\eta_1\,\text{sys}} + a_{7\omega_\zeta}\Delta\omega_{\zeta\,\text{sys}} = \delta\dot{\psi}_{\text{a sys}};\\
&a_{8\psi}\Delta\psi_{\text{a, sys}} + p\Delta\vartheta_{\text{a, sys}} + a_{8\gamma}\Delta\gamma_{\text{a, sys}} + a_{8\omega_\xi}\Delta\omega_{\xi\,\text{sys}} + a_{8\omega_\zeta}\Delta\omega_{\zeta\,\text{sys}} = \delta\dot{\vartheta}_{\text{a sys}};
\end{aligned}
\tag{4.61}
$$

$$
\begin{aligned}
&a_{9\psi}\Delta\psi_{a,\,sys}+a_{9\vartheta}\Delta\vartheta_{a,\,sys}+(p+a_{9\gamma})\Delta\gamma_{a,\,sys}+a_{9\omega_\xi}\Delta\omega_{\xi\,sys}+\\
&\quad+a_{9\omega_\zeta}\Delta\omega_{\zeta\,sys}=\delta\vartheta_{a\,sys};\\
&a_{10\lambda}\Delta\lambda_{arb,\,sys}+p\Delta\varphi_{arb,\,sys}+a_{10\omega_\xi}\Delta\omega_{\zeta}=\delta\omega_{\xi\,sys};\\
&a_{11\lambda}\Delta\lambda_{arb,\,sys}+a_{11\omega_\gamma}\Delta\omega_{\eta}=\delta\omega_{\gamma\,sys};\\
&p\Delta\lambda_{arb,\,sys}+a_{12\omega_\zeta}\Delta\omega_{\zeta}=\delta\omega_{\zeta\,sys},
\end{aligned}
\qquad (4.61)
$$

where equation coefficients (4.61) are determined as: /199

$$
\begin{aligned}
&a_{1V\zeta}=2\Omega_{\gamma_i 0};\\
&a_{1a\xi}=-1;\\
&a_{2V\xi}=-2(\Omega_{\xi 0}\lambda_{arb}+\Omega_{\gamma_i 0});\\
&a_{2a\zeta}=-1;\\
&a_{2\lambda}=-2V_\xi\Omega_{\xi 0};\\
&a_{3a\xi}=1;\\
&a_{3\psi}=a_x\sin\psi_a\cos\vartheta_a-a_y(\cos\psi_a\sin\gamma_a+\sin\psi_a\sin\vartheta_a\cos\gamma_a)-\\
&\quad-a_z(\cos\psi_a\cos\gamma_a-\sin\psi_a\sin\vartheta_a\sin\gamma_a);\\
&a_{3\vartheta}=a_x\cos\psi_a\sin\vartheta_a+a_y\cos\psi_a\cos\vartheta_a\cos\gamma_a\\
&\quad-a_z\cos\psi_a\cos\vartheta_a\sin\gamma_a;\\
&a_{3\gamma}=-a_y(\sin\psi_a\cos\gamma_a+\cos\psi_a\sin\vartheta_a\sin\gamma_a)\\
&\quad-a_z(\cos\psi_a\sin\vartheta_a\cos\gamma_a-\sin\psi_a\sin\gamma_a);\\
&a_{4a\zeta}=1;\\
&a_{4\psi}=a_x\cos\psi_a\cos\vartheta_a-a_y(\cos\psi_a\sin\vartheta_a\cos\gamma_a+\sin\psi_a\sin\gamma_a)+\\
&\quad+a_z(\sin\psi_a\cos\gamma_a+\cos\psi_a\sin\vartheta_a\sin\gamma_a);\\
&a_{4\vartheta}=-a_x\sin\psi_a\sin\vartheta_a-a_y\sin\psi_a\cos\vartheta_a\cos\gamma_a+\\
&\quad+a_z\sin\psi_a\cos\vartheta_a\sin\gamma_a;\\
&a_{4\gamma}=-a_y(\cos\psi_a\cos\gamma_a-\sin\psi_a\sin\vartheta_a\sin\gamma_a)+\\
&\quad+a_z(\sin\psi_a\cos\gamma_a+\sin\psi_a\sin\vartheta_a\cos\gamma_a);\\
&a_{5V\xi}=-\frac{1}{R_{ea}};\qquad a_{6V\zeta}=\frac{1}{R_{ea}};\\
&a_{7\psi}=(\omega_\xi\sin\psi_a+\omega_\zeta\cos\psi_a)\tan\vartheta_a;\\
&a_{7\omega_\xi}=-\cos\psi_a\tan\vartheta_a;\\
&a_{7\omega_\zeta}=\sin\psi_a\tan\vartheta_a;\quad a_{7\omega_\eta}=1;
\end{aligned}
\qquad (4.62)
$$

$$
\begin{aligned}
a_{7\vartheta} &= -\left\{\frac{1}{\cos\vartheta_a}\left[\omega_\xi\cos\psi_a\cos\vartheta_a+\omega_\eta\sin\vartheta_a-\omega_\zeta\sin\psi_a\cos\vartheta_a\right]+\right.\\
&\quad+\frac{\sin\vartheta_a}{\cos^2\vartheta_a}\left(\omega_{ay}\cos\gamma_a-\omega_{az}\sin\gamma_a+\omega_\xi\cos\psi_a\sin\vartheta_a-\right.\\
&\quad\left.\left.-\omega_\eta\cos\vartheta_a-\omega_\zeta\sin\psi_a\sin\vartheta_a\right)\right\};\\
a_{7\gamma} &= \frac{1}{\cos\vartheta_a}\left(\omega_{ay}\sin\gamma_a+\omega_{az}\cos\gamma_a\right);\\
a_{8\psi} &= \omega_\xi\cos\psi_a-\omega_\zeta\sin\psi_a;\\
a_{8\gamma} &= \omega_{az}\sin\gamma_a-\omega_{ay}\cos\gamma_a;\\
a_{9\psi} &= -\omega_\xi\sin\psi_a\sec\vartheta_a-\omega_\zeta\cos\psi_a\sec\vartheta_a;\\
a_{9\vartheta} &= \frac{1}{\cos^2\vartheta_a}\left\{\left(\omega_{ay}\cos\gamma_a-\omega_{az}\sin\gamma_a\right)+\omega_\xi\cos\psi_a\sin\vartheta_a-\right.\\
&\quad\left.-\omega_\zeta\sin\psi_a\sin\vartheta_a\right)\right\};\\
a_{9\gamma} &= -\left(\omega_{az}\cos\gamma_a+\omega_{ay}\sin\gamma_a\right)\tan\vartheta_a;\\
a_{9\omega_\zeta} &= -\sin\psi_a\sec\vartheta_a;\\
a_{10\lambda} &= \Omega_{\tau 0};\\
a_{10\omega_\xi} &= 1;\\
a_{11\lambda} &= -\Omega_{\xi 0};\\
a_{11\omega_\tau} &= 1;\\
a_{12\omega_\zeta} &= 1.
\end{aligned}
\tag{4.62}
$$

Partial systematic errors may be calculated in accordance with the following expressions: /201

$$
\begin{aligned}
\underset{\text{sys}}{\delta}\dot V_\xi &= \frac{V_\xi V_\eta}{R_{ea}+H}+\frac{V_\xi V_\zeta}{R_{ea}+H}\tan\varphi_{arb}-2\left[\left(\Omega_{\tau 0}-\Omega_\eta\right)V_\zeta+\Omega_\zeta V_\eta\right];\\
\underset{\text{sys}}{\delta}\dot V_\zeta &= -\frac{V_\xi^2}{R_{ea}+H}\tan\varphi_{arb}+\frac{V_\eta V_\xi}{R_{ea}+H}+2\Omega_\xi V_\eta;\\
\underset{\text{sys}}{\delta}a_\xi &= 0;\quad \underset{\text{sys}}{\delta}a_\zeta = 0;\\
\underset{\text{sys arb}}{\delta}\dot\lambda &\cong V_\xi\frac{H}{\left(R_{ea}+H\right)R_{ea}\cos\varphi_{arb}}\cong V_\xi\frac{H}{R_{ea}^2};\\
\underset{\text{sys arb}}{\delta}\dot\varphi &= -\frac{H}{R_{ea}\left(R_{ea}+H\right)}V_\zeta\cong -V_\zeta\frac{H}{R_{ea}^2};\\
\underset{\text{sys}}{\delta}\dot\psi_a &= 0;\\
\underset{\text{sys}}{\delta}\dot\vartheta_a &= 0;\\
\underset{\text{sys}}{\delta}\dot\gamma_a &= 0;
\end{aligned}
\tag{4.63}
$$

$$
\begin{aligned}
&\delta_{sys}\Omega_\xi = \Omega_{\xi 0}(1-\cos\lambda_{arb}) - \Omega_{\eta 0}(\lambda_{arb} - \sin\lambda_{arb});\\
&\delta_{sys}\Omega_\eta = \Omega_{\xi 0}(\lambda_{arb} - \sin\lambda_{arb}\cos\varphi_{arb}) + \Omega_{\eta 0}(1-\cos\lambda_{arb}\cos\varphi_{arb}) -\\
&\qquad - \Omega_{\zeta 0}\sin\varphi_{arb}\\
&\delta_{sys}\Omega_\zeta = -(\Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb})\sin\varphi_{arb} + \Omega_{\zeta 0}(1-\cos\varphi_{arb}).
\end{aligned}
\tag{4.63}
$$

In turn, systematic error equations for an INS of the second design variant for medium-range objects (4.56) have the form

$$
\begin{aligned}
&(p + a_{1V_\xi})\Delta V_{\xi\,sys} + a_{1V_\eta}\Delta V_{\eta\,sys} + a_{1V_\zeta}\Delta V_{\zeta\,sys} + a_{1a_\xi}\Delta a_{\xi\,sys} +\\
&\qquad + a_{1\Omega_\eta}\Delta\Omega_{\eta\,sys} + a_{1\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot V_\xi;\\
&a_{2V_\xi}\Delta V_{\xi\,sys} + p\Delta V_{\eta\,sys} + a_{2V_\zeta}\Delta V_{\zeta\,sys} + a_{2a_\eta}\Delta a_{\eta\,sys} + a_{2\Omega_\xi}\Delta\Omega_{\xi\,sys} +\\
&\qquad + a_{2\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot V_\eta;\\
&a_{3V_\xi}\Delta V_{\xi\,sys} + a_{3V_\eta}\Delta V_{\eta\,sys} + (p + a_{3V_\zeta})\Delta V_{\zeta\,sys} + a_{3a_\zeta}\Delta a_{\zeta\,sys} +\\
&\qquad + a_{3\Omega_\xi}\Delta\Omega_{\xi\,sys} + a_{3\Omega_\eta}\Delta\Omega_{\eta\,sys} = \delta_{sys}\dot V_\zeta;\\
&a_{4a_\xi}\Delta a_{\xi\,sys} + a_{4\psi}\Delta\psi_{a,sys} + a_{4\vartheta}\Delta\vartheta_{a,sys} + a_{4\gamma}\Delta\gamma_{a,sys} = \delta_{sys}a_\xi;\\
&a_{5a_\eta}\Delta a_{\eta\,sys} + a_{5\vartheta}\Delta\vartheta_{a,sys} + a_{5\gamma}\Delta\gamma_{a,sys} = \delta_{sys}a_\eta;\\
&a_{6a_\zeta}\Delta a_{\zeta\,sys} + a_{6\psi}\Delta\psi_{a,sys} + a_{6\vartheta}\Delta\vartheta_{a,sys} + a_{6\gamma}\Delta\gamma_{a,sys} = \delta_{sys}a_\zeta;\\
&a_{7V_\xi}\Delta V_{\xi\,sys} + p\Delta\lambda_{arb,sys} + a_{7H}\Delta H_{sys} = \delta_{sys}\dot\lambda_{arb};\\
&a_{8V_\eta}\Delta V_{\eta\,sys} + p\Delta H_{sys} = \delta_{sys}\dot H;\\
&a_{9V_\zeta}\Delta V_{\zeta\,sys} + p\Delta\varphi_{arb,sys} = \delta_{sys}\dot\varphi_{arb};\\
&a_{10\lambda}p\Delta\lambda_{arb,sys} + a_{10\varphi}p\Delta\varphi_{arb,sys} + (p + a_{10\psi})\Delta\psi_{a,sys} + a_{10\vartheta}\Delta\vartheta_{a,sys} -\\
&+ a_{10\gamma}\Delta\gamma_{a,sys} + a_{10\Omega_\xi}\Delta\Omega_{\xi\,sys} + a_{10\Omega_\eta}\Delta\Omega_{\eta\,sys} + a_{10\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot\vartheta_a;\\
&a_{11\lambda}p\Delta\lambda_{arb,sys} + a_{11\varphi}p\Delta\varphi_{arb,sys} + a_{11\psi}\Delta\psi_{arb,sys} + p\Delta\vartheta_{arb,sys} + a_{11\gamma}\Delta\gamma_{arb,sys} -\\
&\qquad + a_{11\Omega_\xi}\Delta\Omega_{\xi\,sys} + a_{11\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\dot\vartheta_a;\\
&a_{12\lambda}p\Delta\lambda_{arb,sys} + a_{12\varphi}p\Delta\varphi_{arb,sys} + a_{12\psi}\Delta\psi_{a,sys} + a_{12\vartheta}\Delta\vartheta_{a,sys} +\\
&\qquad + (p + a_{12\gamma})\Delta\gamma_{a,sys} + a_{12\Omega_\zeta}\Delta\Omega_{\xi\,sys} + a_{12\Omega_\zeta}\Delta\Omega_{\zeta\,sys} =\\
&\quad a_{14\lambda}\Delta\lambda_{arb,sys} + a_{14\varphi}\Delta\varphi_{arb,sys} + a_{14\Omega_\eta}\Delta\Omega_{\eta\,sys} = \delta_{sys}\Omega_\eta;\\
&\quad a_{13\lambda}\Delta\lambda_{arb,sys} + a_{13\Omega_\xi}\Delta\Omega_{\xi\,sys} = \delta_{sys}\Omega_\xi;\\
&a_{15\lambda}\Delta\lambda_{arb,sys} + a_{15\varphi}\Delta\varphi_{arb,sys} + a_{15\Omega_\zeta}\Delta\Omega_{\zeta\,sys} = \delta_{sys}\Omega_\zeta,
\end{aligned}
\tag{4.64}
$$

where the coefficients

$$a_{1V_\xi}=\frac{V_\eta}{R_{ea}};\quad a_{1V_\eta}=\frac{V_\xi}{R_{ea}}-2\Omega_\zeta;\quad a_{1V_\zeta}=2\Omega_\eta;\quad a_{1a_\xi}=-1;$$

$$a_{1\Omega_\eta}=2V_\zeta;\quad a_{1\Omega_\zeta}=-2V_\eta;\quad a_{2V_\xi}=2\left(\Omega_\zeta-\frac{V_\xi}{R_{ea}}\right);$$

$$a_{2V_\zeta}=-2\Omega_\xi;\quad a_{2a_\eta}=-1;\quad a_{2\Omega_\xi}=-2V_\zeta;\quad a_{2\Omega_\zeta}=2V_\xi;$$

$$a_{3V_\xi}=-2\Omega_\eta;\quad a_{3V_\eta}=\frac{1}{R_{ea}}V_\zeta+2\Omega_\xi;\quad a_{3V_\zeta}=\frac{1}{R_{ea}}V_\eta;\quad a_{3a_\zeta}=-1;$$

$$a_{3\Omega_\xi}=2V_\eta;\quad a_{3\Omega_\eta}=-2V_\xi;\quad a_{4a_\xi}=1;$$

$$a_{4\psi}=a_x\sin\psi_a\cos\vartheta_a-a_y(\cos\psi_a\sin\gamma_a+\sin\psi_a\sin\vartheta_a\cos\gamma_a)-$$
$$-a_z(\cos\psi_a\cos\gamma_a-\sin\psi_a\sin\vartheta_a\sin\gamma_a);$$

$$a_{4\vartheta}=a_x\cos\psi_a\sin\vartheta_a+a_y\cos\psi_a\cos\vartheta_a\cos\gamma_a-$$
$$-a_z\cos\psi_a\cos\vartheta_a\sin\gamma_a;$$

$$a_{4\gamma}=-a_y(\sin\psi_a\cos\gamma_a+\cos\psi_a\sin\vartheta_a\sin\gamma_a)-$$
$$-a_z(\cos\psi_a\sin\vartheta_a\cos\gamma_a-\sin\psi_a\sin\gamma_a);$$

$$a_{5a_\eta}=1;\quad a_{5\vartheta}=-a_x\cos\vartheta_a+a_y\sin\vartheta_a\cos\gamma_a-a_z\sin\vartheta_a\sin\gamma_a;$$

$$a_{5\gamma}=a_y\cos\vartheta_a\sin\gamma_a+a_z\cos\vartheta_a\cos\gamma_a;\quad a_{6a_\zeta}=1;$$

$$a_{6\psi}=-a_x\cos\psi_a\cos\vartheta_a-a_y(\cos\psi_a\sin\vartheta_a\cos\gamma_a-\sin\psi_a\sin\gamma_a)+ \qquad (4.65)$$
$$+a_z(\cos\psi_a\sin\vartheta_a\cos\gamma_a+\sin\psi_a\cos\gamma_a);$$

$$a_{6\vartheta}=-a_x\sin\psi_a\sin\vartheta_a-a_y\sin\psi_a\cos\vartheta_a\cos\gamma_a+$$
$$+a_z\sin\psi_a\cos\vartheta_a\sin\gamma_a;$$

$$a_{6\gamma}=-a_y(\cos\psi_a\cos\gamma_a-\sin\psi_a\sin\vartheta_a\sin\gamma_a)+$$
$$+a_z(\cos\psi_a\sin\gamma_a+\sin\psi_a\sin\vartheta_a\cos\gamma_a);$$

$$a_{7V_\xi}=-\frac{1}{R_{ea}+H};\quad a_{7H}=\frac{V_\xi}{(R_{ea}+H)^2};\quad a_{8V_\eta}=-1;\quad a_{9V_\zeta}=\frac{1}{R_{ea}};$$

$$a_{10\lambda}=-\sin\psi_a\tan\vartheta_a;\quad a_{10\varphi}=\cos\psi_a\tan\vartheta_a;$$

$$a_{10\psi}=[(\Omega_\xi-\dot{\varphi}_{arb})\sin\psi_a+(\Omega_\zeta-\dot{\lambda}_{arb})\cos\psi_a]\tan\vartheta_a;$$

$$a_{10\vartheta}=\frac{\sin\vartheta_a}{\cos^2\vartheta_a}[\omega_{ay}\cos\gamma_a-\omega_{az}\sin\gamma_a+(\Omega_\xi-\dot{\varphi}_{arb})\cos\psi_a\sin\vartheta_a-$$

$$-\Omega_\eta \cos\vartheta_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\psi_a \sin\vartheta_a] - \frac{1}{\cos\vartheta_a}[(\Omega_\xi - \dot{\varphi}_{arb})\times$$

$$\times \cos\psi_a \cos\vartheta_a + \Omega_\eta \sin\vartheta_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\psi_a \cos\vartheta_a];$$

$$a_{10\gamma} = \frac{1}{\cos\vartheta_a}(\omega_{ay}\sin\gamma_a + \omega_{az}\cos\gamma_a);\quad a_{10\Omega_\xi} = -\cos\psi_a \tan\vartheta_a;$$

$$a_{10\Omega_\eta} = 1;\quad a_{10\Omega_\zeta} = -\sin\psi_a \tan\vartheta_a;\quad a_{11\dot{\lambda}} = -\cos\psi_a;\quad a_{11\dot{\varphi}} = -\sin\psi_a;$$

$$a_{11\psi} = (\Omega_\xi - \dot{\varphi}_{arb})\cos\psi_a - (\Omega_\zeta - \dot{\lambda}_{arb})\sin\psi_a;$$

$$a_{11\gamma} = \omega_{az}\sin\gamma_a - \omega_{ay}\cos\gamma_a;$$

$$a_{11\Omega_\xi} = \sin\psi_a;$$

$$a_{11\Omega_\zeta} = \cos\psi_a;$$

$$a_{12\dot{\lambda}} = -\sin\psi_a \sec\vartheta_a;$$

$$a_{12\dot{\varphi}} = -\cos\psi_a \sec\vartheta_a;$$

$$a_{12\psi} = -(\Omega_\xi - \dot{\varphi}_{arb})\sin\psi_a \sec\vartheta_a - (\Omega_\zeta - \dot{\lambda}_{arb})\cos\psi_a \sec\vartheta_a; \qquad (4.65)$$

$$a_{12\vartheta} = \frac{1}{\cos^2\vartheta_a}(\omega_{ay}\cos\gamma_a - \omega_{az}\sin\gamma_a) -$$

$$-(\Omega_\xi - \dot{\varphi}_{arb})\cos\psi_a \frac{\sin\vartheta_a}{\cos^2\vartheta_a} + (\Omega_\zeta - \dot{\lambda}_{arb})\sin\psi_a \frac{\sin\vartheta_a}{\cos^2\vartheta_a};$$

$$a_{12\gamma} = -(\omega_{az}\cos\gamma_a + \omega_{ay}\sin\gamma_a)\tan\vartheta_a;$$

$$a_{12\Omega_\xi} = \cos\psi_a \sec\vartheta_a;$$

$$a_{12\Omega_\zeta} = \sin\psi_a \sec\vartheta_a;$$

$$a_{13\lambda} = \Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb};$$

$$a_{13\Omega_\xi} = 1;\quad a_{14\lambda} = -\Omega_{\xi 0}\cos\lambda_{arb} + \Omega_{\eta 0}\sin\lambda_{arb};$$

$$a_{14\varphi} = \Omega_{\zeta 0};\quad a_{14\Omega_\eta} = 1;$$

$$a_{15\varphi} = \Omega_{\xi 0}\sin\lambda_{arb} + \Omega_{\eta 0}\cos\lambda_{arb};$$

$$a_{15\lambda} = -(\Omega_{\xi 0}\cos\lambda_{arb} - \Omega_{\eta 0}\sin\lambda_{arb})\varphi_{arb};\quad a_{15\Omega_\zeta} = 1.$$

Here partial systematic errors are

$$\delta_{sys}\dot{V}_\xi = \frac{V_\xi V_\zeta}{R_{ea}+H}\tan\varphi_{arb} - \frac{V_\xi V_\eta}{R_{ea}^2}H;$$

$$\delta_{sys}\dot{V}_\eta = \frac{V_\zeta^2}{R_{ea}+H} + \frac{V_\xi^2}{R_{ea}^2};$$

$$\delta_{sys}\dot{V}_\zeta = -\frac{V_\xi^2}{R_{ea}+H}\tan\varphi_{arb} - \frac{V_\eta V_\zeta}{R_{ea}^2}H; \qquad (4.66)$$

$$\delta_{sys}\dot{\lambda}_{arb} = \frac{V_\xi}{R_{ea}+H}\left(1 - \frac{1}{\cos\varphi_{arb}}\right);$$

$$
\begin{aligned}
&\dot{\delta}_{\text{sys}}\dot{\varphi}_{\text{arb}} = -\frac{V_\zeta}{R_{\text{ea}}^2}H;\\
&\dot{\delta}_{\text{sys}}\Omega_\xi = 0;\\
&\dot{\delta}_{\text{sys}}\Omega_\eta = (\Omega_{\xi_0}\sin\lambda_{\text{arb}} + \Omega_{\zeta_0}\cos\lambda_{\text{arb}})(1-\cos\varphi_{\text{arb}}) - \Omega_{\zeta_0}(\varphi_0 - \\
&\qquad -\sin\varphi_{\text{arb}});\\
&\dot{\delta}_{\text{sys}}\Omega_\zeta = (\Omega_{\xi_0}\sin\lambda_{\text{arb}} + \Omega_{\zeta_0}\cos\lambda_{\text{arb}})(\varphi_{\text{arb}} - \sin\varphi_{\text{arb}}) + \Omega_{\zeta_0}(1 - \\
&\qquad -\cos\varphi_{\text{arb}});\\
&\dot{\delta}_{\text{sys}}\dot{\psi}_a = -\dot{\lambda}_{\text{arb}}\tan\vartheta_a\sin\psi_a(1-\cos\varphi_{\text{arb}}) + \dot{\lambda}_{\text{arb}}\sin\varphi_{\text{arb}};\\
&\dot{\delta}_{\text{sys}}\vartheta_a = \dot{\lambda}_{\text{arb}}\cos\psi_a(1-\cos\varphi_{\text{arb}});\\
&\dot{\delta}_{\text{sys}}\gamma_a = -\dot{\lambda}_{\text{arb}}\sin\psi_a\sec\vartheta_a(1-\cos\varphi_{\text{arb}}).
\end{aligned}
\qquad (4.66)
$$

INS systematic error equations may be similarly formulated for other design variants as well.

§4.5. INS Instrument Error Equations

Instrument errors in INS's of the class under consideration may be analyzed in a manner similar to that employed for system errors in the first two classes.

In an analysis of INS instrument errors we shall take the simplest error model as a first approximation, just as we did previously: /205

$$
\left.
\begin{aligned}
\Delta a_x &= \Delta a_{x_0} + \delta_{a_x} a_x;\\
\Delta a_y &= \Delta a_{y_0} + \delta_{a_y} a_y;\\
\Delta a_z &= \Delta a_{z_0} + \delta_{a_z} a_z;\\
\Delta\omega_{ax} &= \Delta\omega_{ax_0} + \delta_{\omega_x}\omega_{ax};\\
\Delta\omega_{ay} &= \Delta\omega_{ay_0} + \delta_{\omega_y}\omega_{ay};\\
\Delta\omega_{az} &= \Delta\omega_{az_0} + \delta_{\omega_z}\omega_{az};\\
\Delta\psi' &= \Delta\psi'_0 + \delta_\psi\psi';\\
\Delta\vartheta' &= \Delta\vartheta'_0 + \delta_\vartheta\vartheta';\\
\Delta\gamma' &= \Delta\gamma'_0 + \delta_\gamma\gamma',
\end{aligned}
\right\}
\qquad (4.67)
$$

where Δa_x, Δa_y, Δa_z are errors in the corresponding system accelerometers:
$\Delta\omega_{ax}$, $\Delta\omega_{ay}$; $\Delta\omega_{az}$ are errors in the angular velocity gauges;
$\Delta\psi'$, $\Delta\vartheta'$, $\Delta\gamma'$ are errors in object angular position gauges;
a_{x_0}, Δa_{y_0}, Δa_{z_0}, $\Delta\omega_{ax_0}$, $\Delta\omega_{ay_0}$, $\Delta\omega_{az_0}$, $\Delta\psi'_0$, $\Delta\vartheta'_0$, $\Delta\gamma'_0$ are constant components of guage errors;

δ_{a_x}, δ_{a_y}, δ_{a_z}, δ_{ω_x}, δ_{ω_y}, δ_{ω_z}, δ_ψ, δ_ϑ, δ_γ are relative gauge errors.

The kinematic equations for an INS without a platform for all three design variants, with consideration for element errors, are derived from the corresponding kinematic equation simplifications.

Equations for the second design variant for short-range objects, with consideration for element errors, are derived from (4.42) and will have the following form:

$$
\begin{aligned}
&\dot{V}^*_\xi = a^*_\xi - 2\Omega^*_{\eta_0} V^*_\zeta + \delta_{co}\dot{V}_\xi;\\
&\dot{V}^*_\zeta = a^*_\zeta + 2V^*_\xi(\Omega^*_{\eta_0} + \Omega^*_{\xi_0}\lambda^*_{arb}) + \delta_{co}\dot{V}_\zeta;\\
&a^*_\xi = a^*_x \cos\psi^*_a \cos\vartheta^*_a + a^*_y(\sin\psi^*_a \sin\gamma^*_a - \cos\psi^*_a \sin\vartheta^*_a \cos\gamma^*_a) +\\
&\quad + a^*_z(\sin\psi^*_a \cos\gamma^*_a + \cos\psi^*_a \sin\vartheta^*_a \sin\gamma^*_a) + \delta_{co}a_\xi;\\
&a^*_\zeta = -a^*_x \sin\psi^*_a \cos\vartheta^*_a + a^*_y(\cos\psi^*_a \sin\gamma^*_a + \sin\psi^*_a \sin\vartheta^*_a \cos\gamma^*_a) +\\
&\quad + a_z(\cos\psi^*_a \cos\gamma^*_a - \sin\psi^*_a \sin\vartheta^*_a \sin\gamma^*_a) + \delta_{co}a_\zeta;\\
&\dot{\lambda}^*_{arb} = \frac{1}{R_{ea}} V^*_\xi + \delta_{co}\dot{\lambda}_{arb}\\
&\dot{\varphi}^*_{arb} = -\frac{1}{R_{ea}} V^*_\zeta + \delta_{co}\dot{\varphi}_{arb};\\
&\dot{\psi}^*_a = \frac{1}{\cos\vartheta^*_a}(\omega^*_{ay}\cos\gamma^*_a - \omega^*_{az}\sin\gamma^*_a + \omega^*_\xi \cos\psi^*_a \sin\vartheta^*_a -\\
&\quad - \omega^*_\eta \cos\vartheta^*_a - \omega^*_\zeta \sin\psi^*_a \sin\vartheta^*_a) + \delta_{co}\dot{\psi}_a,\\
&\dot{\vartheta}^*_a = \omega^*_{ay}\sin\gamma^*_a + \omega^*_{az}\cos\gamma^*_a - \omega^*_\xi \sin\psi^*_a - \omega^*_\zeta \cos\psi^*_a + \delta_{co}\dot{\vartheta}_a;\\
&\dot{\gamma}^*_a = \omega^*_{ax} - (\omega^*_{ay}\cos\gamma^*_a - \omega^*_{az}\sin\gamma^*_a)\tan\vartheta^*_a - \omega^*_\xi \cos\psi^*_a \sec\vartheta^*_a +\\
&\quad + \omega^*_\zeta \sin\psi^*_a \sec\vartheta^*_a + \delta_{co}\dot{\gamma}_a;\\
&\omega^*_\xi = \Omega^*_{\xi_0} - \Omega^*_{\eta_0}\lambda^*_{arb} - \dot{\varphi}^*_{arb} + \delta_{co}\omega_\xi;\\
&\omega^*_\eta = \Omega^*_{\xi_0}\lambda^*_{arb} + \Omega^*_{\eta_0} + \delta_{co}\omega_\eta;\\
&\omega^*_\zeta = \Omega^*_{\zeta_0} - \dot{\lambda}_{arb} + \delta_{co}\omega_\zeta,
\end{aligned}
\tag{4.68}
$$

where δ_{co} is relative error in the computers of the corresponding parameters included in the system. /206

In turn, instrument parameter values are

$$
\begin{aligned}
a_x^* &= a_x + \Delta a_x + \delta_{a_x} a_x;\\
a_y^* &= a_y + \Delta a_{y_0} + \delta_{a_y} a_y;\\
a_z^* &= a_z + \Delta a_{z_0} + \delta_{a_z} a_z;\\
\omega_{ax}^* &= a_{ax} + \Delta\omega_{ax_0} + \delta_{\omega_x}\omega_{ax};\\
\omega_{ay}^* &= \omega_{ay} + \Delta\omega_{ay_0} + \delta_{\omega_y}\omega_{ay};\\
\omega_{az}^* &= \omega_{az} + \Delta\omega_{az_0} + \delta_{\omega_z}\omega_{az};\\
\Omega_{\xi_0}^* &= \Omega\cos\varphi_0^*\cos\psi_0^*;\\
\Omega_{\eta_0}^* &= \Omega\sin\varphi_0^*;\\
\Omega_{\zeta_0}^* &= \Omega\cos\varphi_0^*\sin\psi_0^*;\\
\varphi_0^* &= \varphi_0 + \Delta\varphi_0;\\
\psi_0^* &= \psi_0 + \Delta\psi_0;\\
V_\xi^* &= V_{\xi_0} + \int_0^t \dot{V}_\xi^*\,dt + \Delta V_{\xi_0} + \delta_{co} V_\xi;\\
V_\zeta^* &= V_{\zeta_0} + \int_0^t \dot{V}_\zeta^*\,dt + \Delta V_{\zeta_0} + \delta_{co} V_\zeta;\\
\lambda_{arb}^* &= \lambda_{arb_0} + \int_0^t \dot{\lambda}_{arb}^*\,dt + \Delta\lambda_{arb_0} + \delta_{co}\lambda_{arb};\\
\varphi_{arb}^* &= \varphi_{arb_0} + \int_0^t \dot{\varphi}_{arb}^*\,dt + \Delta\varphi_{arb_0} + \delta_{co}\varphi_{arb};\\
\psi_a^* &= \psi_{a_0} + \int_0^t \dot{\psi}_a^*\,dt + \Delta\psi_{a_0} + \delta_{co}\psi_a;\\
\vartheta_a^* &= \vartheta_{a_0} + \int_0^t \dot{\vartheta}_{a_0}^*\,dt + \Delta\vartheta_{a_0} + \delta_{co}\vartheta_a;\\
\gamma_a^* &= \gamma_{a_0} + \int_0^t \dot{\gamma}_{a_0}^*\,dt + \Delta\gamma_{a_0} + \delta_{co}\gamma_a.
\end{aligned}
\tag{4.69}
$$

The solution of system (4.68) with consideration for equations (4.69) makes it possible to determine the instrument values for all kinematic parameters which are included in the INS equation system (which includes both systematic as well as instrument errors in the system). Instrument errors are defined as the difference between the instrument and true values of each parameter. The latter may be determined in advance by solving the system (4.27) in accordance with known values /207

$$\dot{\psi}_a(t),\ \dot{\vartheta}_a(t),\ \dot{\gamma}_a(t),\ \dot{V}_\xi(t),\ \dot{V}_\eta(t),\ \dot{V}_\zeta(t).$$

In turn, the kinematic equations of an INS constructed in accordance with the second design variant for medium-range objects, with consideration for systematic and instrument errors, are derived from the equation system (4.43):

$$\dot{V}_\xi^* = a_\xi^* - \frac{V_\xi^* V_\eta^*}{R_{ea}} - 2(\Omega_\eta^* V_\zeta^* - \Omega_\zeta^* V_\eta^*) + \delta_{in}\dot{V}_\xi;$$

$$\dot{V}_\eta^* = a_\eta^* + \frac{V_\xi^{*2}}{R_{ea}} - 2(\Omega_\zeta^* V_\xi^* - \Omega_\xi^* V_\zeta^*) + g_\eta + \delta_{in}\dot{V}_\eta;$$

$$\dot{V}_\zeta^* = a_\zeta^* - \frac{V_\eta^* V_\zeta}{R_{ea}} - 2(\Omega_\xi V_\eta - \Omega_\eta V_\xi) + \delta_{in}\dot{V}_\zeta;$$

$$a_\xi^* = a_x \cos\psi_a^* \cos\vartheta_a^* + a_y^*(\sin\psi_a^* \sin\gamma_a^* - \cos\psi_a^* \sin\vartheta_a^* \cos\gamma_a^*) +$$
$$+ a_z^*(\sin\psi_a^* \cos\gamma_a^* + \cos\psi_a^* \sin\vartheta_a^* \sin\gamma_a^*) + \delta_{in} a_\xi;$$

$$a_\eta^* = a_x^* \sin\vartheta_a^* + a_y^* \cos\vartheta_a^* \cos\gamma_a^* - a_z^* \cos\vartheta_a^* \sin\gamma_a^* + \delta_{in} a_\eta;$$

$$a_\zeta^* = -a_x^* \sin\psi_a^* \cos\vartheta_a^* + a_y^*(\cos\psi_a^* \sin\gamma_a^* + \sin\psi_a^* \sin\vartheta_a^* \times$$
$$\times \cos\gamma_a^*) + a_z(\cos\psi_a^* \cos\gamma_a^* - \sin\psi_a^* \sin\vartheta_a^* \sin\gamma_a^*) + \delta_{in} a_\zeta;$$

$$\dot{\lambda}_{arb}^* = \frac{1}{R_{ea} + H^*} V_\xi^* + \delta_{in}\dot{\lambda}_{arb}$$

$$\dot{H}^* = V_\eta^* + \delta_{in}\dot{H}; \qquad (4.70)$$

$$\dot{\varphi}^* = -\frac{1}{R_{ea}} V_\zeta^* + \delta_{in}\dot{\varphi}_{arb};$$

$$\dot{\psi}_a^* = \frac{1}{\cos\vartheta_a^*}[\omega_{ay}^* \cos\gamma_a^* - \omega_{az}^* \sin\gamma_a^* + (\Omega_\xi^* - \dot{\varphi}_{arb}^*)\cos\psi_a^* \sin\vartheta_a^* -$$
$$- \Omega_\eta^* \cos\vartheta_a^* - (\Omega_\zeta^* - \dot{\lambda}_{arb}^*)\sin\psi_a^* \sin\vartheta_a^*] + \delta_{in}\dot{\psi}_a;$$

$$\dot{\vartheta}_a^* = \omega_{ay}^* \sin\gamma_a^* + \omega_{az}^* \cos\gamma_a^* - (\Omega_\xi^* - \dot{\varphi}_{arb}^*)\sin\psi_a^* - (\Omega_\zeta^* - \dot{\lambda}_{arb}^*)\cos\psi_a^* +$$
$$+ \delta_{in}\dot{\vartheta}_a;$$

$$\dot{\gamma}_a^* = \omega_{ax}^* - (\omega_{ay}^* \cos\gamma_a^* - \omega_{az}^* \sin\gamma_a^*)\tan\vartheta_a^* - (\Omega_\xi^* - \dot{\varphi}_{arb}^*)\cos\psi_a^* \sec\vartheta_a^* +$$
$$+ (\Omega_\zeta^* - \dot{\lambda}_{arb}^*)\sin\psi_a^* \sec\vartheta_a^* + \delta_{in}\dot{\gamma}_a;$$

$$\Omega_\xi^* = \Omega_{\xi_0}^* \cos\lambda_{arb}^* - \Omega_{\eta_0}^* \sin\lambda_{arb}^*;$$

$$\Omega_\eta^* = \Omega_{\xi_0}^* \sin\lambda_{arb}^* + \Omega_{\eta_0}^* \cos\lambda_{arb}^* - \Omega_{\zeta_0}^* \varphi_{arb}^*;$$

$$\Omega_\zeta^* = (\Omega_{\xi_0}^* \sin\lambda_{arb}^* + \Omega_{\eta_0}^* \cos\lambda_{arb}^*)\varphi_{arb}^* + \Omega_{\zeta_0}.$$

The basic data for the solution of system (4.70) are furnished by the same parameters used for the solution of system (4.68). /208

Cross links in the first three equations of the system (4.70) along the vertical velocity component V_η lead to the fact that INS errors in the determination of this parameter are caused not only by errors in the determination of

altitude H, but also by errors in the determination of horizontal accelerations $\dot{V}_\xi$, $\dot{V}_\zeta$, velocities V_ξ, V_ζ and coordinates λ_{arb}, ϕ_{arb}.

The kinematic equations of an INS of the third design variant may be derived similarly to (4.70) from the (4.55) and (4.56) systems.

We obtain the following instrument errors for short-range objects after linearization of equations (4.42), with consideration for element errors:

$$
\begin{aligned}
&p\Delta V_{\xi\,in} + a_{1V_\zeta}\Delta V_{\zeta\,in} + a_{1a_\xi}\Delta a_{\xi\,in} = \delta_{in}\dot{V}_\xi;\\
&a_{2V_\xi}\Delta V_{\xi\,in} + p\Delta V_{\zeta\,in} + a_{2\lambda}\Delta\lambda_{arb,in} + a_{2a_\zeta}\Delta a_{\zeta\,in} = \delta_{in}\dot{V}_\zeta;\\
&a_{3a_\xi}\Delta a_{\xi\,in} + a_{3\psi}\Delta\psi_{a,in} + a_{3\vartheta}\Delta\vartheta_{a,in} + a_{3\gamma}\Delta\gamma_{a,in} = \delta_{in}a_\xi;\\
&a_{4a_\xi}\Delta a_{\xi\,in} + a_{4\psi}\Delta\psi_{a,in} + a_{4\vartheta}\Delta\vartheta_{a,in} + a_{4\gamma}\Delta\gamma_{a,in} = \delta_{in}a_\zeta;\\
&a_{5V_\xi}\Delta V_{\xi\,in} + p\Delta\lambda_{arb,in} = \delta_{in}\dot{\lambda}_{arb};\\
&a_{6V_\zeta}\Delta V_{\zeta\,in} + p\Delta\varphi_{arb,in} = \delta_{in}\dot{\varphi}_{arb};\\
&(p + a_{7\psi})\Delta\psi_{a,in} + a_{7\vartheta}\Delta\vartheta_{a,in} + a_{7\gamma}\Delta\gamma_{a,in} + a_{7\omega_\xi}\Delta\omega_{\xi\,in} +\\
&\qquad + a_{7\omega_\eta}\Delta\omega_{\eta\,in} + a_{7\omega_\zeta}\Delta\omega_{\zeta\,in} = \delta_{in}\dot{\psi}_a;
\end{aligned}
\qquad (4.71)
$$

$$
\begin{aligned}
&a_{8\psi}\Delta\psi_{a,in} + p\Delta\vartheta_{a,in} + a_{8\gamma}\Delta\gamma_{a,in} + a_{8\omega_\xi}\Delta\omega_{\xi\,in} + a_{8\omega_\zeta}\Delta\omega_{\zeta\,in} = \delta_{in}\vartheta_a;\\
&a_{9\psi}\Delta\psi_{a,in} + a_{9\vartheta}\Delta\vartheta_{a,in} + (p + a_{9\gamma})\Delta\gamma_{a,in} + a_{9\omega_\xi}\Delta\omega_{\xi\,in} +\\
&\qquad + a_{9\omega_\zeta}\Delta\omega_{\zeta\,in} = \delta_{in}\dot{\gamma}_a;\\
&a_{10\lambda}\Delta\lambda_{arb,in} + p\Delta\varphi_{arb,in} + a_{10\omega_\xi}\Delta\omega_{\xi\,in} = \delta_{in}\omega_\xi;\\
&a_{11\lambda}\Delta\lambda_{arb,in} + a_{11\omega_\eta}\Delta\omega_{\eta\,in} = \delta_{in}\omega_\eta;\\
&p\Delta\lambda_{arb,in} + a_{12\omega_\zeta}\Delta\omega_{\zeta\,in} = \delta_{in}\dot{\omega}_\zeta.
\end{aligned}
$$

The equation coefficients (4.69) are determined from expressions (4.60) and the partial instrumental errors, included in the right sides of these equations, are determined as /209

$$
\begin{aligned}
&\delta_{in}\dot{V}_\xi = \delta_{co}\dot{V}_\xi;\\
&\delta_{in}\dot{V}_\eta = \delta_{co}\dot{V}_\eta;\\
&\delta_{in}\dot{V}_\zeta = \delta_{co}\dot{V}_\zeta;\\
&\delta_{in}a_\xi = \cos\psi_a\cos\vartheta_a(\Delta a_{x_0} + \delta_{a_x}a_x) + (\sin\psi_a\sin\gamma_a -\\
&\quad - \cos\psi_a\sin\vartheta_a\cos\gamma_a)(\Delta a_{y_0} + \delta_{a_y}a_y) + (\sin\psi_a\cos\gamma_a +\\
&\quad + \cos\psi_a\sin\vartheta_a\sin\gamma_a)(\Delta a_{z_0} + \delta_{a_z}a_z);
\end{aligned}
\qquad (4.72)
$$

$$
\begin{aligned}
&\delta_{\text{in}} a_\zeta = -\sin\psi_a \cos\vartheta_a (\Delta a_{x_0} + \delta_{a_x} a_x) + (\cos\psi_a \sin\gamma_a + \\
&\quad + \sin\psi_a \sin\vartheta \cos\gamma_a)(\Delta a_{y_0} + \delta_{a_y} a_y) + (\cos\psi_a \cos\gamma_a - \\
&\quad - \sin\psi_a \sin\vartheta_a \sin\gamma_a)(\Delta a_{z_0} + \delta_{a_z} a_z);
\end{aligned}
$$

$$
\begin{aligned}
&\delta_{\text{in arb}} \dot\lambda = \delta_{\text{co arb}} \dot\lambda;\\
&\delta_{\text{in arb}} \varphi = \delta_{\text{co arb}} \varphi;\\
&\delta_{\text{in}} \psi_a = \delta_{\text{co}} \psi_a + \frac{\cos\gamma_a}{\cos\vartheta_a}(\Delta\omega_{ay_0} + \delta_{\omega_y}\omega_{ay}) - \frac{\sin\gamma_a}{\cos\vartheta_a}(\Delta\omega_{az_0} + \delta_{\omega_z}\omega_{az});\\
&\delta_{\text{in}} \dot\vartheta_a = \sin\gamma_a(\Delta\omega_{ay_0} + \delta_{\omega_y}\omega_{ay}) + \cos\gamma_a(\Delta\omega_{az_0} + \delta_{\omega_z}\omega_{az});\\
&\delta_{\text{in}} \dot\gamma_a = (\Delta\omega_{ax_0} + \delta_{\omega_x}\omega_{ax}) - \cos\gamma_a \tan\vartheta_a(\Delta\omega_{ay_0} + \delta_{\omega_y}\omega_{ay}) + \\
&\quad + \sin\gamma_a \tan\vartheta_a(\Delta\omega_{az_0} + \delta_{\omega_z}\omega_{az});\\
&\delta_{\text{in}} \omega_\xi = \delta_{\text{co}} \omega_\xi - \Omega(\sin\varphi_0 \cos\psi_{\gamma 0}\Delta\varphi_0 + \cos\varphi_0 \sin\psi_{\gamma 0}\Delta\psi_{\gamma 0}) - \\
&\quad - \Omega\lambda_{\text{arb}} \cos\varphi_0\Delta\varphi_0;\\
&\delta_{\text{in}} \omega_\eta = \delta_{\text{co}} \omega_\eta + \Omega(\sin\varphi_0 \cos\psi_{\gamma 0}\Delta\varphi_0 + \cos\varphi_0 \sin\psi_{\gamma 0}\Delta\psi_{\gamma 0}) + \\
&\quad + \Omega\cos\varphi_0\Delta\varphi_0;\\
&\delta_{\text{in}} \omega_\zeta = \delta_{\text{co}} \omega_\zeta + \Omega(\sin\varphi_0 \sin\psi_{\gamma 0}\Delta\varphi_0 - \cos\varphi_0 \cos\psi_{\gamma 0}\Delta\psi_{\gamma 0}).
\end{aligned}
\tag{4.72}
$$

Instrument error equations for an INS of the second design variant for medium-range objects (4.43) will have the form: /210

$$
\begin{aligned}
&(p + a_{1V_\xi})\Delta V_{\xi\,\text{in}} + a_{1V_\eta}\Delta V_{\eta\,\text{in}} + a_{1V_\zeta}\Delta V_{\zeta\,\text{in}} + a_{1a_\xi}\Delta a_{\xi\,\text{in}} + \\
&\quad + a_{1\Omega_\eta}\Delta\Omega_{\eta\,\text{in}} + a_{1\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}} = \delta_{\text{in}}\dot V_\xi;\\
&a_{2V_\xi}\Delta V_{\xi\,\text{in}} + (a_{2V_\eta} + p)\Delta V_{\eta\,\text{in}} + a_{2V_\zeta}\Delta V_{\zeta\,\text{in}} + a_{2a_\eta}\Delta a_{\eta\,\text{in}} + \\
&\quad + a_{2\Omega_\xi}\Delta\Omega_{\xi\,\text{in}} + a_{2\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}} = \delta_{\text{in}}\dot V_\eta;\\
&a_{3V_\xi}\Delta V_{\xi\,\text{in}} + a_{3V_\eta}\Delta V_{\eta\,\text{in}} + (p + a_{3V_\zeta})\Delta V_{\zeta\,\text{in}} + a_{3a_\zeta}\Delta a_{\zeta\,\text{in}} + \\
&\quad + a_{3\Omega_\xi}\Delta\Omega_{\xi\,\text{in}} + a_{3\Omega_\eta}\Delta\Omega_{\eta\,\text{in}} = \delta_{\text{in}}\dot V_\zeta;\\
&a_{4a_\xi}\Delta V_{\xi\,\text{in}} + a_{4\psi}\Delta\psi_{\text{a,in}} + a_{4\vartheta}\Delta\vartheta_{\text{a,in}} + a_{4\gamma}\Delta\gamma_{\text{a,in}} = \delta_{\text{in}} a_\xi;\\
&a_{5a_\eta}\Delta a_{\eta\,\text{in}} + a_{5\vartheta}\Delta\vartheta_{\text{a,in}} + a_{5\gamma}\Delta\gamma_{\text{a,in}} = \delta_{\text{in}} a_\eta;\\
&a_{6a_\zeta}\Delta a_{\zeta\,\text{in}} + a_{6\psi}\Delta\psi_{\text{a,in}} + a_{6\vartheta}\Delta\vartheta_{\text{a,in}} + a_{6\gamma}\Delta\gamma_{\text{a,in}} = \delta_{\text{in}} a_\zeta;\\
&a_{7V_\xi}\Delta V_{\xi\,\text{in}} + p\Delta\lambda_{\text{arb,in}} + a_{7H}\Delta H_{\text{in}} = \delta_{\text{in}}\dot\lambda_{\text{arb}};\\
&a_{8V_\eta}\Delta V_{\eta\,\text{in}} + p\Delta H_{\text{in}} = \delta_{\text{in}}\dot H;\\
&a_{9V_\zeta}\Delta V_{\zeta\,\text{in}} + p\Delta\varphi_{\text{arb,in}} = \delta_{\text{in arb}}\dot\varphi;\\
&a_{10\lambda}p\lambda_{\text{arb,in}} + a_{10\varphi}p\Delta\varphi_{\text{arb,in}} + (p + a_{10\psi})\Delta\psi_{\text{arb,in}} + a_{10\vartheta}\Delta\vartheta_{\text{a,in}} + a_{10\gamma}\Delta\gamma_{\text{a,in}} + \\
&\quad + a_{10\Omega_\xi}\Delta\Omega_{\xi\,\text{in}} + a_{10\Omega_\eta}\Delta\Omega_{\eta\,\text{in}} + a_{10\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}} = \delta_{\text{in}}\dot\psi_a;\\
&a_{11\lambda}p\Delta\lambda_{\text{arb,in}} + a_{11\varphi}p\Delta\varphi_{\text{arb,in}} + a_{11\psi}\Delta\psi_{\text{a,in}} + p\Delta\vartheta_{\text{a,in}} + a_{11\gamma}\Delta\gamma_{\text{a,in}} + \\
&\quad + a_{11\Omega_\xi}\Delta\Omega_{\xi\,\text{in}} + a_{11\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}} = \delta_{\text{in}}\dot\vartheta_a;
\end{aligned}
\tag{4.73}
$$

$$
\begin{aligned}
&a_{12\lambda}p\Delta\lambda_{\text{arb, in}}+a_{12\varphi}p\Delta\varphi_{\text{arb, in}}+a_{12\psi}\Delta\psi_{\text{a, in}}+a_{12\vartheta}\Delta\vartheta_{\text{a, in}}+\\
&\qquad+(p+a_{12\gamma})\Delta\gamma_{\text{a, in}}+a_{12\Omega_\xi}\Delta\Omega_{\xi\,\text{in}}+a_{12\Omega_\zeta}\Delta\Omega_{\zeta\,\text{in}}=\delta_{\text{in}}\dot{\gamma}_{\text{a}};\\
&a_{13\lambda}\Delta\lambda_{\text{arb, in}}+a_{13\Omega_\xi}\Delta\Omega_{\xi\,\text{in}}=\delta_{\text{in}}\Omega_\xi;\\
&a_{14\lambda}\Delta\lambda_{\text{arb, in}}+a_{14\varphi}\Delta\varphi_{\text{arb, in}}+a_{14\omega_\eta}\Delta\Omega_{\eta\,\text{in}}=\delta_{\text{in}}\Omega_\eta;\\
&a_{15\lambda}\Delta\lambda_{\text{arb, in}}+a_{15\varphi}\Delta\varphi_{\text{arb, in}}+a_{15\omega_\zeta}\Delta\Omega_{\zeta\,\text{in}}=\delta_{\text{in}}\Omega_\zeta.
\end{aligned}
\tag{4.73}
$$

where equation coefficients (4.73) are determined from equations (4.65); partial instrument errors are

$$
\begin{aligned}
&\delta_{\text{in}}\dot{V}_\xi=\delta_{\text{co}}\dot{V}_\xi;\\
&\delta_{\text{in}}\dot{V}_\eta=\delta_{\text{co}}\dot{V}_\eta;\\
&\delta_{\text{in}}\dot{V}_\zeta=\delta_{\text{co}}\dot{V}_\zeta;\\
&\delta_{\text{in}}a_\eta=\sin\vartheta_{\text{a}}(\Delta a_{x_0}+\delta_{a_x}a_x)+\cos\vartheta_{\text{a}}\cos\gamma_{\text{a}}(\Delta a_{y_0}+\delta_{a_y}\Delta a_y)-\\
&\qquad-\cos\vartheta_{\text{a}}\sin\gamma_{\text{a}}(\Delta a_{z_0}+\delta_{a_z}a_z);\\
&\delta_{\text{in}}\Omega_\xi=-\Omega[\cos\lambda_{\text{arb}}(\sin\varphi_0\cos\Psi_0\Delta\varphi_0+\cos\varphi_0\sin\Psi_0\Delta\Psi_0)-\\
&\qquad-\sin\lambda_{\text{arb}}\cos\varphi_0\Delta\varphi_0+\delta_{\text{co}}\Omega_\xi;\\
&\delta_{\text{in}}\Omega_\eta=-\Omega(\sin\lambda_{\text{arb}}(\sin\varphi_0\cos\Psi_0\Delta\varphi_0+\cos\varphi_0\sin\Psi_0\Delta\Psi_0)+\\
&\qquad+\cos\lambda_{\text{arb}}\cos\varphi_0\Delta\varphi_0]+\delta_{\text{co}}\Omega_\eta;\\
&\delta_{\text{in}}\Omega_\zeta=-\Omega[\sin\lambda_{\text{arb}}\sin\varphi_0\cos\Psi_0\Delta\varphi_0+\cos\varphi_0\sin\Psi_0\Delta\Psi_0)+\\
&\qquad+\cos\lambda_{\text{arb}}\cos\varphi_0\Delta\varphi_0]\Delta\varphi_{\text{arb}}+\Omega(\sin\varphi_0\sin\Psi_0\Delta\varphi_0-\\
&\qquad-\cos\varphi_0\cos\Psi_0\Delta\Psi_0),
\end{aligned}
\tag{4.74}
$$

and the remaining errors are determined from equations (4.72). /211

System instrument errors for other design variants of the INS may be derived in a similar fashion.

§4.6. Systematic and Instrument Errors in an INS Equipped Short-Range Object.

Systematic and instrument errors in an INS without a platform will be calculated for the number 2 variant as applied to the same object and motion trajectory as in Section 2.7 and 3.6.

On the basis of the simplifications already accomplished of the full kinematic equation system for an INS of the second design variant, the system (4.42) was recommended.

It is apparent that an INS constructed in accordance with the kinematic equation system mentioned may be a two-component system, since in the first two equations the object vertical velocity component (V_η) is lacking; however, in

order to determine the instantaneous parameter values a_ξ and a_ζ, it is necessary to have three accelerometers in the system in order to measure accelerations a_x, a_y, a_z as well as three angular velocity sensors (for the measurement of ω_{ax}, ω_{ay}, ω_{az}).

The kinematic equations for the INS under consideration, allowing for system element errors, are presented in expression (4.68).

The INS block diagram for the given object, corresponding to the kinematic equation system (4.42), is shown in Figure 4.12.

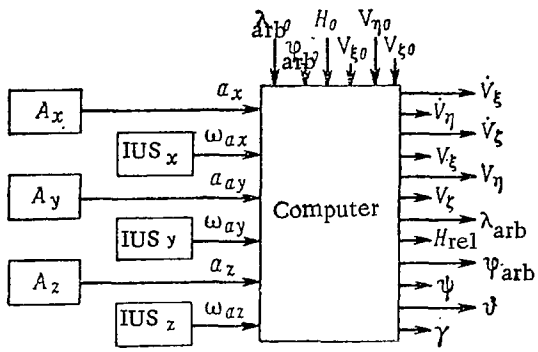

Figure 4.12. INS Block Diagram for a Short-Range Object.

It is apparent from the block diagram that the INS must contain three accelerometers A_x, A_y, A_z and three angular velocity gauges IUS_x, IUS_y, IUS_z, installed directly on the object. At the starting point it is necessary to display the measurement axes of these instruments (x, y, z) in the plane of the horizon and to direct them along the arbitrary equator. At the moment of start the measurement axes of the instruments are bound to the object.

The calculator (VM) must calculate the instantaneous values of the horizontal acceleration components $\dot{V}_\xi$, $\dot{V}_\zeta$, the velocities V_ξ, V_ζ, the angular coordinates λ_{arb}, ϕ_{arb} as well as yaw angle ψ, pitch angle ϑ, and bank angle γ, which characterize object motion with respect to the surface of the earth. The values of angles ψ, ϑ, γ in a first approximation are equal to ψ_a, ϑ_a, γ_a, respectively, if we assume zero initial values for the latter (ψ_{a0}, ϑ_{a0}, γ_{a0}) at the moment of start.

The functional diagram of the system computer is shown in Figure 4.13. /213

In contrast to an INS with a corrected platform, in which the position of the $\xi\eta\zeta$ angular coordinate trihedron is physically modelled on the object, and with a platform which is fixed in inertial space, in which the position of the $\xi_1\eta_1\zeta_1$ coordinate trihedron, bound with inertial space, is modelled physically, in an INS without a platform the position of the trihedrons indicated is determined mathematically in accordance with data obtained from the accelerometers and angular velocity gauges. This is apparent from the block diagram of the INS computer (See Figure 4.13).

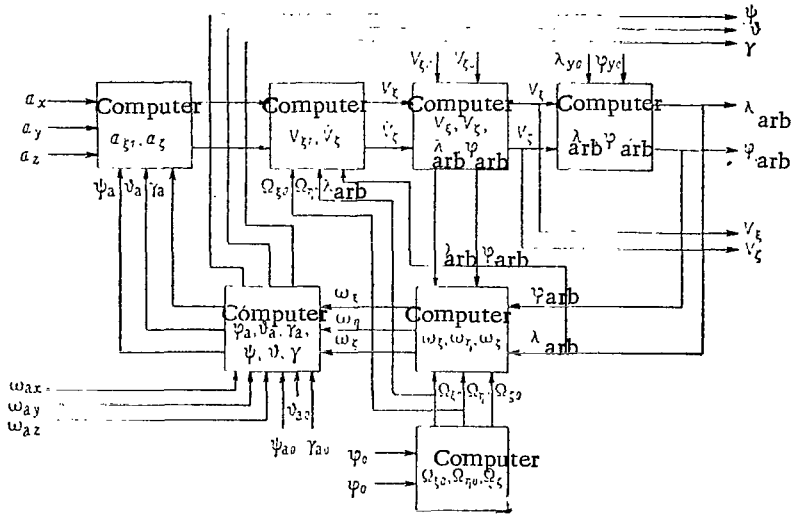

Figure 4.13. The Functional Diagram of an INS Computer.

In an INS without a platform, the computers of parameters ψ_a, ϑ_a, γ_a, ω_ξ, ω_η, ω_ζ, Ω_{ξ_0}, Ω_{η_0}, Ω_{ζ_0} mathematically model the kinematics of the $\xi\eta\zeta$ trihedron, and the a_ξ and a_ζ computers mathematically stabilize the system accelerometers in azimuth and with respect to the plane of the horizon.

It is not difficult to note that in an INS of the class under consideration feedback occurs along the ϕ_{arb} and λ_{arb} coordinates as well as along their first derivatives $\dot{\phi}_{arb}$, $\dot{\lambda}_{arb}$. In addition, there are cross links in the computer system with respect to λ_{arb}, Ω_{ξ_0}, Ω_{η_0}. In order to improve operating conditions, it is desirable to install the INS operating instruments in a special frame.

It is advantageous to provide a rotational capability for the frame or for the instrument platform with respect to the three mutually perpendicular axes in order to display the measurement axes of the instruments at the point of start. It is also possible to display automatically the system measuring instruments in accordance with the indications of the accelerometers and the supplementary course sensor by the use of appropriate tracking systems for the inclination of the frame or the platform. In addition, INS display at the starting point is possible by introducing into the computer the initial values of the angles ψ_{a0}, ϑ_{a0}, γ_{a0}, which characterize the position of the accelerometer trihedron with respect to the arbitrary trihedron at the starting point.

The technique for the preparation of the initial data used to determine system errors was discussed previously (Section 2.7), therefore problems associated with the specific application of the technique indicated will be examined in this section.

The problem of preparing the initial data includes the determination of the true values of all the kinematic parameters of system (4.42), including vector acceleration components a_x, a_y, a_z in absolute angular velocity components ω_{ax}, ω_{ay}, ω_{az}.

The object motion trajectory in the vertical plane was divided by flight time into a number of segments in order to simplify the problem.

Constancy of the values of vector components with respect to accelerations $\dot{V}_\xi$, $\dot{V}_\eta$, $\dot{V}_\zeta$ was assumed on each trajectory segment. The values of the velocity /214
vector components $V_\xi(t)$, $V_\eta(t)$, $V_\zeta(t)$ and of the arbitrary coordinates $\lambda_{arb}(t)$, $\phi_{arb}(t)$ of the object are found by single and double integration, respectively, of the acceleration components.

After the start of the object, the measurement instruments of the system are tightly bound to the object. In the process of motion the object is under the influence of external perturbations and accomplishes damped oscillations with amplitude $\psi_m = \vartheta_m = \gamma_m = 5°$, with period T = 12 sec and attenuation $\sigma = 0.2231$.

$$\left.\begin{aligned}\psi(t) &= \psi_m e^{-\sigma t}\sin\omega t;\\ \vartheta(t) &= \vartheta_m e^{-\sigma t}\sin\omega t;\\ \gamma(t) &= \gamma_m e^{-\sigma t}\sin\omega t.\end{aligned}\right\} \qquad (4.75)$$

These effects are repeated every 20 sec, which permits us to make a more objective evaluation of the capability of an INS of the given class.

By solving the system (4.27) in accordance with known and parameter values $\dot{V}_\xi(t)$, $\dot{V}_\eta(t)$, $\dot{V}_\zeta(t)$, we determine the true values of all the remaining kinematic parameters which enter into the system (4.42), including the vector components of absolute acceleration $a_\xi(t)$, $a_\eta(t)$, $a_\zeta(t)$, of absolute angular velocity $\omega_{ax}(t)$, $\omega_{ay}(t)$, $\omega_{az}(t)$, of parameters a_x, a_y, a_z and of the angles $\psi_a(t)$, $\vartheta_a(t)$, $\gamma_a(t)$.

In agreement with the technique indicated, the systematic error values may be determined with a sufficient degree of accuracy by solving system (4.57).

The basic data for the solution of this system include instantaneous values of accelerations $a_x(t)$, $a_y(t)$, $a_z(t)$ and angular velocities $\omega_{ax}(t)$, $\omega_{ay}(t)$, $\omega_{az}(t)$, as well as the initial values of the kinematic parameters of object motion, geographic latitude ϕ_0 and course angle Ψ_0 of the programmed orthodrome (of the arbitrary equator) at the starting point.

With no element errors present, the system (4.57) was solved on a type M-20 computer (for a clarification of systematic errors only).

INS systematic errors in the determination of parameter $V_\xi(t)$, $V_\zeta(t)$, $\lambda_{arb}(t)$, $\phi_{arb}(t)$, were calculated in accordance with equations (2.189), in which a parameter values with asterisks were taken from the results of the solution of system (4.57). The systematic errors $\Delta V_{\xi\ sys}(t)$, $\Delta V_{\zeta\ sys}(t)$, $\Delta\lambda_{arb\ sys}(t)$, $\Delta\phi_{arb\ sys}(t)$ involved are shown in Figure 4.14.

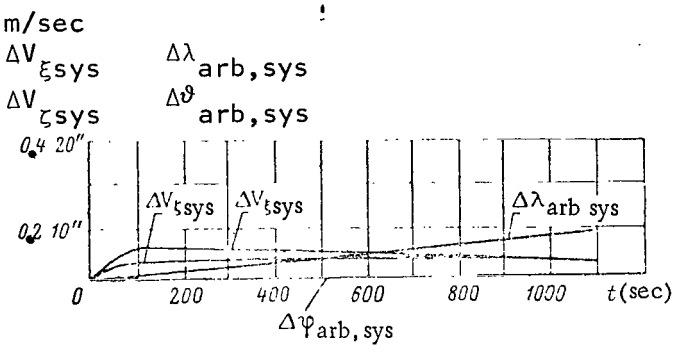

Figure 4.14. Time Function Graphs of INS Systematic Errors.

It is apparent from the graphs that INS systematic errors increase monotonically with time, but do not exceed the following values:

$$\Delta V_{\xi_{sys}} \leqslant 0{,}067' \text{ m/sec};$$
$$\Delta V_{\zeta_{sys}} \leqslant 0{,}0368 \text{ m/sec};$$
$$\Delta\lambda_{arb,sys} \leqslant 0{,}25\cdot 10^{-4} \text{ rad } (\approx 10');$$
$$\Delta\varphi_{arb,sys} \leqslant 0{,}0542\cdot 10^{-4} \text{ rad } (\approx 1'').$$

Based on the results obtained, we may assume that the proposed technique /215 for the simplification of INS kinematic equations is completely applicable to engineering calculations and that the kinematic equations recommended for an INS of the given class of objects fully satisfy accuracy requirements.

The INS kinematic equations system (4.68), with consideration for element errors, is shown in Section 3.7.

The values of partial errors included in the system (Section 4.5) and caused by element errors are shown in Table 4.1.

TABLE 4.1. ELEMENT INSTRUMENT ERRORS

Name of Error	Designation	Error Value
Constant accelerometer components	Δa_{x0}, Δa_{y0}, Δa_{z0}	$0{,}5 \cdot 10^{-4}$ g
Relative accelerometer error	δ_a	0,1%
Angular velocity gauge constant error component	$\Delta\omega_{ax0}$, $\Delta\omega_{ay0}$, $\Delta\omega_{az0}$	$0{,}5 \cdot 10^{-6}$ 1/ sec (0,18 deg/hr)
Relative angular velocity gauge error	δ_ω	0,1%
Error in the determination and in the introduction of the initial values of object motion	$\Delta V_{\xi 0}$, $\Delta V_{\zeta 0}$	1 m/sec
Errors in the initial display of inst. measurement axes	$\Delta\psi_{a0}$, $\Delta\vartheta_{a0}$, $\Delta\gamma_{a0}$	1′
Computer error in $\dot{V}_\xi$, $\dot{V}_\zeta$	$\delta_{co}\,\dot{V}$	0,1%
Computer error in a_ξ, a_ζ	$\delta_{co}\,a$	0,1%
Computer error in $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$, ψ_a, ϑ_a, γ_a	δ_{co}	0,1%
Computer error in ω_ξ, ω_η, ω_ζ	$\delta_{co\omega}$	0,1%
Computer error in $\dot{V}_\xi$, V_ζ, λ_y, φ_y	δ_{coV}, $\delta_{co\lambda}$, $\delta_{co\phi}$	0,1%

Tr. Note: Commas indicate decimal points.

The true values of accelerations $a_x(t)$, $a_y(t)$, $a_z(t)$ and of angular velocities $\omega_{ax}(t)$, $\omega_{ay}(t)$, $\omega_{az}(t)$, measured by the appropriate system instruments, remain the same as in the determination of systematic errors. /216

The limiting values of INS instrument errors caused by errors in each of the system elements were calculated in accordance with equations (2.215), in which the parameters with asterisks for each moment of time were taken from the solution of the system of nonlinear kinematic equations (4.68).

The initial data for the solution of system (4.68) were the true values of accelerations $a_x(t)$, $a_y(t)$, $a_z(t)$ and of angular velocities $\omega_{ax}(t)$, $\omega_{ay}(t)$, $\omega_{az}(t)$, as well as initial values ϕ_0, Ψ_0 and partial errors of the corresponding INS elements (see Table 4.1).

In an evaluation of the influence of individual system elements on the accuracy of the determination of the true kinematic parameters of object motion, system (4.68) was solved for several variants in initial data, which differ from each other by the values of the partial instrument errors. /217

In accordance with the results of these solutions, a summary table was prepared of the INS instrument errors in the determination of the appropriate parameters of object motion for the end of the trajectory (Table 4.2).

Figure 4.15-4.18 show graphs of the INS error relationships in the determination of each of the kinematic parameters ($\Delta V_{\xi in}$ m/sec, $\Delta V_{\zeta in}$ m/sec, $\Delta\lambda_{arb}$ 10^{-4}rad, $\Delta\phi_{arb}$ 10^{-4}rad) for all of the variants of the solution. The numbers of the curves on the graphs correspond to the numbers of the variants in the system solution (4.68) indicated in Table 4.2.

TABLE 4.2. INSTRUMENTAL ERRORS IN INS'S OF THE III CLASS.

Name of Error	Partial instrument error values	Number of Variant	INS Instrument Errors			
			V_ξ in m/sec	V_ζ in m/sec	λ_{arb} rad	ϕ_{arb} rad
Error in the initial display of system measuring instruments in the longitudinal plane	$\Delta\vartheta_0=2,9\cdot10^{-4}$ (1′)	12	—2,253 —2,224	—2,253	—2,286·10⁻⁴ (0,8′)	0,1615·10⁻⁴ (≈3″)
Error in the initial display of measuring instruments in the lateral plane	$\Delta\gamma_0=2,9\cdot10^{-4}$ (1′)	13	—0,234	2,2159	0,154·10⁻⁴ (≈3″)	—2,282·10⁻⁴ (≈0,8′)
Error in the object angular velocity gauge with respect to the longitudinal axis (IUS)	$\Delta\omega_{ax}=0,5\cdot10^{-6}$ 1/sec $\delta_\omega=0,1\%$	6	—0,216	2,7793	—0,094·10⁻⁴ (≈2′)	—1,6833·10⁻⁴ (0,6′)
Error in the introduction of initial longitudinal velocity component values	$\Delta V_{\xi 0}=1$ m/sec	9	—0,205	0,0662	1,235·10⁻⁴ (≈0,5′)	0,0871·10⁻⁴ (≈1,8″)

Tr. Note: Commas indicate decimal points.

TABLE 4.2. INSTRUMENTAL ERRORS IN INS'S OF THE III CLASS. (Con't)

Error in the object angular velocity gauge with respect to the lateral axis (IUS_z)	$\Delta\omega_{az}=0,5\cdot10^{-6}$ 1/sec $\delta_\omega=0,1\%$	8	—1,892	—0,154	$-1,193\cdot10^{-4}$ (≈0,6′)	$0,682\cdot10^{-4}$ (≈1,2″)
Error in the introduction of initial lateral flight speed component values	$V_{\zeta 0}=1$ m/sec	10	—0,061	0,204	$-0,078\cdot10^{-4}$ (≈1,5″)	$-1,2342\cdot10^{-4}$ (≈0,5′)
Accelerometer error A_x	$\Delta a_x=0,5\cdot10^{-4}$ $\delta_a=0,1\%$	3	0,415	0,0558	$0,881\cdot10^{-4}$ (≈0,3′)	$-0,0461\cdot10^{-4}$ (≈1″)
Accelerometer error A_z	$\Delta a_z=0,5\cdot10^{-4}$ $\delta_a=0,1\%$	5	—0,017	0,1859	$-0,013\cdot10^{-4}$ (0,2″)	$0,2934\cdot10^{-4}$ (0,1′)
Computer error in v_ξ	$\delta_{co}=0.1\%$	14	0,174	0,013	$0,551\cdot10^{-4}$ (≈10″)	$-0,0352\cdot10^{-4}$ (0,8″)
Computer error in a_ξ	$\delta_{co}=0.1\%$	15	0,184	0,0313	$0,552\cdot10^{-4}$ (10″)	$-0,0363\cdot10^{-4}$ (0,3″)
Computer error in $\omega_{a\zeta}$	$\delta_{co}=0.1\%$	25	—0,453	—0,0315	$-0,270\cdot10^{-4}$ (≈0,1′)	$-0,0139\cdot10^{-4}$ (≈0,2″)
Error in the initial display of the platform in azimuth	$\Delta\psi_0=2,9\cdot10^{-4}$ $\delta_{co}=(1)$	11	0,038	—0,1826	$0,023\cdot10^{-4}$ (≈0,5″)	$0,2375\cdot10^{-4}$ (≈0,1′)
Computer error in γ_a	$\delta_{co}=0.1\%$	18	0,161	0,0124	$0,074\cdot10^{-4}$ (≈1,5″)	$-0,0049\cdot10^{-4}$ (≈0,1″)

Tr. Note: Commas indicate decimal points.

TABLE 4.2. INSTRUMENTAL ERRORS IN INS'S OF THE III CLASS. (Con't)

Name of Error	Partial instrument error values	Number of Variant	INS Instrument Errors			
			V_ξ in m/sec	V_ζ in m/sec	λ_{arb} rad	ϕ_{arb} rad
Computer error in γ_a	$\delta_{co} = 0.1\%$	19	—0,012	0,1607	$0,00016 \cdot 10^{-4}$ (0,003″)	$-0,0736 \cdot 10^{-4}$ (1,5″)
Accelerometer error f_y	$\Delta a_y = 0,5 \cdot 10^{-4}\ g$	4	0,155	0,182	$0,071 \cdot 10^{-4}$ (≈1,5″)	$-0,0837 \cdot 10^{-4}$ (≈1,6″)
Computer error in ω_{ax}	$\delta_{co} = 0.1\%$	20	0,013	0,1641	$-0,005 \cdot 10^{-4}$ (≈0,1″)	$-0,0757 \cdot 10^{-4}$ (≈0,15″)
Computer error in ω_{az}	$\delta_B = 0,1\%$	22	0,157	0,0119	$0,071 \cdot 10^{-4}$ (≈0,15″)	$-0,0045 \cdot 10^{-4}$ (≈0,1″)
Error in ω_{ay} Ang. vel. gauge	$\Delta\omega_{ay} = 0,5 \cdot 10^{-6}$ 1/sec, $\delta_{co} = 0.1\%$	7	—0,010	—0,1178	$0,002 \cdot 10^{-4}$ (≈0,04″)	$0,0678 \cdot 10^{-4}$ (≈1,2″)
Error in the integrator of longitudinal acceleration $\dot{V}_\xi$	$\delta_{co} = 0.1\%$	26	—0,018	0,0031	$0,055 \cdot 10^{-4}$ (≈1″)	$-0,0036 \cdot 10^{-4}$ (≈0,07″)
Error in the integrator of longitudinal velocity V_ξ	$\delta_{co} = 0.1\%$	30	—0,032	—0,0014	$0,055 \cdot 10^{-4}$ (≈1″)	$0,0006 \cdot 10^{-4}$ (≈0,01″)

Tr. Note: Commas indicate decimal points.

TABLE 4.2. INSTRUMENTAL ERRORS IN INS'S OF THE III CLASS. (Con't).

Error in $\dot{V}_\xi, \dot{V}_\eta, \dot{V}_\zeta$ computers	δ_{co} = 0.1%	16	0,003	—0,0449	0,002·10−4 (≈0,04″)	0,0421·10−4 (≈1″)
Error in $\omega_{э\xi}$ computer	δ_{co} = 0.1%	25	0,008	—0,0210	0,004·10−4 (0,08″)	0,0756·10−4 (1,5″)
Error in $\omega_{э\eta}$ computer	δ_{co} = 0.1%	24	—0,004	0,0613	—0,002·10−2· (≈0,04″)	—0,0499·10−4 (≈1″)
Error in the ϑ_a integrator	δ_{co} = 0.01%	29	0,001	0,0161	0	—0,0074·10−4 (0,15″)
Error in the $\dot{\vartheta}_a$ integrator	δ_{co} = 0.01%	28	0,016	0,0012	0,007·10−4 (0,07″)	0;0005·10−4 (0,01″)
Error in the ω_{ax} computer	δ_{co} = 0.1%	21	0,004	0,0028	0,001·10−4	0,0066·10−4 (0,1″)
Error in the ψ_a computer	δ_{co} = 0.1%	17	0	0,0058	0	0,0048·10−4 (≈0,1″)
Error in the $\dot{\psi}_a$ integrator	δ_{co} = 0.01%	28	0	0,0002	0	0,0006·10−4 (0,01″)

Tr. Note: Commas indicate decimal points.

It is apparent from Table 4.2 and Figures 4.15-4.18 that INS errors may be divided into the following three groups.

I Group - INS errors caused by errors in initial display of measuring instruments in the longitudinal and lateral planes (of accelerometers and object angular velocity measurement gauges with respect to x and z axes) and in the introduction of initial values of the longitudinal velocity component. The values of these errors in the determination of the ϕ_{arb} and λ_{arb} coordinates amount to approximately one angular minute, and in the determination of the V_ξ and V_ζ horizontal velocity components, to meters per second. /222

II Group - INS errors caused by computer errors in ω_ζ, $\dot{V}_\xi$, a_ξ parameters, in the introduction of initial lateral flight speed component values and in the initial display of the frame in azimuth. The values of these errors in the determination of the ϕ_{arb} and λ_{arb} coordinates are tenths of an angular minute, and in the determination of velocity components, tenths of meters per second.

III Group - INS errors caused by errors in accelerometer A_y, in the object angular velocity gauge with respect to the oy axis, in the integrators for longitudinal accelerations $\dot{V}_\xi$ and velocity V_ξ and in the $\dot{\vartheta}_a$, $\dot{\gamma}_a$ parameter computers. /223 The values of these errors in the determination of the λ_{arb} and ϕ_{arb} coordinates lie within the limits of hundredths of an angular minute, and in the determination of V_ξ and V_ζ velocity components, within the limits of tenths and hundredths of meters per second.

For an evaluation of the influence of angular object motion on INS errors in the determination of $\Delta\lambda_{arb}$ and $\Delta\phi_{arb}$ coordinates, of yaw angle $\Delta\psi$, pitch angle $\Delta\vartheta$ and bank angle $\Delta\gamma$ of the object, and also in the determination of ΔV_ξ and ΔV_ζ velocity components, a large-scale graph was plotted of the dependencies of the parameters indicated on time at the initial t = 0-25 sec and the final t = 1,060-1,097 sec segments of the flight trajectory (Figure 4.19). /224 It is apparent that object oscillations in the yaw, bank and pitch angles do not exert a substantial influence on the accuracy of the determination of the coordinates and the velocity components (since the errors $\Delta V_{\xi\ in}(t)$, $\Delta V_{\eta\ in}(t)$, $\psi(t)$, $\vartheta(t)$, $\gamma(t)$ have no periodic components caused by a change in $\psi(t)$, $\vartheta(t)$, $\gamma(t)$), which makes it possible to include the INS of the given class in a system of automatic objects flight control.

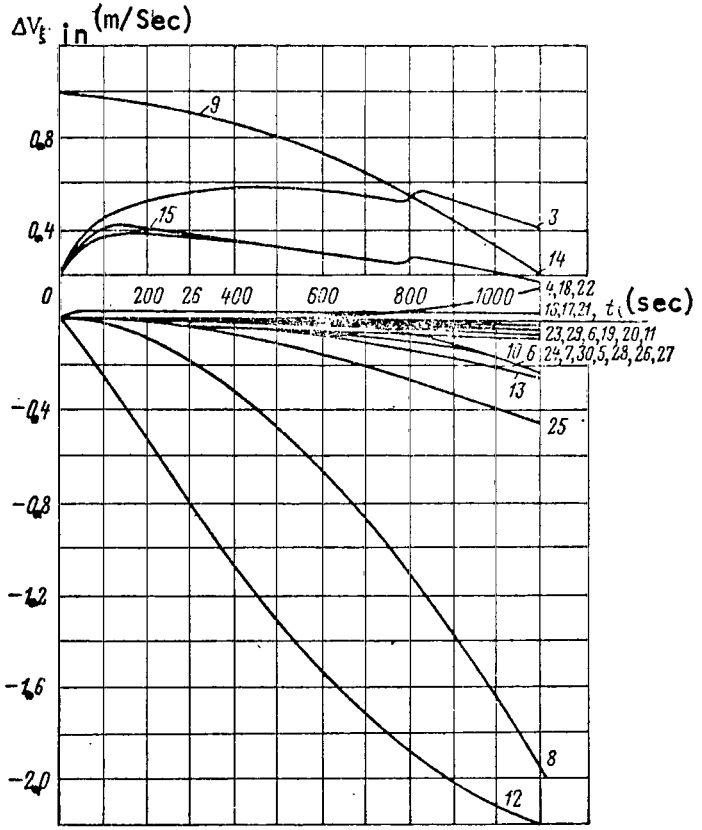

Figure 4.15. Graph of $\Delta V_{\xi\ in}(t)$ Function.

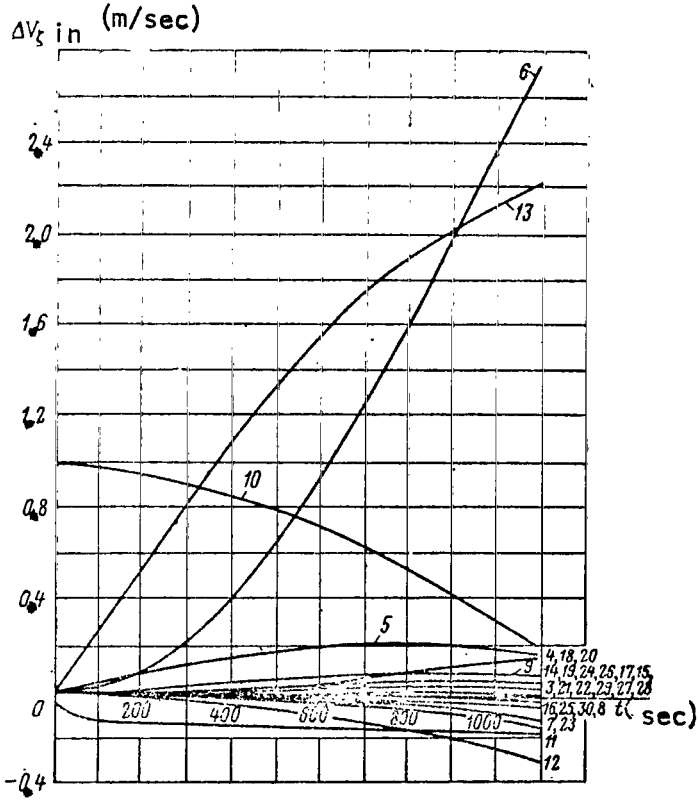

Figure 4.16. Graph of $\Delta V_{\zeta\ in}(t)$ Function.

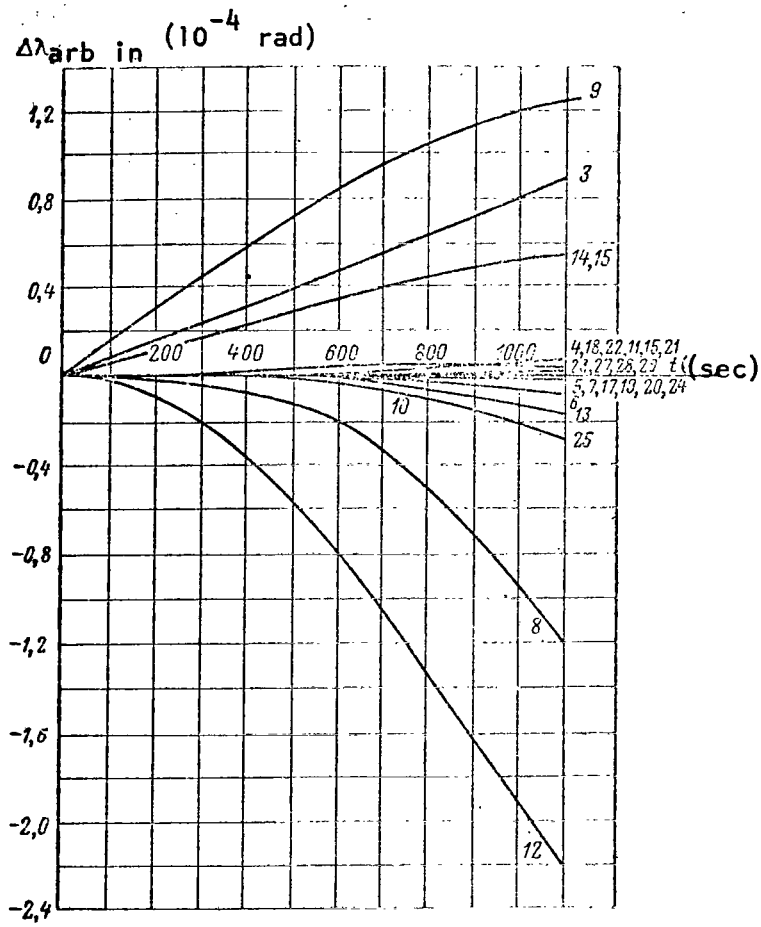

Figure 4.17. Graph of $\Delta\lambda_{arb\ in}(t)$ Function.

Tr. Note: Commas indicate decimal points.

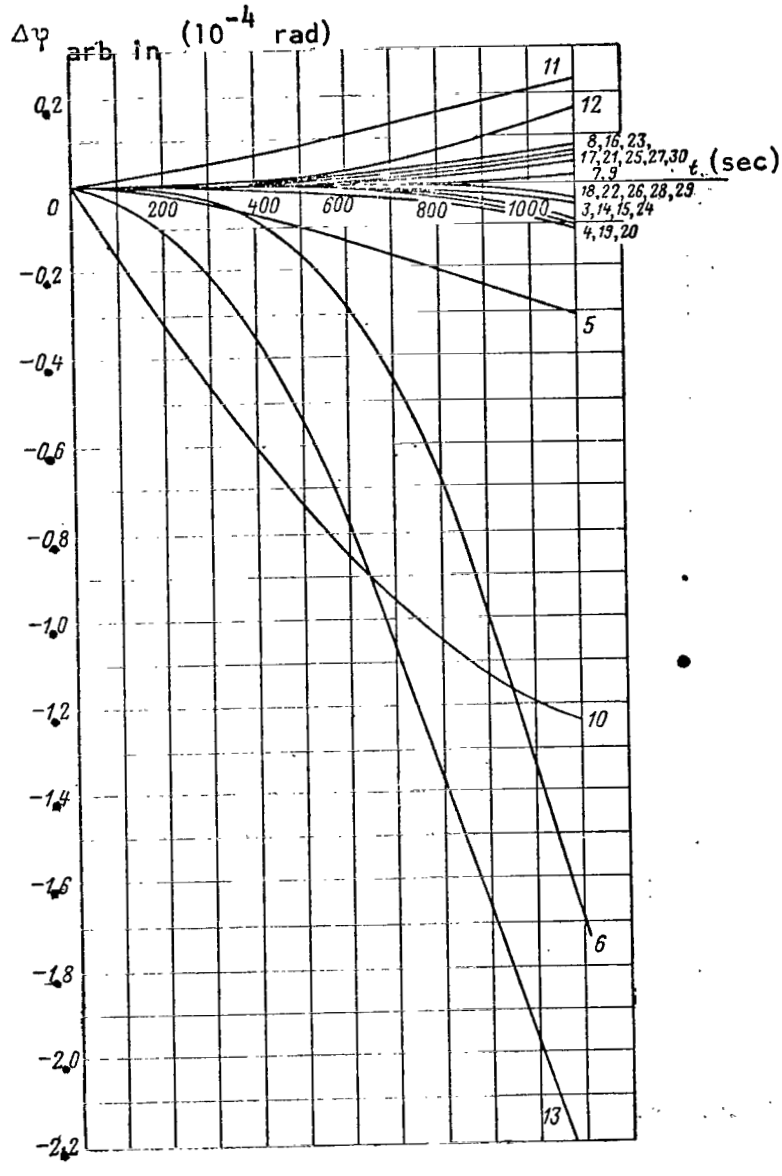

Figure 4.18. Graph of $\Delta\phi_{arb\ in}(t)$ Function.

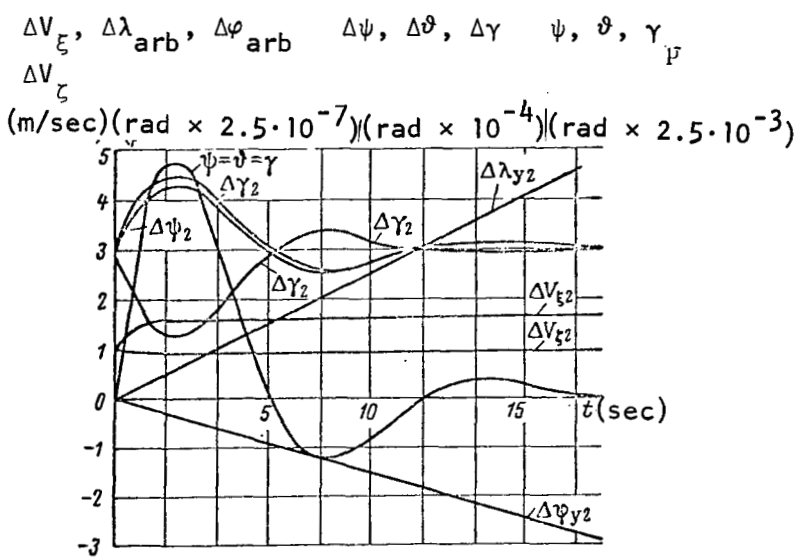

Figure 4.19. INS Errors on the Initial and Final Segments of the Trajectory.

CHAPTER V

A COMPARATIVE ANALYSIS OF THE KINEMATICS OF INERTIAL NAVIGATION SYSTEMS

§5.1. A Comparative Analysis of INS Kinematic Equations.

The systems of kinematic equations for INS's of the I (2.50), II and III (3.34), (4.29) classes, derived in Section 2.3, 3.2, 4.2, differ from each other mainly by supplemental equations required for the conversion of vector components of linear acceleration, measured by the accelerometers, from the accelerometer trihedron to the earth's trihedron (to the NHE geographic or the $\xi\eta\zeta$ arbitrary trihedrons). /226

As has been shown previously, the xyz accelerometer trihedron, depending on INS class, may be stabilized either in the earth coordinate system (INS of the I class), in inertial space (INS of the II class), or along the axes of an object in motion (INS III class), the kinematic parameters of which are subject to measurement.

Correspondingly, the kinematic equation system for an INS of the II class (3.34) differs from the kinematic equation system for an INS of the I class (2.50), in that the system has supplementary equations relative to the acceleration projections on the a_ξ, a_η, a_ζ axes of the arbitrary trihedron. These equations are required for the conversion of the indications a_x, a_y, a_z of the accelerometers, the measurement axes of which in an INS of the II class are fixed in inertial space, to the $\xi\eta\zeta$ arbitrary trihedron, which is bound with the earth coordinate system. In addition, in order to convert the accelerometer indications in the INS of the II class, it is necessary to have equations for the angles of rotation ψ_a, ϑ_a, γ_a of the axes of the arbitrary trihedron with respect to the inertial.

Thus in an INS of the II class, the equations pertaining to a_ξ, a_η, a_ζ and ψ_a, ϑ_a, γ_a mathematically model the kinematics of $\xi\eta\zeta$ arbitrary trihedron motion in inertial space, whereas in an INS of the I class, the position of this trihedron in inertial space is modeled physically with the aid of an appropriate platform which is inclined in the plane of the horizon and in azimuth in proportion to object motion.

In an INS of the II class, only the $\xi_1\eta_1\zeta_1$ inertial trihedron is modeled physically (by means of gyroscopes). /227

In turn, in an INS of the III class, in contrast to an INS of the II class, not only is the position of the $\xi\eta\zeta$ arbitrary trihedron mathematically modeled in inertial space, but the position of the $\xi_1\eta_1\zeta_1$ inertial trihedron is modeled in the same way.

It should be noted that the angular position of the arbitrary trihedron in inertial space is determined by angular motions of the accelerometer trihedron with subsequent conversion of these motions.

There motions are determined by the indications of angular acceleration gauges, angular velocity gauges or by direct measurement. Therefore, as was pointed out in Section 4.2, an INS of the III class may be constructed in accordance with all three design variants which differ from each other both by their equations and by their measurement instruments.

But regardless of the design variant, in an INS of the III class, the kinematics of the arbitrary and inertial trihedrons are modeled mathematically.

Correspondingly, INS systems of the III class contain equations required for the conversion of indications a_x, a_y, and a_z of the accelerometers tightly bound with the object to the a_ξ, a_η, a_ζ axes of the arbitrary trihedron and equations for the determination of deviation angles ψ_a, ϑ_a, γ_a of the accelerometer trihedron with respect to the arbitrary trihedron (in INS's of the first and second design variants), or angles ψ', ϑ', γ', ψ'', ϑ'', γ'', which characterize the positions of the accelerometer trihedron with respect to the inertial and the inertial with respect to the arbitrary, respectively (in an INS of the third design variant).

It is apparent that the accuracy of the mathematical modelling of the position of the $\xi_1\eta_1\zeta_1$ inertial trihedron will be determined not only by computer, but also by the corresponding object angular movement gauges (accelerometers, angle and angular velocity gauges). Therefore the gain in accuracy in an INS of the III class may be reduced as a result of the somewhat poorer operating conditions of the system measuring instruments (which are installed directly on the object) in comparison with the operating conditions of these instruments in an INS of the II class (where the installation is on a stabilized platform which decouples the instruments from angular object motion).

If we consider that for a majority of automatically controlled objects, the ranges of the change in the angular motion parameters of aircraft is relatively small, and the accuracy of operation of object angular position gauges in inertial space as they are developed increases with an increase in the limits of measurements, then the shortcoming indicated for an INS of the III class will be eliminated, and these systems, which possess a number of significant advantages, will occupy an important place in the complex of automatic control equipment for objects in motion.

§5.2. A Comparative Analysis of INS Systematic Errors.

Systematic errors in INS's of the I, II and III classes are determined by /228
the simplifications of their kinematic equations.

The difference between the systematic errors of INS's of the II and III classes and errors of the I class are caused by supplementary simplifications of

the equations associated with the mathematical modeling of the kinematics of arbitrary trihedron motion and the conversion of accelerometer errors to the trihedron axes.

Systematic errors in INS's of the II class, in contrast to errors in INS's of the I class, are also caused by simplifications in the equations pertaining to a_ξ, a_η, a_ζ and $\dot{\psi}_a$, $\dot{\vartheta}_a$, $\dot{\gamma}_a$.

These systematic errors are similar to errors in platform inclination along the axes of the arbitrary trihedron in INS's of the I class.

Whenever required the equations for the mathematical modeling of the kinematics of arbitrary trihedron motion may remain unsimplified, and then the systematic errors of INS's of the II class indicated will not occur, whereas as in INS's of the I class platform inclination errors will always occur.

In turn, systematic errors of INS's of the III class, in contrast to system errors in the II and I classes, are caused by supplementary equation simplifications pertaining to the modeling of the position of the inertial and arbitrary trihedrons and also by simplifications in the equations for converting accelerometer indications to the inertial trihedron axes. These errors are also similar to inclination errors in the Cardan joint of the platform of INS's of the I class.

If the equations for the mathematical modeling of the kinematics of the arbitrary and inertial trihedrons are not simplified, then naturally, systematic errors in INS's of the III class will be lacking.

An analysis of systematic errors in INS's of the I, II and III classes, made for the same object, reveals (Figure 5.1), that the time variation of these errors in identical and their values at each given moment differ somewhat from each other.

§5.3. A Comparative Analysis of INS Instrument Errors.

Instrument errors in INS's of all three classes are caused by INS display errors at the starting point and by errors in the elements which make up the systems, including accelerometers, gyroscopes, computers, etc.

In INS's of the I, II and III classes, the nature of the time relationship of instruments errors caused by accelerometer errors is practically identical.

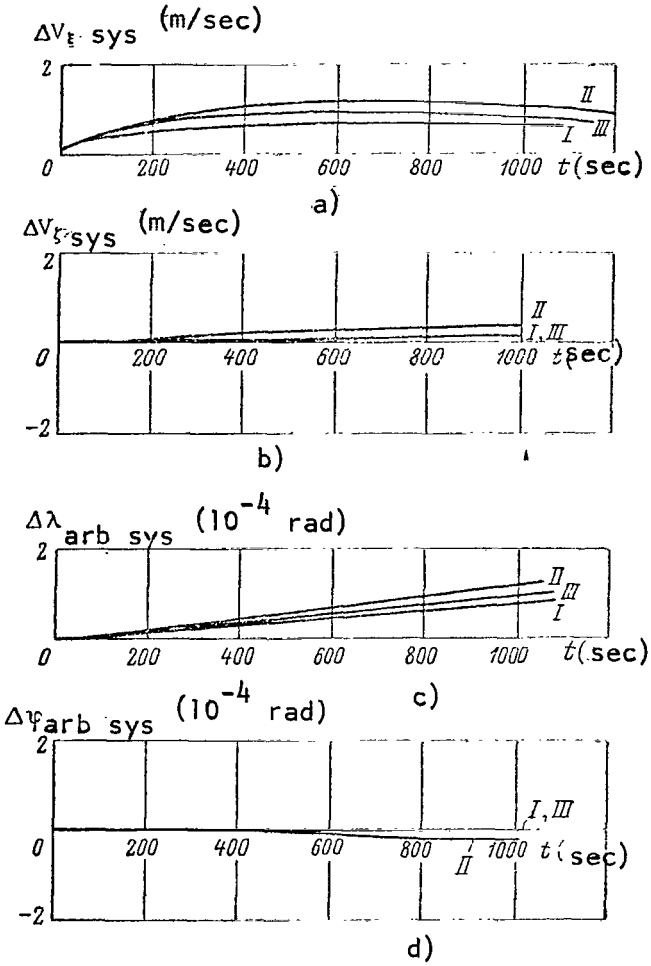

Figure 5.1. Graphs of the Time Relationships of Systematic Errors in INSs of the I, II and III Classes.

Error time variations $\Delta V_{\xi\ in}(t)$, $\Delta\lambda_{arb\ in}(t)$ in INS's of the I, II and III classes as a result of errors in longitudinal accelerometer A_x /230

$$\Delta a_0 = 5\cdot 10^{-4}\ m/sec^2,\ \delta_a = 0{,}1\%$$

(for the same object) are shown in Figure 5.1.

Figure 5.2 shows errors in INS's of the I, II and III classes, caused by error in lateral accelerometer A_z, $\Delta a_0 = 5\cdot 10^{-4}$ m/sec^2, $\delta_a = 0.1\%$.

It is apparent from Figures 5.1 and 5.2 that minimum values of the errors under consideration will occur in an INS of the I class, and the maximum values will occur in an INS of the II class.

The time variations in instrument errors as a result of errors in the longitudinal and lateral acceleration integrators $\delta_{\dot V} = 0.01\%$ and velocity integrators $\delta_V = 0.01\%$ are identical for all INS classes.

When platform drift $\Delta\dot\vartheta_{a0} = 5\cdot 10^{-7}$ 1/sec is present relative to the plane of the horizon in INS's of the I and II classes, or if zero drift of the angular velocity gyroscope gauges is present (for INS's of the III class), errors will occur; graphs of these errors are shown in Figures 5.3 a, b, c, d.

It is apparent from the graphs that the INS errors under consideration are of approximately the same order of magnitude for all classes.

In the INS classes under consideration, errors caused by initial display errors also have similar values.

With errors present in platform inclination in azimuth and in the plane of the horizon (for INS's of the I class) or errors in the calculation of angles ψ_a, ϑ_a, γ_a (for INS's of the II and III classes), which characterize the position

of the accelerometer trihedron with respect to the arbitrary trihedron, the errors in these INS's in the determination of the true parameters will also change identically with time.

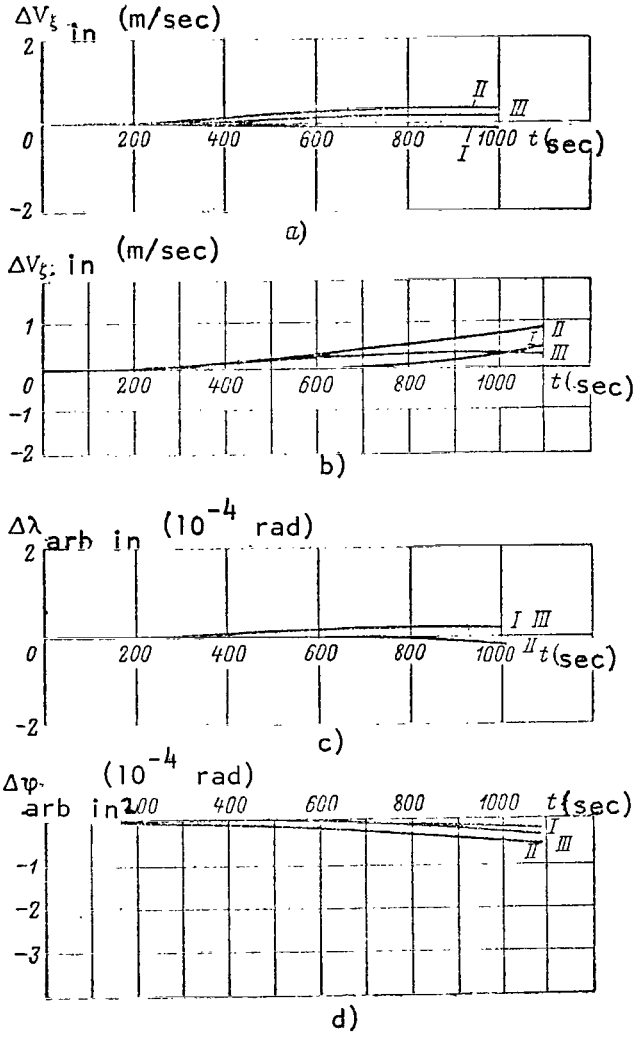

Figure 5.2. Time Variations of INS Instrument Error Caused by Accelerometer Errors.

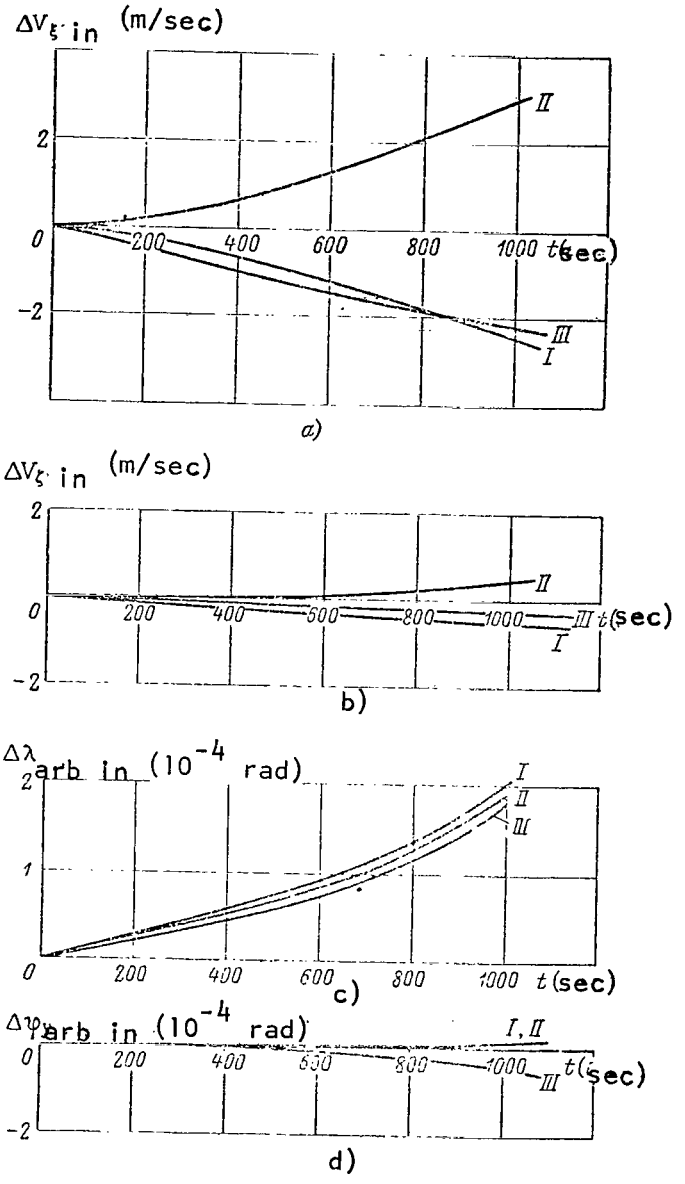

Figure 5.3. Graphs of the Time Variations of INS Errors Caused by Errors in Gyroscope Elements.

It should be noted that with the selection of a computer of appropriate accuracy, errors in the calculation of angles ψ_a, ϑ_a, γ_a, which determine the position of the arbitrary trihedron (in INS's of the II and III classes) in principle may be reduced to any established values, whereas platform inclination accuracy in INS's of the I class is limited by technological production capabilities.

In addition, control actions for platform inclination corresponding to angular trihedron rotation are applied to the gyroscope elements of INS's of the I class. This increases somewhat the drift of performance in inertial space in comparison with elements of INS's of the II class, which we shall not deal with here.

Errors in INS's of the II class may be reduced to a significant degree (especially with a comparatively long system operational time period) as a result of the elimination of platform drift in inertial space by means of linking the platform with the stars by means of appropriate devices.

§5.4. A Comparative Evaluation of Several INS Operational Characteristics Based on the Kinematics of these systems.

The operating characteristics of INS's of the I, II and III classes are determined first of all by the starting process and by system display at the starting point of the object. /233

INS display provides for the orientation of the measurement axes of the instruments along the axes of the appropriate coordinate trihedron (depending on the coordinate system chosen for navigation) and for the introduction into the system of initial kinematic parameter values.

In INS's of the I and II classes, the measurement axes of the instruments are oriented by the display of their platforms along the axes ξ_0, η_0, ζ_0 of the arbitrary trihedron at the starting point, whereas in INS's of the III class the orientation of the measurement axes are determined by their angles established with respect to object axes and by object angular position with respect to the surface of the earth. These angles may be automatically determined and introduced into the system during the object starting operation.

In INS's of the I, II and III classes initial kinematic parameters values are introduced identically at the start (into the computer system).

The time required for preparing the INS for operation is determined by the starting and display time of the measurement elements of the system, of the platform, and also by the preparation time required for the system computer. In this case the INS of the III class is in the most advantagous position.

The weight and size characteristics of INS's are also important operational characteristics.

It is apparent that the weight and dimensions of INS's of the III class in comparison with INS's of the I and II classes may be reduced by a value approximately equal to the weight and dimensions of the gyroscope platforms.

With the utilization of prospective small-size elements of computer technology, the weight of INS's of the III class may be reduced to kilogram units, and the dimensions to cubic decimeter units.

The operational reliability of INS's of the III class in comparison with INS's of the I and II classes may be increased by means of increasing the operational reliability of the computer.

The manufacturer of INS's of the III class in comparison with INS's of the I and II classes may be simplified as a result of the fact that in INS's of the III class the precision gyroscope platform is not present, and the remaining elements may be standardized.

INS's of the III class will find ever-increasing application as the accuracy and reliability of measurement instruments and computer devices are improved.

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Translated for the National Aeronautics and Space Administration under Contract No. NASw-1695 by Techtran Corp., P. O. Box 729, Glen Burnie, Md. 21061

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